Network Programming Exam 2
CSE 8374 (NTU# QN 721M)

November 26, 2000
Updated December 4, 2000

Exam Instructions

- This “take-home” exam is open book and notes.
- The submitted exam must be your original work, achieved with no help from others. You are not to discuss your work with any person other than your instructor. Include a statement to this effect with your exam.
- The exam should be turned in to the instructor by December 12, 2000. Any faxes should be sent to (214) 768-3085.
- Updates made to the original exam are shown in boldface.

Assignment:

1. Consider the uncapacitated network flow problem in Figure 1. Supply and demand values are shown next to the nodes, and unit costs next to each arc. Consider also the following basis:

   $$\mathbf{x}_B = \{x_{13}, x_{42}, x_{25}, x_{64}, x_{36}, x_{67}\}$$

   All other arcs are non-basic at zero.

   (a) For this basis, show the solution as a graph, rooted at node 6, including all basic flows.

   (b) Write out the dual problem for the same network (in Figure 1).

   (c) Show a dual solution. Is it (dual) feasible? Is it unique? In each case, explain why or why not.

   (d) For the basis graph in part 1a, indicate in a table, such as Table 1, the values for the corresponding node labels.$^1$

   (e) Show the basis matrix, $\mathbf{B}$, in upper- or lower-triangular form. Label the rows and columns of the matrix appropriately. (Hint: drop the root-node constraint.)

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$^1$Flow(i) is the flow on arc $(i,p(i))$ or arc $(p(i),i)$. 

1
Figure 1: Uncapacitated network with supplies, demands, and unit costs

<table>
<thead>
<tr>
<th>Node</th>
<th>Predecessor</th>
<th>Flow</th>
<th>Dual</th>
<th>Thread</th>
<th>Distance</th>
<th>Cardinality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>4</td>
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<td>6</td>
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<tr>
<td>7</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Table of node labels
Figure 2: Transshipment network basis graph

(f) Show the simplex tableau corresponding to the solution from part 1a.

(g) If \( x_{35} \) were pivoted into the solution, what would be the change in the objective-function value?

(h) **Pivot** \( x_{34} \) **into the basis** and show the new basis graph and updated table of node labels. Circle all node labels that changed from the previous solution.

(i) Characterize, in words, the individual node labels that changed in the following lists: **flow, dual, distance, and cardinality.** (For example, the predecessor labels changed for those nodes that were both in the basis equivalent path of the incoming arc and in the lower subtree formed by removing the leaving arc from the basis.)

(j) Is the new solution from part 1h optimal? Why or why not?

2. Given the network basis in Figure 2, shown with flows and true arc orientation, and the fact that arc \( x_{26} \) is non-basic at its upper bound of 5.

(a) What is the right-hand-side for this problem?

(b) Assume the following right-hand-side for the same problem: \( b = \{b_1, \ldots, b_6\} = \{-2, -67, 47, -23, 77, 2, -46, -24, 20, 33, -17\} \), where positive (negative) values indicate supply (demand). What is the corresponding set of flows on the basic variables? Is the solution primal feasible?

3. Given the generalized transportation problem in Figure 3 and the
fact that the cost of slacks $s_1$ and $s_3$ is 0 and $s_2$ is 1. Answer the following.

(a) Using the following starting basic solution, show the basis graph.

$$
\mathbf{x}_B = \begin{pmatrix}
 x_{14} \\
 x_{15} \\
 x_{24} \\
 x_{25} \\
 x_{36} \\
 x_{36}
\end{pmatrix} = \begin{pmatrix}
 0 \\
 10 \\
 6 \\
 0 \\
 4 \\
 10
\end{pmatrix}
$$

(b) Using the graph from 3a, construct a full simplex tableau for that basis, showing your work on the graph calculations of representations.

(c) Solve the problem beginning with the solution from part 3a, using network graphs and the generalized transportation algorithm described in class. Indicate all basis graphs, arc flows,
node duals, and incoming and leaving arcs. Always pivot on
the arc with the most negative $c_j - z_j$ value.

4. Twenty million barrels of oil must be transported from Dhahran in
Saudi Arabia to the ports of Rotterdam, Marseilles, and Naples in
Europe. The demands of these ports are, respectively, 4, 12, and 4
million barrels. The following three alternative routes are possible
(see Figure 4).

- From Dhahran, around Africa to Rotterdam, Marseilles, and
  Naples. The average transportation and handling cost per bar-
 rel is $1.20, $1.40, and $1.40, respectively.
- From Port Said. From Port Said the oil is shipped to Rotter-
  dam, Marseilles and Naples. The average transportation and
  handling cost from Dhahran to the city of Suez is $0.30, and
  the additional unit cost of transporting through the canal is
  $0.20. Finally, the unit transportation costs from Port Said to
  Rotterdam, Marseilles, and Naples are $0.25, $0.20, and $0.15,
  respectively.
- From Dhahran to the city of Suez, and then through the pro-
  posed pipeline system from Suez to Alexandria. The average
  transportation cost per barrel through the pipeline is $0.15, and
  the unit transportation costs from Alexandria to Rotterdam,
  Marseilles, and Naples are $0.22, $0.20, and $0.15, respectively.

Thirty percent of the oil is transported by large tankers that can-
not pass through the Suez Canal. Also the pipeline from Suez to
Alexandria has a capacity of 10 million barrels of oil.

(a) Formulate the problem as a network flow model and draw the
associated diagram, indicating clearly the meaning of the nodes,
and showing the supplies, demands, costs, upper bounds, and
lower bounds.

(b) Solve using the network simplex algorithm for capacitated trans-
shipment networks. Use a modified node-most-negative starting
solution.\footnote{Identical to the modified row-most-negative rule for transportation problems, except that
arcs are evaluated in from-node groups instead of rows, and arc capacities may influence the
assigned flows.} Indicate all basis graphs, arc flows, node duals, and
incoming and leaving arcs. Always pivot on the arc with the
most attractive $c_j - z_j$ value.

5. Formulation. (Exercise 6.13 from text.) A bus company has three
different types of buses (standard, limousine, and shuttle) that it
seeks to assign to four routes. Table 2 gives the capacity limits (in
passengers), numbers of buses available, numbers of trips that can
be made on different routes, and the daily passenger demand for
each route. Operating costs on different routes and the opportunity
cost (lost profit) for not carrying a passenger are given in Table 3.
Figure 4: Oil Transportation Routes

<table>
<thead>
<tr>
<th>Bus Type</th>
<th>Passenger Capacity</th>
<th>No. of Buses Available</th>
<th>Maximum Daily Trips Per Bus on Route</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>40</td>
<td>6</td>
<td>2 3 1 2</td>
</tr>
<tr>
<td>Limo</td>
<td>12</td>
<td>11</td>
<td>3 4 2 3</td>
</tr>
<tr>
<td>Shuttle</td>
<td>9</td>
<td>13</td>
<td>5 5 2 4</td>
</tr>
</tbody>
</table>

Demand in passengers

180 70 90 85

Table 2: Bus operating data

6
<table>
<thead>
<tr>
<th>Bus Type</th>
<th>Dollar Operating Cost Per Trip on Route</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Standard</td>
<td>220</td>
</tr>
<tr>
<td>Limo</td>
<td>180</td>
</tr>
<tr>
<td>Shuttle</td>
<td>160</td>
</tr>
</tbody>
</table>

| Opportunity cost per customer | 12 | 9  | 14 | 10 |

Table 3: Operating and opportunity costs

Provide an integer netform to determine how many buses to assign to each route to minimize total cost. Clearly label the components of the model and any assumptions made.