Linear Programming Exercises

September 19, 2000

1. Given the following linear programming problem.

Maximize
\[ x_1 + \frac{1}{2}x_2 \]

subject to
\[
\begin{align*}
3x_1 + 2x_2 & \leq 12 \\
5x_1 & \leq 10 \\
x_1, x_2 & \geq 0
\end{align*}
\]

(a) Solve the problem using the simplex method and give its optimal solution.
(b) (skip this) What is the inverse of the basis matrix in the optimal tableau?
(c) Check three of the \(c_j - z_j\) values in the optimal tableau using the fact that \(c_j \hat{A}_j = z_j\).
(d) Show, using partial vector notation, how column \(A_4\) can be reconstructed by its representation in the optimal basis.

2. Solve the following linear programming problem using the simplex method and give its optimal solution.

Maximize
\[ 2x_1 + 3x_2 \]

subject to
\[
\begin{align*}
5x_1 - x_2 & \geq -1 \\
x_1 + 2x_2 & \leq 5 \\
2x_1 + x_2 & \leq 4 \\
x_1, x_2 & \geq 0
\end{align*}
\]
3. Given the following problem

Maximize

\[ x_1 + x_2 \]

subject to

\[ x_1 + 2x_2 + x_3 = 6 \]
\[ 2x_1 + x_2 + x_4 = 4 \]
\[ 3x_1 - x_2 + x_5 = 1 \]
\[ x_1, x_2, x_3, x_4, x_5 \geq 0 \]

(a) Complete the following tableau.

<table>
<thead>
<tr>
<th>( c_B )</th>
<th>Basis</th>
<th>( x_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e )</td>
<td></td>
<td>( 0 )</td>
<td>( 1/5 )</td>
<td>( 1/5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e )</td>
<td></td>
<td>( 0 )</td>
<td>( 3/5 )</td>
<td>( -2/5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( e )</td>
<td></td>
<td>( 1 )</td>
<td>( -7/5 )</td>
<td>( 3/5 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c_j - \bar{z}_j )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Is the solution optimal?
(c) What is the representation of \( A_4 \) with respect to the current basis?

4. Solve the following problem using the Big M method.

Maximize

\[ 2x_1 + 5x_2 + 3x_3 \]

subject to

\[ x_1 + 2x_2 + x_3 \leq 5 \]
\[ 2x_1 - x_2 = 4 \]
\[ x_1, x_2, x_3 \geq 0 \]

5. Solve the following problem using the Phase I-II method.

Maximize

\[ x_1 + \frac{1}{4}x_2 + 5x_3 \]

subject to

\[ 3x_1 + 2x_2 + x_3 = 6 \]
\[ 2x_1 + x_2 + 5x_3 = 4 \]
\[ x_1, x_2, x_3 \geq 0 \]