MATH 4325
Taylor Series

1)

(a) $f(x) = 2e^{x+1}$ $\quad f(0) = 2e$

$f'(x) = 2e^{x+1}$ $\quad f'(0) = 2e$

$f''(x) = 2e^{x+1}$ $\quad f''(0) = 2e$

$f(x) = 2e + 2e(x-0) + \frac{1}{2} 2e (x-0)^2 + \cdots$

$= 2e(1 + x + \frac{1}{2}x^2 + \cdots)$

(b) $f(x) = \cos x \sin 2x$ $\quad f(\pi/4) = \cos \pi/4 \sin \pi/2 = \frac{1}{\sqrt{2}} \cdot 1$

$f'(x) = -\sin x \sin 2x$ $\quad f'(\pi/4) = -\frac{1}{\sqrt{2}} \cdot 1 + 0 = -\frac{1}{\sqrt{2}}$

$+ \cos x \, 2\cos 2x$

$f''(x) = -\cos x \sin 2x$ $\quad f''(\pi/4) = -\frac{1}{\sqrt{2}} \cdot 1$

$- \sin x \, 2\cos 2x$ $\quad - 0$

$- \sin x \, 2\cos 2x$ $\quad - 0$

$- \cos x \cdot 4 \sin 2x$ $\quad - \frac{1}{\sqrt{2}} \cdot 4 \cdot 1 = -\frac{5}{\sqrt{2}}$

$f(x) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}(x - \pi/4) - \frac{1}{2}\frac{5}{\sqrt{2}}(x - \pi/4)^2 + \cdots$

(c) $f(x) = x^{-1/2}$ $\quad f(1) = 1$

$f'(x) = -\frac{1}{2} x^{-3/2}$ $\quad f'(1) = -\frac{1}{2}$

$f''(x) = \frac{3}{4} x^{-5/2}$ $\quad f''(1) = \frac{3}{4}$

$f(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{2}\frac{3}{4}(x-1)^2 + \cdots$

2) $f(x) = \sin x$

$f(0) = 0 \quad f'(0) = 1 \quad f''(0) = 0 \quad f'''(0) = -1$

1 term: $f(x) \approx 0 + \cdots$

2 term: $f(x) \approx 0 + x$

3 term: $f(x) \approx 0 + x + 0$

4 term: $f(x) \approx 0 + x + 0 - \frac{1}{6}x^3$