Lecture 11

Multiple Regression Model:

$$y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_k X_{ik} + u_i$$

(where $K = k+1$).

To obtain the ordinary least squares estimators of $\beta_0, \beta_1, \cdots, \beta_k$ we have to minimize the following sum of squared residuals

$$\sum_1^N (y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik})^2$$

by solving the normal (first order) equations

$$\sum_1^N (y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}) = 0$$

$$\sum_1^N X_{i1}(y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}) = 0$$

$$\sum_1^N X_{i2}(y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}) = 0$$

$$\vdots$$

$$\sum_1^N X_{ik}(y_i - \hat{\beta}_0 - \hat{\beta}_1 X_{i1} - \cdots - \hat{\beta}_k X_{ik}) = 0$$

for the K unknowns $\hat{\beta}_0, \hat{\beta}_1, \cdots, \hat{\beta}_k$.

This is easily done by means of matrix algebra. Of course, the computer is quite good at doing this.

"Partially Out" Interpretation of Multiple Regression

Consider the case where $k = 2$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 X_{i1} + \hat{\beta}_2 X_{i2}$$

One way to express $\hat{\beta}_1$ is

$$\hat{\beta}_1 = \frac{\sum_1^N \hat{r}_{i1} y_i}{\sum_1^N \hat{r}_{i1}^2},$$

where $\hat{r}_{i1}$ are the OLS residuals one would obtain by regressing X_1 on X_2 and a constant term. Therefore r_{i1} is the part of X_1 that remains after X_2 has been "netted" out of it.

$\hat{\beta}_1$ is then the effect that X_1 has on y after the effect of X_2 on y has been netted out.

In this sense we can talk about $\hat{\beta}_1$ being the effect that a one unit change in X has on y after controlling what the effect of X_2 would otherwise be on y if it was also to change.

So $\hat{\beta}_1$ represents the effect of X_1 on y "holding all other ~~factors~~ independent variables fixed" (i.e. ceteris paribus in economic terms.)