Applied Mathematics & Modeling

Given some problem to solve.
  • May be from science, engineering, economics, finance, . . .

Model (describe) the problem with a differential equation.
  • This can be quite difficult and time consuming.

Solve the DE-model
  • Exactly
  • Approximately
  • Numerically/computationally
  • All of the above

Evaluate the solution
  • Does the solution described previously observed behavior?
  • Should the model be modified?
  • Is the model good in some restricted set of cases?
  • Can the model predict behavior not yet observed?

Are there other ways to model the problem? DEs are just one tool of many.

Philosophy on the application of mathematics:
  • It is not “exact.”
  • Requires judgment and imagination.
  • Requires knowledge of both the application and mathematics.
  • Requires collaboration and communication across disciplines.

All of this is why it is fun!
0.1 Growth & Decay

...or Death & Birth, or Deposit & Withdrawal.

There is a branch of mathematical biology called “Population Dynamics,” where the competition between species is study. This is important in environmental resource management. “Epidemiology” is very similar in that there is a competition between those who are susceptible, those who are infected and those who are recovered, from a particular disease.

Let $P$ be the number of items of interest, for example, “population.”

\[
\frac{dP}{dt} = \text{Rate of change of the population.}
\]

\[
\frac{dP}{P} = \frac{\text{Rate of change}}{\text{population}} = \frac{1}{P} \frac{dP}{dt} = \text{Growth Rate}
\]

Growth rate measures the Rate of change with respect to the population size, i.e., the relative rate of change. The distinction is important.

- Suppose $\frac{dP}{dt} = k$.
  - No matter what the size of the population, there are always $k$ new elements in a given time.
  - Solve by integration.
  - $P(t) = kt + C$, apply IC $C = P(0)$, so $P(t) = P(0) + kt$.
  - Results in linear growth.
  - Models number of people in Bush Stadium. No matter how many have been admitted, the turnstile gates allow only a fixed number to enter over a given time.

- Suppose $\frac{dP}{dt} = kP$.
  - The rate of change depends on the population size. For example, this reflects the fact that with more members in a population there can be more births.
  - Solve by separation.
  - $P(t) = Ce^{kt}$, apply IC $C = P(0)$, so $P(t) = P(0)e^{kt}$.
  - Results in exponential grown

- Which model is more accurate depends on the application.

Decay is just the negative of growth.

- $\frac{dP}{dt} = -k$ so $P(t) = P(0) + -kt$.
  - Linear decrease.
  - People leave Bush Stadium at a fixed rate.

- $\frac{dP}{dt} = -kP$ so $P(t) = P(0)e^{-kt}$.
  - Exponential decay.
  - The more members of a population, the more deaths there are.