

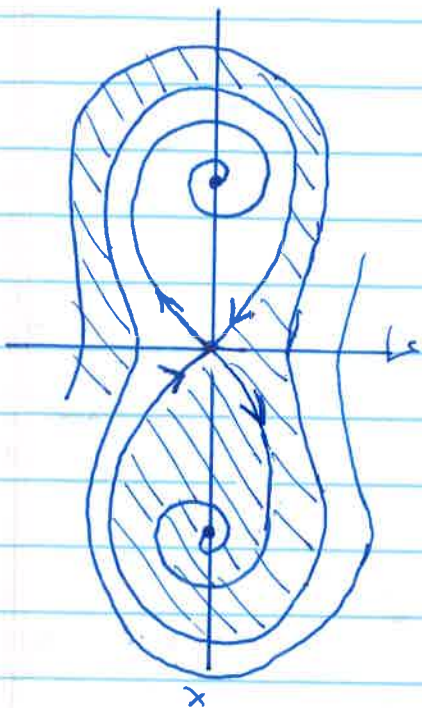
PHASE PLANE

(6.5.19)

(6.5.11) (0,0) is a SADDLE
 (0, ±1) are STABLE FOCUS

$$\frac{R}{R} = a - bF \quad \frac{F}{F} = -c + dR$$

↑ Growth & Death FOR FOXES ↑ LUNCH



• other than ignoring other outside influences, and the function form of growth rates, the model has no "self regulation". That is, the rabbits have an infinite food supply and the foxes do not compete with each other.

• let $T = at$ $\frac{b}{a}F = y$ $\frac{d}{c}R = x$

$$\frac{dx}{dT} = x(1-y) \quad \frac{dy}{dT} = \frac{c}{a}y(x-1)$$

$$\frac{dy}{dx} = \frac{c(x-1)}{x(1-y)} \Rightarrow$$

$$E = cx - y - c \ln(x) - \ln(y)$$

• There are no other fixpts but (0,0) & (1,1). Near (1,1)...

let $x = 1+u$ $y = 1+v$ $u, v \ll 1$

$$E \approx u^2 + v^2$$

→ Circles!

(6.7.1) let $x = \theta$
 $y = \dot{\theta}$
 $\dot{x} = \dot{\theta} = y$
 $\dot{y} = \ddot{\theta} = -by - \sin x$

Fixpts: $y=0$ $x = n\pi$ $n=0, \pm 1, \pm 2, \pm 3, \dots$

For $x = 0, 2n\pi$ (EVEN)

$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & -b \end{pmatrix} \cdot x \Rightarrow$ FOCUS if $b < 2$ NODE if $b > 2$ STABLE

For $x = (2n+1)\pi$ (ODD)

$\dot{x} = \begin{pmatrix} 0 & 1 \\ 1 & -b \end{pmatrix} \Rightarrow$ ALWAYS a SADDLE

For $b < 2$

