Math 3313: Homework - Direction Fields

Instructions:

- Hand-drawn sketches should be neat, clear, of reasonable size, with axis and tick marks appropriately labeled. All figures should have a short caption explaining what they show and describe. Any figure without a caption will not be graded.
- Staple or bind all pages together. DO NOT dog ear pages as a method to bind.

Important concepts:

- Given an ODE you should be able to sketch the direction field.
- How are the dashes on the direction field related to the solution?
- What is the relationship of the ICs and the different solution curves?

Problems:

1. (Computer not needed.) Sketch the direction field for the following two problems by hand. Do this by drawing short lines of the appropriate slope centered at each of the integer valued coordinates (t, x), where $-2 \le t \le 2$ and $-1 \le x \le 1$. Be sure to sketch some solutions curves.

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(a) x' = x + t
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(b)
$$x' = x^2 - t$$

- 2. (Computer not needed.)
 - (a) What is the slope of a solution curve to an ODE when its tangent is horizontal?
 - (b) For each problem in (1), find the equation for the curve where the solutions all have slope m = 0 and m = 1. Add them to your sketches in (1) using a dotted line.
 - (c) What is the behavior of the solution on either side of curve where m=0 isocline? Increasing? Decreasing? Changing slope? Explain?
- 3. (Use online Blufton Univ. direction field program.) For each of the ODEs below (a)-(c), generate the direction field. For each ODE, click the mouse on each of the ICs below to generate solution curves passing through that point. Sketch your results; you may print out the plot that the program makes or you may simply make a neat sketch of what is on the screen.

 ODEs:

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(a) x' = \cos(t),
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(b)
$$x' = \cos(x)$$
,

(c)
$$x' = \cos(tx)$$
,

ICs:
$$x(-2) = -2$$
, $x(-2) = -1$, $x(-2) = 0$, $x(-2) = 1$.

- (d) For each ODE above, determine the equation for where the tangent is horizontal, x' = 0 and add these to your sketch.
- Your sketches should be accurate but don't overdo the detail. For example, you don't need to sketch 500000 little dashes for the direction field. Plot enough so it is clear what is happening.