

Name: \_\_\_\_\_

## Math 3313 Homework - *Euler Numerical Method*

Instructions:

- Hand-drawn sketches should be neat, clear, of reasonable size, with axis and tick marks appropriately labeled. All figures, hand drawn computer generated, should have a short caption explaining what they show and describe. Any figure without a caption will not be graded.
- *Staple or bind* all pages together. *DO NOT* dog ear pages as a method to bind.

Important Concepts:

- How does stepsize affect numerical accuracy?
- Practice with matlab.

Problems:

1. For the example in class,  $\frac{dx}{dt} = 2x + 5$ , if  $t_0 = -1$ ,  $t_f = 2$ , and  $x(-1) = e^{-2} - (5/2) \dots$ 
  - (a) and  $h = 0.1$ , what should  $N$  be?
  - (b) and  $h = 0.05$ , what should  $N$  be?
  - (c) and  $N = 100$ , what should  $h$  be? Print this figure and turn in.
  - (d) Simulate the ODE for each case (a), (b) and (c). Which fits the analytical solution the best?  
*Matlab tips: The syntax for the initial condition is "x(-1)=exp(-2) - (5/2)". Note, you can use the "data cursor" to select the last computed point on the curve to obtain  $x(2)$ .*
2. Consider  $\frac{dx}{dt} = \sin(x + t)$ ,  $x(0) = 1$ .
  - (a) Let  $h = 0.5$  and solve numerically from  $t_0 = 0$  to  $t_f = 10$ .
  - (b) Decrease  $h$  by half until you can't see a change in the result from one value of  $h$  to the next. Print a figure for the last case and indicate what stepsize you used.
  - (c) For each value of  $h$  you used in b, how many steps ( $N$ ) were required.
3. Consider  $\frac{dx}{dt} = \frac{\cos t}{x}$ ,  $x(0) = x_0$ .
  - (a) Solve analytically. What restriction is necessary on  $x_0$  so that the solution is real?For (b) through (d) use  $h = 0.1$  and  $N = 100$  to simulated from  $t = 0$  to  $10$ .
  - (b) Simulate numerically with  $x(0) = 2$ .
  - (c) Simulate numerically with  $x(0) = 1.5$ .
  - (d) Simulate numerically with  $x(0) = 1$ . What happened in this case that is different than in (b) and (c)? Print this figure and turn in.

Note, for each of the figures turned in, be sure to adjust the axis (*axis([ $t_{min}$   $t_{max}$   $x_{min}$   $x_{max}$ ])*) so that the solution fills most of the figure.