A Demonstration of an

Additive Time Series Decomposition

Based on the SAS program Decomposition.sas

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In traditional time series analysis it is often assumed that a time series y_t can be **additively decomposed** into four components, namely, **trend**, **season**, **cycle**, and **irregular** components as in

$$y_t = T_t + S_t + C_t + I_t \qquad (1)$$

where T_t represents the trend in y_t at time t, S_t the seasonal effect at time t, C_t the cyclical effect at time t and I_t the irregular effect at time t.

To demonstrate this decomposition, consider the following characterizations of trend, cycle, seasonal, and irregular components that have been encoded in a SAS program entitled Decomposition.sas that is available on the class website.

$T_t = 100 + 4.0 * t$ (deterministic trend: intercept = 100, slope = 4)

$C_t = 50 * \cos(3.1416 * t / 10)$ (deterministic cycle: amplitude = 50, Period = 20 months, phase = 0)

S_t = {fixed seasonal effects: -50, -25, 25, -25, -50, 50, 75, 50, 5, -25, -50, 20}

(i.e. Jan. effect = -50, Feb. effect = -25, … , Dec. effect = 20)

$I_t \rightarrow NIID(0,100)$

Therefore, the actual population model can be written as

$$\begin{aligned} y_t &= T_t + C_t + S_t + I_t \\ &= 100 + 4.0t + 50\cos(0.31416t) - 50 * gamma1 - 25 * gamma2 \\ &\quad + 25 * gamma3 - 25 * gamma4 - 50 * gamma5 + 50 * gamma6 \\ &\quad + 75 * gamma7 + 50 * gamma8 + 5 * gamma9 - 25 * gamma10 \\ &\quad - 50 * gamma11 + 20 * gamma12 + \varepsilon_t . \end{aligned}$$

The above seasonal dummies are defined $gammai$ = 1 if the observation is in the i-th month and zero otherwise. Obviously the intercept of the month varies by month. For example, the intercept for all of the January months is (100 – 50 = 50), the intercept for the February months is (100 – 25 = 75), etc. The irregular component is represented by the unobserved error ε_t which is normally and independently distributed with mean zero and variance of 100. The time index t ranges from 1 for the first observation and t = T for the last observation.

Each of these components is plotted in order in the following figures below:

Figure 1

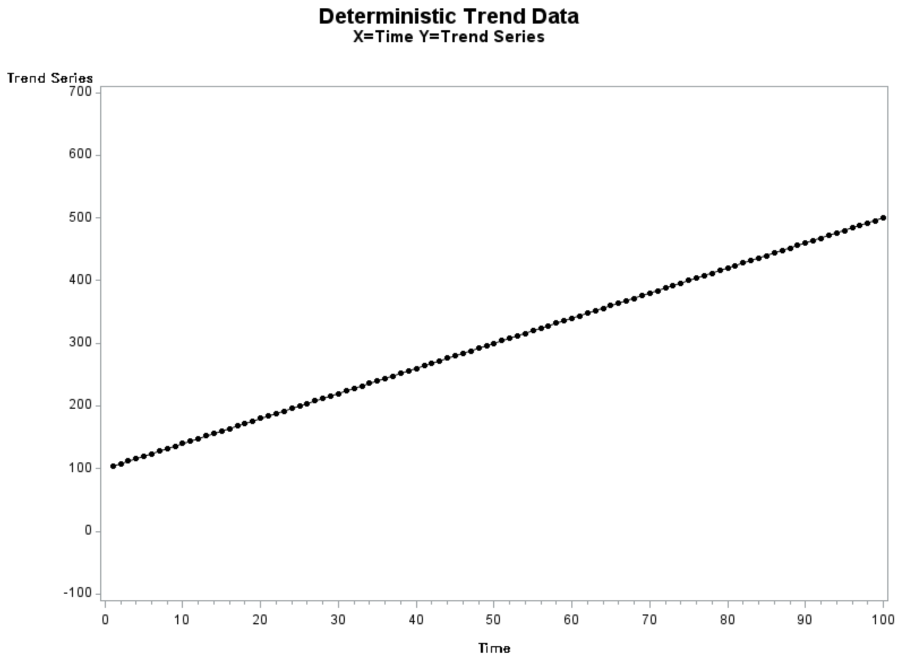


Figure 2

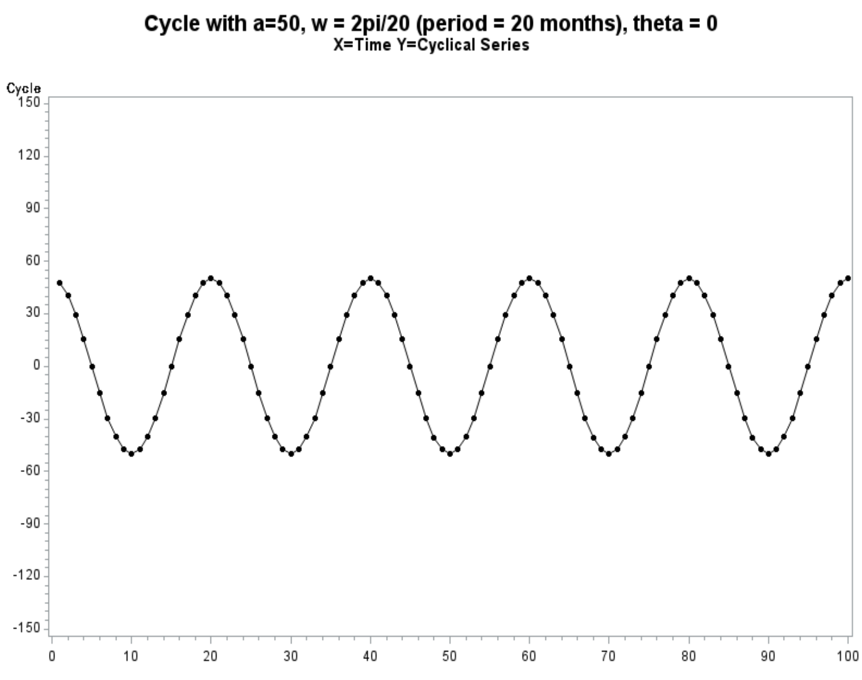


Figure 3

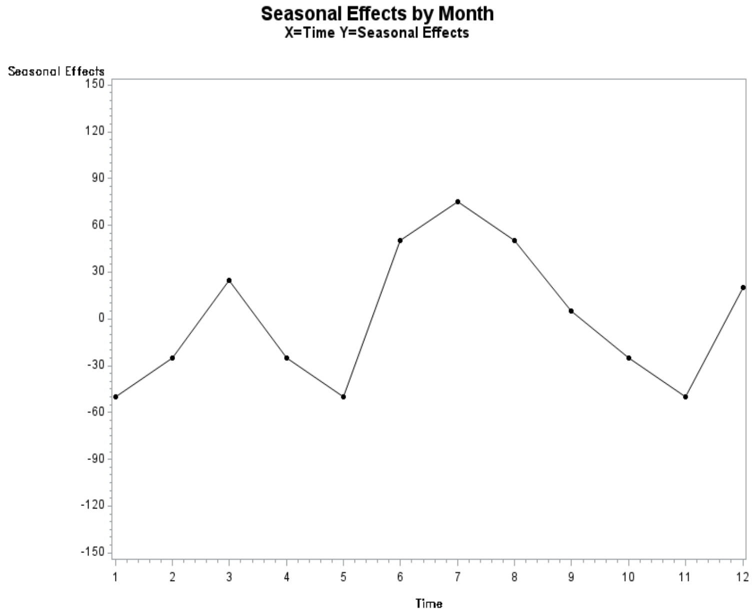
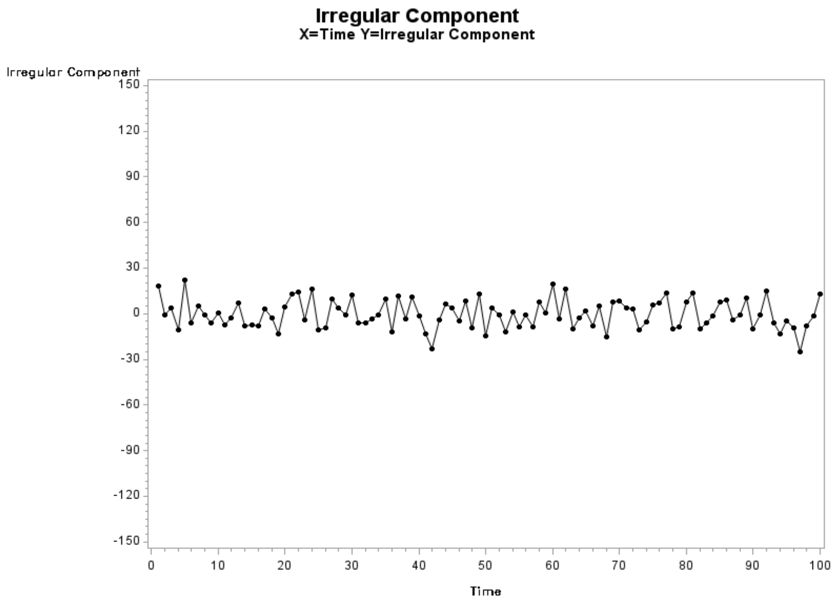


Figure 4



In the following graphs we sum these components up as is intended in the additive decomposition (1).

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Figure 5

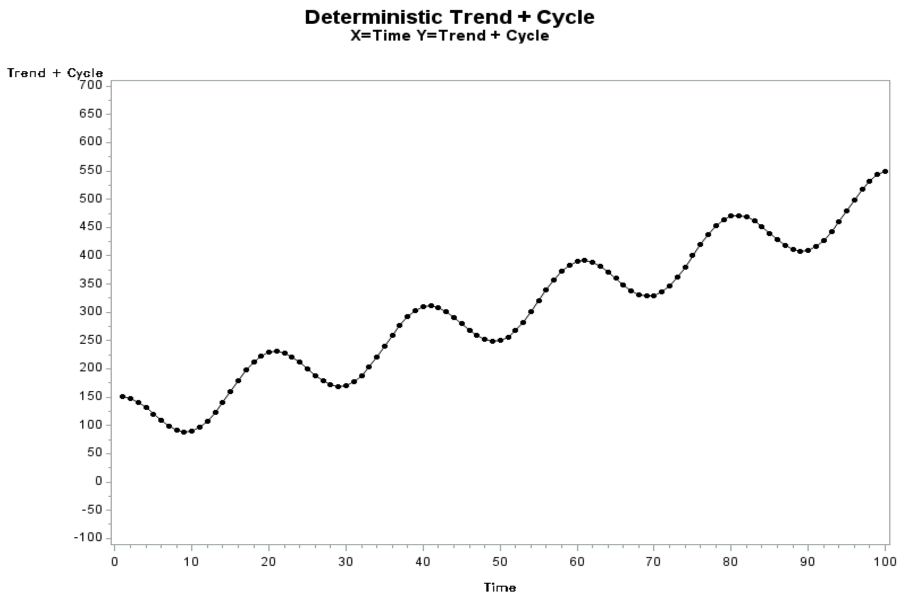


Figure 6

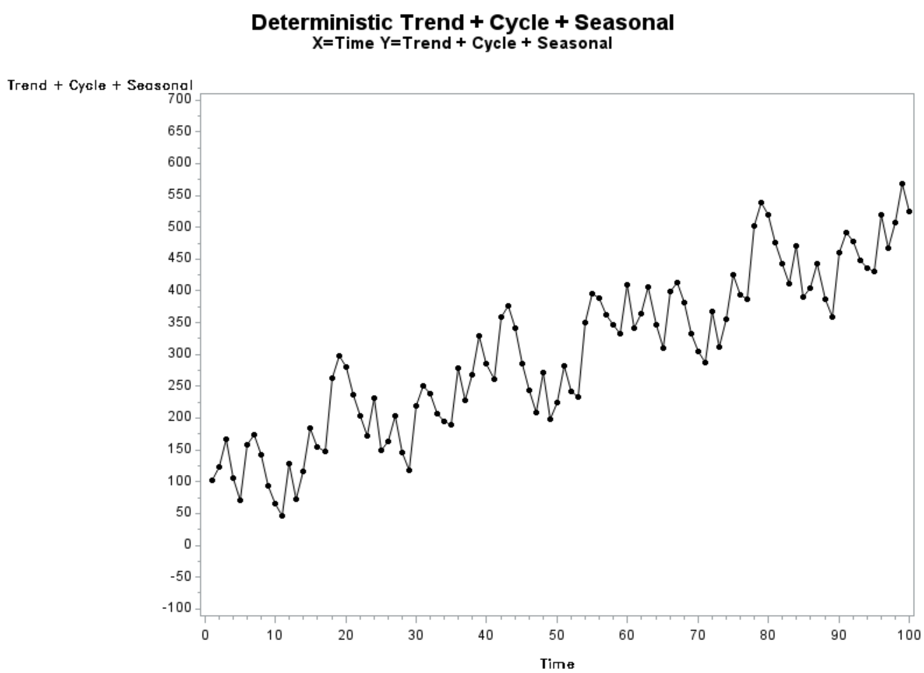
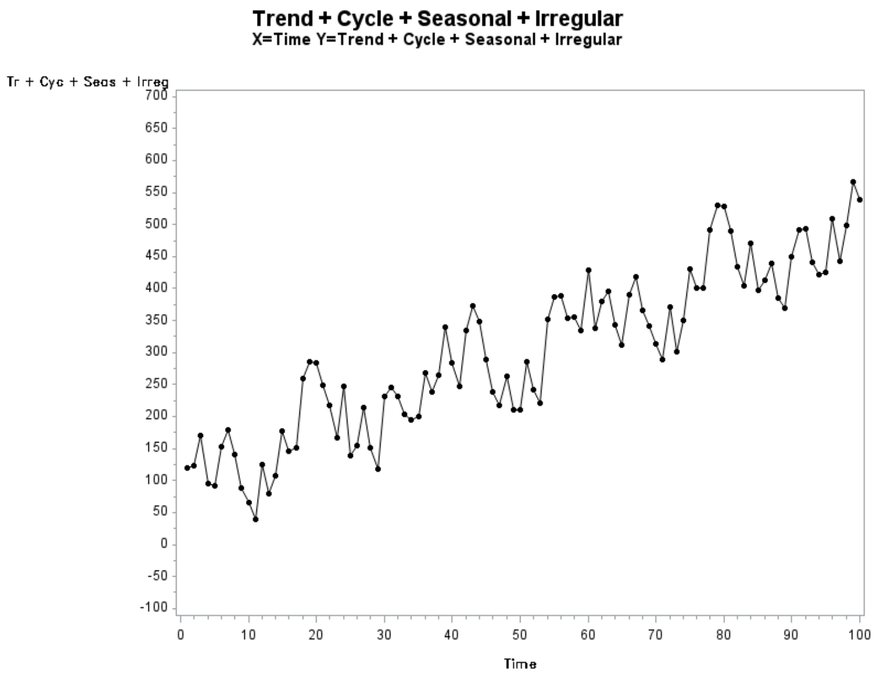
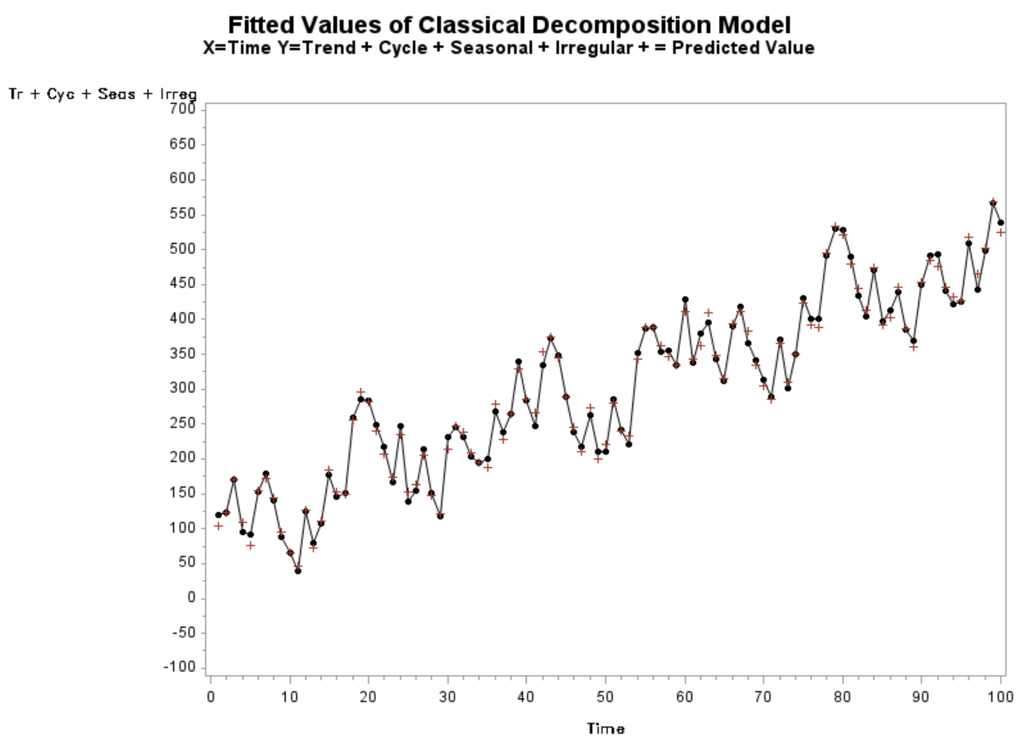


Figure 7



And in Figure 8 below we have the representation of the fitted model obtained by Proc Nlin (a Nonlinear Least Squares procedure).

Figure 8



The fitted model is

$$y_t = 51.74 + 3.98t + 50.76\cos(0.3140t - 0.0314) + 21.36 * gamma2 + 75.21 * gamma3$$
$$+ 23.99 * gamma4 + 2.21 * gamma5 + 93.26 * gamma6 + 120.3 * gamma7$$
$$+ 100.6 * gamma8 + 56.16 * gamma9 + 24.69 * gamma10 - 0.58 * gamma11$$
$$+ 69.97 * gamma12 + \hat{\varepsilon}_t$$

which closely matches the population parameters. Obviously, the January intercept is estimated as 51.74, the February intercept is 51.74 + 21.36 = 73.1, etc. The slope of the trend is 3.98 while the amplitude of the cycle is 50.76, the phase is -0.0314, and the period is $p = 2\pi / w = 2 * 3.1416 / 0.3140 = 20.01$ months.