Name: ______________________________

Math 3313 Homework - *Autonomous DE & Equilibrium*

Instructions:

- Hand-drawn sketchs should be neat, clear, of reasonable size, with axis and tick marks appropriately labeled. All figures should have a short caption explaining what they show and describe. Any figure without a caption will not be graded.
- *Staple or bind* all pages together. *DO NOT* dog ear pages as a method to bind.

Important Concepts:

- What is the different between an *autonomous* and *non-autonomous* ODE?
- For an autonomous ODE you should be able to:
  - Identify *equilibrium points*.
  - Sketch the *phase line*.
  - Determine *stability* of the equilibrium points.
  - Very important... Understand how the phase line relates to solutions families $x(t)$.

Problems:

1. In class we examine the ODE $x' = x^2 - 1$. Now consider
$$x' = f(x) = x^2 + r$$
where r is a parameter that can take the values $r = -1, -0.5, -0.1, 0.1$. For each value of r:
(a) Sketch $f(x) = x^2 + r$ and determine the equilibrium points.
(b) Draw the phase line.
(d) Determine the stability of the equilibrium points.
(d) Plot solutions families $x(t)$.
(e) Describe how location of the equilibrium points and their stability change as you increase the parameter r.

2. Consider the ODE
$$x' = f(x) = \sin(x) - rx \quad \text{for} \quad r = 1, 0.25, 0.1.$$
For each value of r
(a) First sketch the curve $f_1(x) = \sin(x)$. Then, on the same graph, sketch the line $f_2(x) = rx$ for the different values of r. Because $f = f_1 - f_2$ is the difference between the two curves. Where the two curves intersect $f = 0$; these are the equilibrium points. Label these points.
(c) Draw the phase line.
(d) Determine the stability of the equilibrium points.
(d) Plot solutions families $x(t)$.
(e) Describe how location of the equilibrium points and their stability change as you increase the parameter r.

- $x' = dx/dt$ NOT dx/dr. r is a fixed parameter that you set. It is not the independent variable.
- Double check that your phase line is consistent with your direction field. They should both indicated the same stability information and $t \to \infty$ behavior of solutions.
- There is still a phase line when there are no equilibrium points.