1) \( \frac{dx}{dt} = e^t + x \)

\[
\begin{align*}
\int e^{-x} \, dx &= \int e^t \, dt \\
- e^{-x} &= e^t + C \\
x &= - \ln(C - e^t)
\end{align*}
\]

2) \( \frac{dx}{dt} = t + 4x + 3t + 12 \\
\Rightarrow (x+3)(t+4) \\
\int \frac{dx}{x+3} = \int (t+4) \, dt \\
\ln(x+3) = \frac{1}{2} t^2 + 4t + C \\
x = Ce^{\frac{1}{2} t^2 + 4t} - 3
\]

Note: \( x+3 = 0 \) if \( x = -3 \)

\( \frac{dx}{dt} = 0 \)

\( \Rightarrow x = -3 \) is also a solution.

3) \( \frac{dx}{dt} = t \cos(\frac{x}{2}) \), \( x(0) = 2 \)

\[
\int \frac{1}{\cos(\frac{x}{2})} \, dx = \int t \, dt
\]

\[
\int \frac{1}{\cos(\frac{t}{2})} \, dt = \frac{1}{2} t^2 - \frac{1}{2}
\]

Implicit sol.

5) \( \frac{du}{dt} = \frac{t^2 + 1}{u^2 + 4} \), \( u(0) = 1 \)

\[
\int (u^2 + 4) \, du = \int (t^2 + 1) \, dt
\]

\[
\int (u^2 + 4) \, du = \int (t^2 + 1) \, ds
\]

\[
\frac{1}{2} u^3 + 4u \bigg|_1 = \frac{1}{3} s^3 + s \bigg|_0
\]

\[
\frac{1}{2} u^3 + 4u - \frac{1}{3} t^2 - t - \frac{13}{2} = 0
\]

Implicit sol.

Note:

- \( x = 0 \) is a solution but does not satisfy the IC.
- Not a solution to the IVP.