The SMU honor code applies.

This homework is due at the beginning of class on Tuesday 10 April.

1. (From Exercise 3.2.39) You are to calculate the amount of work (number of flops) in the following set of matrix operations. Throughout, $B$ is an $n \times m$ matrix, and $u$ and $v$ are vectors with $n$ components.
   
   (a) Show that if we compute $(uv^T)B$ then the intermediate result is an $n \times n$ matrix and the total computation requires about $(2m + 1)n^2$ flops. [Hint: the intermediate result is the quantity in parentheses.]
   
   (b) Show that if we compute $u(v^TB)$ then the intermediate result is a $1 \times m$ matrix and the total computation requires about $3nm$ flops.
   
   (c) Show that when $Q = I - uv^T$ the computation of the matrix $QB = B - uv^TB$ requires about $4nm$ flops if executed efficiently.
   
   (d) How many flops are needed to compute $QB$ if the matrix $Q$ (as defined in part (c)) is computed (assembled) and stored as an $n \times n$ matrix then multiplied into the $n \times m$ matrix $B$?

2. (From Exercise 3.2.41)

   (a) Find a reflector $Q$ that maps the vector $x = (3, 4, 1, 3, 1)^T$ to a vector $(-\tau, 0, 0, 0, 0)$. Write $Q$ in two ways: (i) in the form $Q = I - \gamma uu^T$ where $\gamma = \frac{2}{||u||^2}$; and (ii) as a completely assembled matrix.

   (b) Let $z = (0, 2, 1, -1, 0)^T$. Calculate $Qz$ in two ways: (i) the efficient way using $Q = I - \gamma uu^T$; and (ii) using the assembled matrix $Q$.

3. (Exercise 3.2.4)

   Use the QR decomposition to solve the linear system
   
   $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \underline{x} = \begin{pmatrix} 12 \\ 29 \end{pmatrix}$

   [Hint: Use a rotation matrix to compute the QR decomposition.]