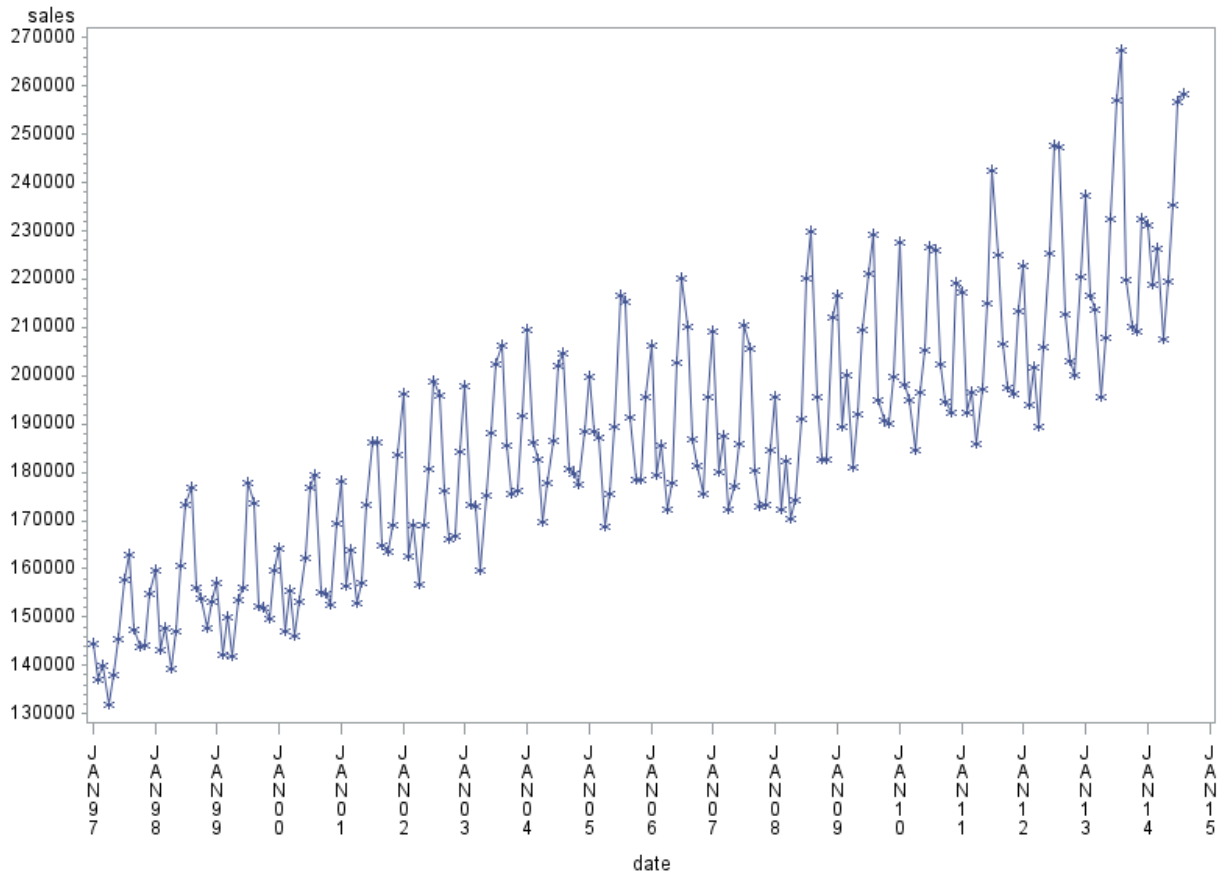


**Optimal Inventory Control:
Minimizing the Effects of Stock-outs**

Here is a plot of Fomby Inc's monthly sales of widgets from January 1997 to August 2014. As it turns out this series has a trend in it and seasonality as well. The trend in the data is pretty obvious but the seasonality is not so obvious. (This graph and following graphs and computer output are produced by the SAS program sales_forecast_1.sas.)

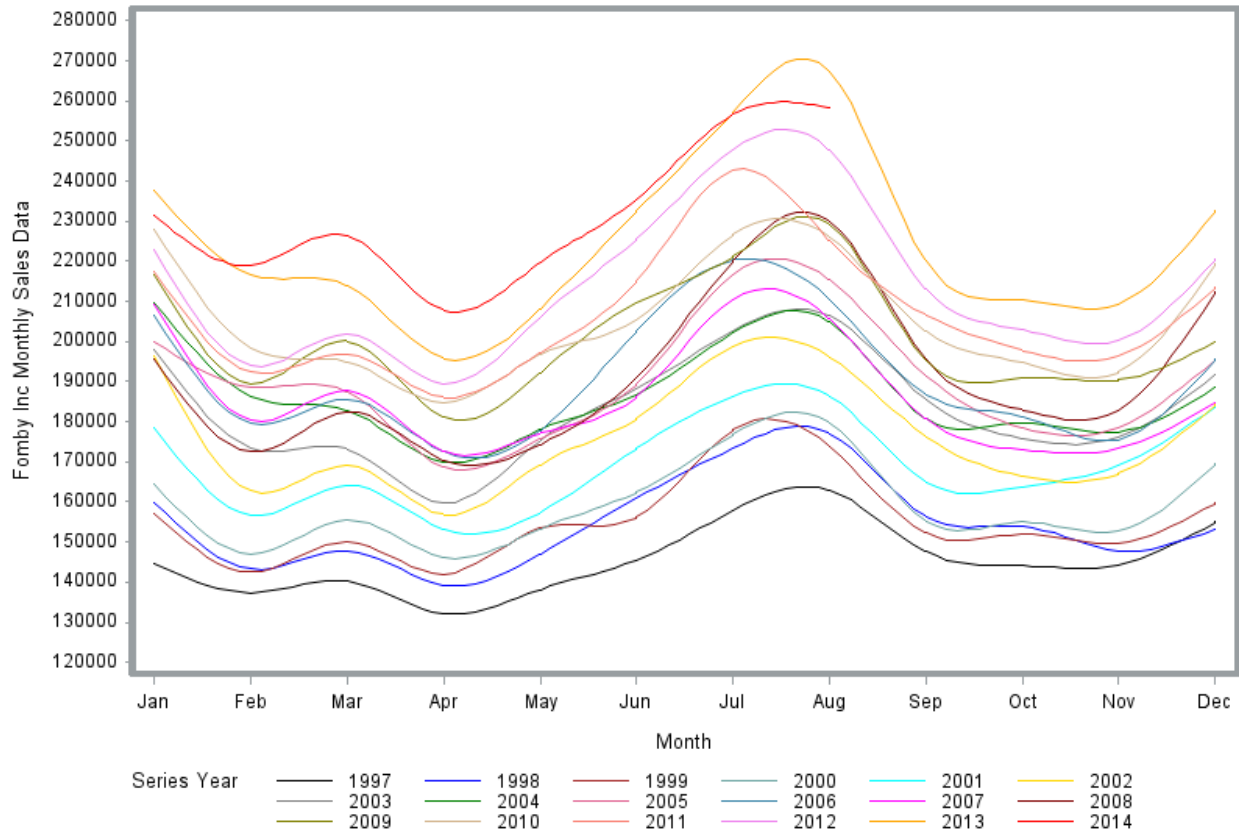
Fomby Inc Monthly Sales of Widgets in quantity units



However, the so-called **Buyers-Ballot "Seasons-by-Year" plot** reveals the nature of the seasonality in the sales data. See the graph below. There are 18 separate colored curves in the plot, each curve representing the sales within the seasons (months) of a given year. As you can see, there is a similar pattern in each of the yearly curves. The local minima in the curves are in February, April, and November while local maxima exist at January, March, August, and December. Also note that the Buyers-Ballot plot indicates a trend in the data because with the increase in the years, the yearly curves are at increasing higher levels. In contrast, if the data had trend but no seasonality, the yearly curves would

have been stacked at successively higher levels but with no discernable systematic minima and maxima within the yearly curves.

Fomby Inc Monthly Sales of Widgets
Buys-Ballot "Seasons By Year" Plot



Now let us assume we want to build an optimal inventory model for this data. First, let us define some terms: R = **reorder point**; D_L = forecasted sales over the lead period, sometimes called **Lead Demand**; the **lead period** is the time it takes to receive an order for additional inventory; σ_L = the standard error of the forecast of sales over the lead period; $\Phi^{-1}(p)$ = the inverse of the standard normal cumulative distribution function; p = **the service level**, i.e., the probability that the product will not “stock out” during the lead period.

Then the so-called **reorder point** is given by the following formula:

$$R = D_L + \sigma_L \Phi^{-1}(p) . \tag{1}$$

In words, the reorder point equals the forecasted sales (lead demand) over the lead period plus the safety stock, $\sigma_L \Phi^{-1}(p)$. The **safety stock** is an amount of additional inventory that is ordered over and above the mean forecast of sales (D_L) for the lead period that would ensure that the probability of not stocking out during the lead period is at least p , the service level.

Now to put formula (1) into operation let I_C denote the current inventory that is available (measured in production units). Then a) if $I_C < R$, no additional inventory is needed right now while b) if $I_C > R$, the inventory controller needs to order $Q = R - I_C$ more units of product.

Let us consider the following example: Let the lead period be two months long and consider the above widget data. Then the lead period sales forecast is given by the following output as produced by the SAS program sales_forecast_1.sas.

Forecast Summary		
Variable	SEP2014	OCT2014
sales	223576.6	215343.6

Forecast Summation				
Variable	Forecast	Standard Error	Confidence Limits	
sales	438920	8581.99	422100	455741

The end of our monthly data is August, 2014. The forecasted sum of sales for the next two periods, September and October, 2014, is $D_L = 438,920$ while the standard error of the lead demand forecast is $\sigma_L = 8,581.99$. This standard error of the lead demand forecast is calculated as $\text{sqrt}[\text{Var}(e_1 + e_2)] = \text{sqrt}[\text{Var}(e_1) + \text{Var}(e_2) + 2\text{Cov}(e_1, e_2)]$ where e_1 and e_2 are the errors associated with the one- and two-step ahead forecasts, respectively. Let us assume the proper service level (p) for the problem at hand is $p = 0.90$. Then the reorder point for the next two months is

$$R = D_L + \sigma_L \Phi^{-1}(p) = 438,920 + (8,581.99)(1.281552) = 449,918.3.$$

Obviously, the safety stock is given by $(8,581.99)(1.281552) = 10,998.27$. If one is using EXCEL, the computation of $\Phi^{-1}(p)$ can be obtained by using the Normsinv function as in $\text{normsinv}(0.9) = 1.281552$.

So suppose that $I_C = 500,000$. Then, at this point there is no need to order additional inventory of widgets. On the other hand, if $I_C = 400,000$, the inventory controller needs to order $Q = R - I_C = 449,918.3 - 400,000 = 49,918.3$ widgets to bring inventory to the proper level for servicing the next two months of product demand.

Now one might ask, "Is there some way to determine what an optimal service level should be for a particular case?" The answer to this question is yes. For the following discussion of optimal service level we draw on the discussion at the URL <http://www.lodak.com/service-level-definition-and-formula>. Then given certain assumptions (see the above URL), the optimal service level is given by the formula

$$p = \Phi \left[\sqrt{2 \ln \left(\frac{1}{\sqrt{2\pi}} \frac{M}{H} \right)} \right] \quad (2)$$

where Φ = the cumulative distribution for the standard normal distribution, p = the optimal service level, H = carrying cost per unit for the duration of the lead time, and M = marginal unit cost of stock-out. Sometimes H is calculated as $H = \frac{d}{365}H_y$ where d = the number of days in the lead time period and H_y is the annual carrying cost per unit. M includes the gross margin, i.e., instant profit per unit that would have been generated if no stock-out had occurred. However, the loss of gross margin is not the only cost: for example, customer frustration and loss of customer loyalty should also be taken into account. As a rule (according to the above URL), many food retailers consider M to be equal to 3 times gross margin.

In formula (2) above, one should note that $M \geq \sqrt{2\pi}H$ is required for formula (2) to be applicable. Otherwise, the formula would result in computing the square root of a negative number. This implies that the optimal service level will naturally fall in the interval, $0.5 \leq p \leq 1$.

Let us take an example. Suppose that $M/H = 8$. This certainly satisfies the required inequality since $\sqrt{2\pi} \approx 2.5$ and $8 \gg 2.5$. In this case, the optimal service level is computed as

$$p = \Phi \left[\sqrt{2 \ln \left(\frac{1}{2.5} 8 \right)} \right] = \Phi \left[\sqrt{2 \ln(3.2)} \right] = \Phi(1.525222) = 0.936398.$$

The latter number is generated in EXCEL by using the function Normsdist and getting normsdist(1.52522) = 0.936398. In terms of our example above, the optimal service level would be

$$\begin{aligned} R &= D_L + \sigma_L \Phi^{-1}(p) = 438,929 + 8,581.99 \Phi^{-1}(0.936398) = \\ &= 438,920 + 8,581.99(1.525221) = 452,009.4. \end{aligned}$$

Looking at the formula (2), as $\frac{M}{H}$ rises, the optimal service level, p , rises. In contrast, as $\frac{M}{H}$ falls, the optimal service level, p , falls. This makes sense. As the gross margin adjusted for customer loyalty increases (decreases) relative to holding costs, the inventory controller should increase (decrease) the service level (p) so as to reduce (increase) the probability of a stock-out, $1 - p$, during the lead time period.

For more discussion on the optimal inventory problem see www.lokad.com/support which is the Lokad company's "Knowledge Base" website.