Name: ______________________________

Math 3313 Homework -*Damped Mass-Spring*

Instructions:

- Hand-drawn sketchs should be neat, clear, of reasonable size, with axis and tick marks appropriately labeled. All figures, hand drawn computer generated, should have a short caption explaining what they show and describe. Any figure without a caption will not be graded.
- *Staple or bind* all pages together. *DO NOT* dog ear pages as a method to bind.

Important Concepts:

- You should be able to differentiate between *overdamped*, *critically damped*, and *underdamped* from the roots of the characteristic equation or from a plot of the solution.
- You should be able to describe in words the effect that changing the mass, damping or spring constant has on the resulting motion.

Problems:

1. Consider the unforced mass-linear spring system

$$mx'' + bx' + kx = 0, \quad x(0) = x_0, \quad x'(0) = v_0$$

(a) Solve the characteristic equation for the values of r..
(b) For the simulations below assume that the mass is pulled down 1 unit, held at rest, then released. (This information sets your ICs).
(c) Using matlab set $k = 1$ and $b = 0.1$ and simulate for $m = 0.5,\ 1,\ 2$.

(c-1) Does the system exhibit underdamped or overdamped behavior?

(c-2) Describe how the resulting motion changes as m is increased making note of the frequency and the evolution of the amplitude. Two or three sentences will suffice.

(c-3) Explain how your observations are consistent with the analytical solution found in (a).

(c-4) Plot and turn-in the case of $m = 2$.

(d) Using matlab set $k = 1$ and $b = 2$ and simulate for $m = 0.5,\ 1,\ 2$.

(d-1) Does the system exhibit underdamped or overdamped behavior?

(d-2) Describe how the resulting motion changes as m is increased making rate of decay.

(d-3) Explain how your observations are consistent with the analytical solution found in (a).

(c-4) Plot and turn-in the case of $m = 0.5$.

2. Consider the unforced and undamped mass-spring.

$$x'' + x = 0, \quad x(0) = x_0, \quad x'(0) = v_0 \tag{1}$$

(a) Solve the equation by hand, apply the initial conditions, and rewrite in amplitude-phase form. Suppose $x_0 = 1$ and $v_0 = 0$, what is the resulting amplitude? Suppose $x_0 = 0$ and $v_0 = 1$, what is the resulting amplitude? Simulate using matlab for each case and confirm your simulations are consistent with your analysis (no plots to turn in).

(b) Because there is no friction/resistence/damping ($b = 0$) the *Energy* in the system is conserved. To see this:

- Multiply the differential equation (1) by $x'(t)$ to obtain

$$x'x'' + xx' = 0 \tag{2}$$

- Notice that $x'x'' = 1/2[(x')^2]'$ and $xx' = 1/2(x^2)'$. Thus, show that the equation can be integrated to obtain

$$\frac{1}{2}[x'(t)]^2 + \frac{1}{2}x(t)^2 = C \tag{3}$$

- The Kinetic Energy is $1/2[x']^2$ and the Potential Energy is $1/2x^2$. How does the result in Eq. (3) examplify *conservation of energy*.

(c) Apply the initial conditions $x(0) = x_0$ and $x'(0) = v_0$ to determine the constant C. Discuss why the motions in (a) are either the same or different based on their energy.

Note, for each of the figures turned in, be sure to adjust the axis (*axis([* t_{min} t_{max} x_{min} x_{max}*])*) so that the solution fills most of the figure.