Name: ____

Math 3313 Homework -*Forced-Response*

Instructions:

- Hand-drawn sketches should be neat, clear, of reasonable size, with axis and tick marks appropriately labeled. All figures should have a short caption explaining what they show and describe.
- *Staple or bind* all pages together with *this page on top.DO NOT* dog ear pages as a method to bind.

Important Concepts:

- You understand the amplitude response curve for a periodically forced linear oscillator.
  - Sketch the curve for different values of damping.
  - Describe in words what it indicates about the dependence of the solution on frequency. What is resonance?
  - Be able to analytically derive the formula for the curve.
  - How do the cases of zero vs. non-zero damping differ both in the solution method, the solution, and the resulting sketch.

Problems:

1. Use the secondorderrk45.m to investigate the the frequency response of a forced, linear, mass-spring system subject to different levels of damping. Set $m = 1$, $k = 1$ and $b = 0.1$. Set the ICs however you please. To put the forcing in the matlab problem, in the function "dxdt = f(x,t)", set f with the following two commands:
   w = 1
   *f = cos(w*t)*
   (a) Create a plot of the Amplitude vs the frequency w by doing the following:
   - Let w = 0.2, 0.5, 0.8, 0.9, 1.0, 1.1, 1.2, 1.5, 2.0 and simulate.
   - For each value of w, find the record the amplitude of the oscillations.
   - Plot each data point on a graph of amplitude vs. frequency.

   Make sure you set the time interval long enough so that the amplitude is the same from one maximum to the next.
   (b) For what value of the forcing frequency is the amplitude a maximum? How is this consistent with the theory of resonance?
   (c) Repeat the experiment in (a) but with $b = 0.5$ (but you don't have to make the plot.) What is the result? Is this consistent with the theory of resonance?

2. The Van der Pol equation is a nonlinear oscillator that has both positive and negative damping.

$$x'' + e(x^2 - 1)x' + x = 0$$

where x is the current in the circuit. In the Van der Pol circuit the resistance term depends on the current x, that is, the damping "coefficient" is $e(x^2 - 1)$. If $|x| > 1$ then damping is positive and energy is removed from the system. If $|x| < 1$ then the damping is negative and energy is supplied to the system.

- Let $x' = y$ and write as a system of 2 first order differential equations.
- Program this system into the function "dxdt = f(x,t)" of secondorderrk45.m with x = x(1) and y = x(2).
- Set $e = 0$. Describe is the output?
- What happens to the output as you increase e to 0.5, 1, 1.5 and 2.? Print a plot of the last case when $e = 2$. Zoom in so that the plot clearly shows "non-harmonic" oscillations.

- Notice that the system has sustained oscillations as $t \to \infty$ but there is no forcing.
  - If $|x| > 1$ the dissipation is positive and will tend to bring the solution back to zero.
  - However, if $|x| < 1$ the dissipation is negative, which means it is actually a gain. This pushes the system away from zero.

Van der Pol study published his observations about the behavior of this circuit in 1927 (see me if you would like to see the paper). He did not have an oscilloscope or any way to see the current oscillations. Instead, he monitored the output with a speaker driven by the circuit current.

The nonlinear "resistor" of the Van der Pol can be implemented with a triode-tube circuit element and the circuit easily built in undergraduate circuit labs. While most modern electronics are solid-state (chips) tubes are still used in high-end stereo amplifiers.