

# Strategic Immigration Policies and Welfare in Heterogeneous Countries\*

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## Abstract

In this paper we consider a model with two industrialized countries and immigrants that come from “the rest of the world.” The countries are distinguished on the basis of three parameters: population size, bias toward immigrants, and production complementarity between native population and immigrants. We consider a non-cooperative game where each country makes a strategic choice of its immigration quota. We first show that our game admits a unique pure strategy Nash equilibrium and then study the welfare implications of countries’ choices. It turns out that a country with a higher degree of production complementarity and a higher level of tolerance towards immigrants would allow a larger immigration quota and achieve a higher welfare level. Our results call for coordinated and harmonized immigration policies that may improve the welfare of both countries.

**Key Words:** Immigration Quotas, Heterogeneity, Production Complementarity, Welfare, Policy Harmonization.

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# 1 Introduction

In describing an array of industries across countries, one can immediately come to the conclusion that different production technologies impose distinct requirements on the level and distribution of labor skills and the way workers in those industries interact with each other. For example, over the years Japan has achieved a very high level of performance in the industries (cars, sophisticated consumer goods) that require a high level of precision and consistent quality control. These industries are characterized by a large number of production stages and technological progress is usually achieved through the series of small but incessant improvements called “kaizen” (see, e.g., Imai (1989)). This type of production requires not only highly educated and able workers, but also a consistent and extensive level of interaction between them. This results in emergence of a labor force that is quite homogeneous in its educational, cultural, and linguistic background.

On the other hand, the United States specializes in “knowledge”, and especially, software industries that rely on talents and abilities of individuals coming from a wide range of diverse educational and cultural environments. The success of Silicon Valley in the late nineties is often attributed to the diversified backgrounds of scientists, engineers and entrepreneurs who arrived from places like India, China, Taiwan, Israel, among others. In fact, Saxenian (1999) points out more than 30 percent of new businesses in Silicon Valley had an Asian-born co-founder.<sup>1</sup> However, this diversity did not prevent, and, in fact, even reinforced the commonality of workers’ purpose and goals. Saxenian (1996) describes how workers in Silicon Valley enjoyed frequent and intensive exchange of information through a variety of formal and informal contacts. The exchange was facilitated by the frequent moves of workers from one firm to another (the average time spent by an individual in one firm was about two years), and a flexible industry structure (it has been often claimed that in Silicon Valley a firm is simply a vehicle allowing an individual to work.)

The nature of knowledge production indicates the importance of interaction between different workers and, especially, complementarity of their talents and skills, that is quite different from the multi-stage technological process in high-precision manufacturing (see Milgrom and Roberts (1990)

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<sup>1</sup>The openness to immigrants is historically a relatively new phenomena. Chinese immigration (forbidden in the U.S. in 1880) and Japanese immigration (forbidden in 1905) were considered incompatible with American cultural foundations and unwarranted from the economic point of view (Maignan et al (2003)).

and Kremer (1993)).

In general, the production complementarity is based on two sources, *internal heterogeneity*, that describes the diversity of talents within the existing group of workers engaged in a given industry, and *external heterogeneity*, that captures the diversity between the existing group of workers and “newcomers” to the industry. The first type of heterogeneity has been the focus of the Grossman and Maggi (2000) two-country analysis, which introduced a model with a diverse talent pool within each country and examined, among other issues, an assignment of different individuals to complementary tasks, and its impact on trade patterns between two countries. Our goal is to examine an external heterogeneity between “native” population and immigrant workers in an industry that exhibits a production complementarity.

We consider three groups of individuals, citizens of two countries,  $A$  and  $B$ , and immigrants, denoted by  $I$ . Since our focus is on heterogeneity between  $A$  and  $B$  in terms of their production complementarities with the immigrant population, we assume that each of the three groups is homogeneous in nature and consists of identical individuals. Using the language of Esteban and Ray (1994) in their study of polarization, we focus on heterogeneity across three clusters and assume a complete homogeneity within each of them.

In our model there are three parameters that distinguish between two countries. The first, mentioned above, is the degree of production complementarity between countries’ native population and immigrants. Secondly, countries vary in their population size. Finally, the countries may differ with respect to their “bias” towards immigrants. Indeed, one may accept the fact the United States has a lower bias or higher degree of tolerance towards immigrant population than Europe and Japan. The distinctive difference in attitudes towards immigrants can be explained by a variety of historical, cultural, ethnic, religious, geographic, and economic reasons. We do not discuss this issue here and simply accept various degrees of tolerance towards immigrants.

We consider a non-cooperative game between two countries strategically choosing their immigration quotas. We first prove that for any set of parameters of our model there exists a unique pure strategies Nash equilibrium. We further investigate the equilibrium levels of welfare and show that a country with a higher production complementarity and lower immigration basis would enjoy a higher

welfare and accept a larger number of immigrants than its counterpart. It turns out that while a more populous country would attract a larger number of immigrants, its relative immigration quota would nevertheless be lower than in a smaller country.

We also discuss a possible coordination and harmonization of immigration policies that may improve the welfare of both countries. Even though the coordinated reduction of immigration quotas may be beneficial for both countries, one should realize that this conclusion has been stated with respect to equilibrium levels of immigration. In terms of empirical implications, one can justifiably argue that immigration quotas in industrialized countries are far away from the equilibrium levels, and, therefore, a raise of quotas, rather than their reduction, would be a prudent policy recommendation.

The paper is organized as follows. In the next section we introduce the model and provide examples that illuminate our assumptions. In Section 3 we introduce the immigration game and state our result on the existence and uniqueness of a Nash equilibrium. In Section 4 we compare the welfare of two countries as well as the level of the immigration quotas chosen by the countries in equilibrium. Section 5 is devoted to discussion on harmonization of immigration policies aimed at the welfare improvement of two countries. Finally, we provide the concluding results. All the proofs of our results are relegated to the Appendix.

## 2 The Model

There are two “industrialized” countries,  $A$  and  $B$ , and unlimited source of immigration from the “rest of the world”. Both countries possess a degree of complementarity in production between native population (natives) and immigrants and one of the main features of our model is that we allow for heterogeneity of degree of complementarity. Thus two countries may face different effects of immigrants’ contribution towards its production capabilities. More specifically, the production function of country  $j = A, B$  is given by

$$Q_j = (N_j^{\alpha_j} + I_j^{\alpha_j})^{\frac{1}{\alpha_j}},$$

where  $N_j$  is the country population of natives and  $I_j$  is the number of immigrants to country  $j$ . The parameter  $\alpha_j$  represents the reverse measure of production complementarity between natives and

immigrants in  $j$ . We assume that  $0 < \alpha_j < 1$ . Within this range, smaller values of  $\alpha_j$  reflect a higher degree of production complementarity.

To further comment on our choice of the range  $\alpha_j$ 's, note that when  $\alpha_j \leq 0$ , the complementarity is so strong that the output  $Q_j$  tends to zero when the number of immigrants  $I_j$  approaches zero. This would imply that the country would be actually unable to survive without the influx of immigrants. In order to avoid this unrealistic situation, we rule out all non-positive values of  $\alpha_j$ . On the other hand, the iso-quant curves of country  $j$  are strictly concave when  $\alpha_j > 1$ , implying that the mix of natives and immigrants is actually harmful for production purposes. This may happen if the cultural gap between two populations is too wide, which makes it difficult to integrate the heterogeneous population into the production process. In the case when  $\alpha_j = 1$ , the mix of two populations has a neutral effect and has neither positive nor negative benefit in production. Summarizing all these arguments, our analysis is focused on the interesting and meaningful case of  $0 < \alpha_j < 1$ , where natives and immigrants possess a sufficient degree of heterogeneity to enhance the productivity. At the same time, the degree of heterogeneity is sufficiently small to allow beneficial integration of two populations into the production process.

The immigrant wage,  $w_I$ , in both countries is determined via supply function given by

$$w_I = c + \gamma(I_A + I_B),$$

where  $c$  and  $\gamma$  are positive constants.

In addition to their size and complementarity parameters, two countries are distinguished by their bias towards immigrants. That is, for  $j = A, B$ , the welfare of country  $j$  is given by<sup>2</sup>

$$W_j = Q_j - w_I I_j - b_j \frac{I_j}{N_j},$$

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<sup>2</sup>Here we implicitly assume a circular flow of migration between country  $j$  and the rest of the world (called “temporary migration” (Wong (1995))), when immigrants do not stay in  $j$  for “too long”. Thus, the welfare of country  $j$  is that of its natives only. More generally, we may replace the term  $w_I I_j$  by  $\theta_j w_I I_j$ , where  $\theta_j \in [0, 1]$  is a parameter reflecting the degree of integration of immigrants in country  $j$ 's society.  $\theta = 1$  corresponds to our model whereas other extreme case  $\theta = 0$  represents the case of the complete integration of immigrants where their earnings are fully accounted in the country welfare.

where  $b_j$  is a (positive) bias parameter of country  $j$ . In economic terms, the bias effect can be compared to heterogeneous congestion effects imposed on the native population.

Denote by  $x_j$  the (relative) immigration quota of country  $j = A, B$ :

$$x_j = \frac{I_j}{N_j}.$$

Then

$$Q_A = N_A(1 + x_A^\alpha)^{\frac{1}{\alpha}}, \quad Q_B = N_B(1 + x_B^\beta)^{\frac{1}{\beta}},$$

$$w_I = c + \gamma(N_A x_A + N_B x_B),$$

$$W_A = Q_A - w_I N_A x_A - b_A x_A, \quad W_B = Q_B - w_I N_B x_B - b_B x_B,$$

where, for simplicity of notation, the degrees of complementarity  $\alpha_A$  and  $\alpha_B$  are replaced by  $\alpha$  and  $\beta$ , respectively.

To illustrate the features of our model, consider the following examples:

**Example 2.1:** Let country  $A$  be the United States and  $B$  Japan. Suppose that all immigrants are from China. Given closeness of Chinese and Japanese cultures, the degree of complementarity of Chinese immigrants in Japan is relatively low. The situation is obviously different in the U.S., where, after receiving an appropriate education, Chinese immigrants exhibit a higher degree of complementarity. Thus, the reverse degree of complementarity of “natives” and immigrants in the U.S.,  $\alpha$ , is lower than  $\beta$ , the corresponding degree in Japan. One may also accept the view that the U.S. is more open to immigration than Japan, implying  $b_A < b_B$ . Finally, a larger population in the U.S. yields  $N_A > N_B$ . To summarize, this example satisfies the following relationship between the parameters of the model:

$$\alpha < \beta, \quad b_A < b_B, \quad N_A > N_B. \tag{1}$$

**Example 2.2:** The relationship indicated by (1) can be obtained from a slightly different story, where the degree of complementarity of two populations in production depends not only on their cultural heterogeneity but also on what industry they work in. As in Example 2.1, let country  $A$  be the United States and  $B$  is Japan, but suppose that all immigrants come now from

India. We may assume that the cultural gap between Americans and Indians is roughly the same as between Indians and Japanese. One can also assume that while Japan specializes in the production of high-quality manufacturing, the U.S. specialization lies in software development. Then the mix of heterogeneous populations of Japanese and Indians may be rather harmful in refining the high-quality manufacturing through incessant “kaizen” in the production process. In contrast, mixing appropriately heterogeneous populations of Americans and Indians will yield a higher complementarity in software development. Thus again the *reversed* degree of complementarity in the U.S.,  $\alpha$ , would be lower than that in Japan,  $\beta$ . The other inequalities in (1) are held for the same reasons as in Example 2.1.

As it is commonly known, the number of immigrants in the U.S. is larger than that in Japan. In Section 3 we shall re-examine the relationship described in (1) and demonstrate that our theoretical conclusions are consistent with the existence of the immigration gap between two countries.

In the next section we examine strategic interaction between two countries in determining their immigration and show the existence and uniqueness of a pure strategies Nash equilibrium in the two-country non-cooperative game.

### 3 The Immigration Game

To proceed with the formal framework, we consider a multidimensional parameter space  $P$ , where each point  $p \in P$  represents degrees of production complementarity, population size, and immigration bias of two countries. That is,

$$P \equiv \{p = (\alpha, \beta, N_A, N_B, b_A, b_B) | 0 < \alpha, \beta < 1, N_A, N_B, b_A, b_B > 0\}.$$

Let point  $p = (\alpha, \beta, N_A, N_B, b_A, b_B)$ , that describes the parameters of the “world economy”, be given. We consider a game  $\Gamma(p)$  between two countries,  $A$  and  $B$ , whose strategic choices are determined by their relative immigration quotas,  $x_A$  and  $x_B$ , respectively. Countries’ payoffs are represented by their welfare levels,  $W_A(x_A, x_B)$  and  $W_B(x_A, x_B)$ , that depend on their production levels, immigrant wages, and the immigration bias of this country’s population. Specifically, for

country  $j = A, B$  we assume that

$$W_j(x_A, x_B) = Q_j - w_I N_j x_j - b_j x_j.$$

Thus, we have the following expressions for players' payoffs:

$$W_A(x_A, x_B) = N_A(1 + x_A^\alpha)^{\frac{1}{\alpha}} - [c + \gamma(N_A x_A + N_B x_B)]N_A x_A - b_A x_A, \quad (2)$$

$$W_B(x_A, x_B) = N_B(1 + x_B^\alpha)^{\frac{1}{\alpha}} - [c + \gamma(N_A x_A + N_B x_B)]N_B x_B - b_B x_B. \quad (3)$$

Since both payoff functions are continuously differentiable and concave in  $x_A$  for any  $x_B$ , we determine countries' best responses to their rival's choices by solving the first order conditions:

$$\frac{\partial W_A}{\partial x_A} = 0 \quad \text{and} \quad \frac{\partial W_B}{\partial x_B} = 0.$$

It is easy to verify that the best response of countries  $A$  and  $B$ , respectively, is determined by the following equations:

$$f_\alpha(x_A) - 2\gamma N_A x_A = c + \gamma N_B x_B + \frac{b_A}{N_A} \quad (4)$$

$$f_\beta(x_B) - 2\gamma N_B x_B = c + \gamma N_A x_A + \frac{b_B}{N_B}, \quad (5)$$

where

$$f_\delta(x) \equiv (1 + x^\delta)^{\frac{1}{\delta}-1} x^{\delta-1} = (1 + x^{-\delta})^{\frac{1-\delta}{\delta}}$$

for every two positive numbers  $\delta$  and  $x$ .

The following lemma summarizes the property of the function  $f_\delta(x)$  that will be useful for our analysis:

**Lemma  $\mathcal{A}$ :** (i) The function  $f_\delta(x)$  is decreasing in  $x$  on  $\mathfrak{R}_{++}$  for every  $\delta \in (0, 1)$ .

(ii) The function  $f_\delta(x)x$  is increasing in  $x$  on  $\mathfrak{R}_{++}$  for every  $\delta \in (0, 1)$ .

(iii) The function  $f_\delta(x)$  is decreasing in  $\delta$  on  $\mathfrak{R}_{++}$  for every positive  $x$ .

(iv)  $\lim_{x \rightarrow 0} f_\delta(x) = +\infty$  for every  $\delta \in (0, 1)$ .

(v)  $\lim_{x \rightarrow +\infty} f_\delta(x) = 1$  for every  $\delta \in (0, 1)$ .

Lemma  $\mathcal{A}$  guarantees (see Figure 1) that the solution to (4),  $x_A^*(x_B)$  is well-defined, positive-valued, continuous and strictly decreasing in  $x_B$ , and approaches zero as  $x_B$  tends to infinity.



**Insert Figure 1.**

Since the same properties hold for the best response of country  $B$ ,  $x_B^*(x_A)$ , determined by (5), we obtain our first result:

**Proposition 3.1:** For every  $p \in P$ , the immigration game  $\Gamma(p)$  admits an equilibrium in pure strategies.

Our main result is actually stronger as we are able to demonstrate the uniqueness of a Nash equilibrium for every choice of the model parameters:

**Proposition 3.2:** For every  $p \in P$ , the immigration game  $\Gamma(p)$  has a unique equilibrium in pure strategies. Moreover, in equilibrium, both countries choose strictly positive immigration quotas.

The proof of this proposition is relegated to the Appendix. We would like to point out that the reason for uniqueness of an equilibrium is the fact that the best response curves,  $x_A^*(x_B)$  and  $x_B^*(x_A)$  are negatively sloped, and, moreover, have derivatives between 0 and  $-1$ . That is, a raise by  $\epsilon$  of an immigration quota in country  $A$  would trigger a decline in immigration quota in  $B$  by the amount which is less than  $\epsilon$  (see Figure 2).

**Insert Figure 2.**

Thus, two best response curves cannot have more than one point of intersection, which, together with Proposition 3.1, guarantees uniqueness of an equilibrium.

In the next section we compare the welfare and equilibrium levels of immigration quotas in two countries. We shall examine how differences in population size, degree of complementarity and immigration bias impact the variance in welfare and immigration quotas chosen by countries  $A$  and  $B$ .

## 4 Cross-Country Comparison of Welfare and Immigration Quotas

For every choice of the model parameters  $p \in P$ , the (unique) Nash equilibrium will be denoted by  $(x_A^e(p), x_B^e(p))$ .

We begin this section by providing a welfare comparison between the countries when they differ in their production complementarity, population size and immigration bias.

Consider the point  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$ . The welfare levels of two countries in the equilibrium of the game  $\Gamma(p)$ ,  $W_A(x_A^e(p), x_B^e(p))$  and  $W_B(x_A^e(p), x_B^e(p))$ , will be denoted simply by  $W_A^e(p)$  and  $W_B^e(p)$ , respectively.

The next result shows that a country with a lower immigration bias attains a higher level of welfare:

**Proposition 4.1:** Assume that  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  is such that  $\alpha = \beta$ ,  $N_A = N_B$  but  $b_A < b_B$ . Then  $W_A^e(p) > W_B^e(p)$ .

Furthermore, a country with a higher degree of complementarity has a higher level of welfare:

**Proposition 4.2:** Assume that  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  is such that  $\alpha < \beta$ ,  $N_A = N_B$ , and  $b_A = b_B$ . That is, the countries differ only with respect to their degree of complementarity. Then  $W_A^e(p) > W_B^e(p)$ .

Finally, a more populous country is better off relatively to its smaller counterpart:

**Proposition 4.3:** Assume that  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  is such that  $\alpha = \beta$ ,  $N_A > N_B$ , and  $b_A = b_B$ . That is, the countries differ only with respect to their population size. Then  $W_A^e(p) > W_B^e(p)$ .

Now let us turn to the comparison of the equilibrium immigration quotas. First, we state that if the countries differ only with respect to their immigration tolerance, then the less biased country would accept a larger number of immigrants:

**Proposition 4.4:** Let  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  is such that  $\alpha = \beta$ ,  $N_A = N_B$  but  $b_A < b_B$ . That is, the countries differ only with respect to their “bias” with  $A$  being less biased country. Then  $A$  would accept a larger number of immigrants, i.e.,  $x_A^e(p) > x_B^e(p)$ .

The next proposition shows that the country with a higher degree of complementarity would choose a higher immigration quota:

**Proposition 4.5:** Assume that  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  is such that  $N_A = N_B = N$ , and  $b_A = b_B$ ,  $\alpha < \beta$ . That is, the countries differ only with respect to their degree of complementarity, with  $A$  exhibiting a higher level of complementarity degree. (Recall that  $\alpha$  and  $\beta$  are reverse measures of complementarity.) Then  $x_A^e(p) > x_B^e(p)$ .

Since the size of two countries was identical in Propositions 4.4 and 4.5, there was no need to distinguish between the relative and absolute number of immigrants to  $A$  and  $B$ . We now turn to the case where the countries are heterogeneous with respect to their population size. First, we compare the number of immigrants to  $A$  and  $B$ :

**Proposition 4.6:** Assume that  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  is such that  $\alpha = \beta$ ,  $b_A = b_B = b$ , but  $N_A > N_B$ . That is,  $A$  is a more populous country. Then the number of immigrants  $I_A$  in  $A$  would exceed the number of immigrants  $I_B$  in  $B$ , i.e.,  $I_A = x_A^e(p)N_A > I_B = x_B^e(p)N_B$ .

The next corollary examines the aggregate effect of differences in population size, degree of complementarity and immigration bias. We consider the case where, as in Examples 2.1 and 2.2, country  $A$  has a larger population size, higher degree of complementarity and lower immigration bias than country  $B$ . Then the number of immigrants to country  $A$  exceeds the number of immigrants to country  $B$ , which is consistent with the fact that the number of immigrants in the U.S. is larger than that in Japan.

**Proposition 4.7:** Let  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  be such that  $\alpha < \beta$ ,  $b_A < b_B$  and  $N_A > N_B$ . Then  $I_A > I_B$ .

To state our next result regarding a relative number of immigrants, we consider countries with a large native population. Formally, define a subset  $P'$  of the parameter space  $P$  by:

$$P' \equiv \{p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P \mid c + \frac{2b_A}{N_A} + \frac{2b_B}{N_B} < 1\}.$$

Obviously, a sufficient condition for a point  $p = (\alpha, \beta, N_A, N_B, b_A, b_B)$  to belong to the set  $P'$  is that both countries have a large population.

It is interesting to note that a smaller country would impose a higher immigration quota than its larger counterpart:

**Proposition 4.8:** Assume that  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P'$  is such that  $\alpha = \beta$ ,  $N_A > N_B$ , and  $b_A = b_B$ . That is, the countries differ only with respect to their population size. Then the relative number of immigrants is negatively correlated with the size of a country, i.e.,  $x_A^e(p) < x_B^e(p)$ .

## 5 Harmonization of Immigration Policies

In this section we address the question how coordinated and harmonized immigration policies of countries  $A$  and  $B$  may improve their welfare. We consider the first best outcome when countries jointly choose their immigration quotas. It turns out that the equilibrium immigration levels yield an excessive number of immigrants. This result clearly calls for a need for coordinated immigration policies that might be beneficial for both countries. To reinforce this point, we show that a harmonized reduction, both relative and absolute, of equilibrium immigration levels would raise countries' welfare.

For every point  $p$  in the parameter space  $P$  we consider a cooperative outcome  $(x_A^c(p), x_B^c(p))$  that maximizes the joint welfare of two countries. That is, for a given  $p \in P$ , the countries attempt to find:

$$\max_{(x_A, x_B) \in \mathfrak{R}_{++}^2} \{W_A(x_A, x_B) + W_B(x_A, x_B)\}$$

The first result of this section is surprisingly strong. It states that, regardless of differences in size, bias toward immigrants and degree of complementarity between natives and immigrants, the total number of immigrants allowed under the non-cooperative regime is *excessive* as compared with the first-best outcome.

**Proposition 5.1:** For every point  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  the total number of immigrants in equilibrium is larger than that generated by the cooperative outcome:

$$N_A x_A^c(p) + N_B x_B^c(p) < N_A x_A^e(p) + N_B x_B^e(p).$$

Proposition 5.1 obviously rules out the situation where both countries raise their immigration levels under cooperative outcome. However, it is still possible that in the case of two countries with vastly different characteristics, one of the countries would raise its immigration quota under first best

solution. This situation is clearly impossible for two countries with similar characteristics: then their immigration quotas are excessive with respect to the first best:

**Proposition 5.2:** Let point  $p = (\alpha, \alpha, N, N, b, b) \in P$  represent two countries with identical characteristics. Then there exist positive numbers  $\epsilon_1, \epsilon_2$  and  $\epsilon_3$  such that every point  $p' = (\alpha, \beta, N_A, N_B, b_A, b_B)$ , satisfying  $|\alpha - \beta| < \epsilon_1, |N_A - N_B| < \epsilon_2, |b_A - b_B| < \epsilon_3$ , the following inequalities hold:

$$x_A^c(p') < x_A^e(p') \quad \text{and} \quad x_B^c(p') < x_B^e(p')$$

We complete this section by analyzing how the countries can increase their welfare by implementing harmonized immigration policies. We consider two approaches, harmonized relative reduction and harmonized absolute reduction. Under the first policy, both countries reduce their immigration levels by the same percentage point. Alternatively, they may agree on the same number of immigrants eliminated from their respective equilibrium quotas.

**Proposition 5.3: - Relative Reduction:** For every point  $p \in P$  there is  $\lambda, 0 < \lambda < 1$  such that for all  $\lambda', \lambda < \lambda' < 1$ :

$$W_A(\lambda' x_A^e(p), \lambda' x_B^e(p)) > W_A^e(p), \quad W_B(\lambda' x_A^e(p), \lambda' x_B^e(p)) > W_B^e(p).$$

**Proposition 5.4: - Absolute Reduction:** For every point  $p \in P$  there is  $\mu > 0$  such that for all  $\mu', 0 < \mu' < \mu$ :

$$W_A(x_A^e(p) - \mu', x_B^e(p) - \mu') > W_A^e(p), \quad W_B(x_A^e(p) - \mu', x_B^e(p) - \mu') > W_B^e(p).$$

## 6 Concluding Remarks

In this paper we consider a model with two industrialized countries and a homogeneous mass of immigrants. The countries' characteristics may vary with respect to their population size, bias toward immigrants, and production complementarity between native population and immigrants. The latter is an outcome of distinctive production processes in two countries: one (e.g., software industry) is rooted in a heterogeneous labor force with a wide range of cultural, ethnic and educational

backgrounds, and the other (e.g., high-precision manufacturing) is based on homogeneity and a high degree of interaction between workers.

We consider a non-cooperative game between two countries where each of them makes a strategic decision by choosing its immigration quota. We first show that our game admits a unique pure strategies Nash equilibrium and then study the welfare implications of countries' choices. It turns out that a country with a higher degree of production basis and a higher level of tolerance towards immigrants would allow a larger immigration quota. In addition, we show that while a more populous country allows more immigrants, it would establish a lower ratio between immigrants and natives. We also argue that both countries can benefit by coordinating their strategies, and our results call for harmonized immigration policies aimed at improving the welfare of both countries.

To focus our analysis on difference in production complementarities between the native population in countries  $A$  and  $B$ , we assumed a complete homogeneity within each of the three groups, natives in  $A$  and  $B$ , and immigrants. The next natural step, left for future research, would be to generalize this model by allowing heterogeneity, both across countries and within immigrant population. It is especially important in analysis of high-tech knowledge industries, where ethnic, cultural, and social diversity play an even more important role. Indeed, as Florida and Gates (2001) and Florida (2002) show in their studies of metropolitan areas in the U.S., population diversity is a strong indicator of a metropolitan area's high-technology success. They argue that such indicators as the percentage of gay population, number of artists and "bohemians", as well as a high concentration of foreign-born residents are closely linked with the area's high-technology concentration and growth (see also Saxenian (1999)). Another important direction of future research would be an investigation of international trade consequences between countries as a function of their distinct industrial specialization and distribution of skills and talents across population (for the latter see Grossman and Maggi (2000) and Grossman (2002)).

## 7 Appendix

**Proof of Lemma A:** (i), (iv) and (v) are straightforward. For (ii) note that

$$\frac{d[f_\delta(x)x]}{dx} = (1+x^{-\delta})^{\frac{1}{\delta}-2}[-(1-\delta)x^{-\delta} + 1 + x^{-\delta}] = (1+x^{-\delta})^{\frac{1}{\delta}-2}[\delta x^{-\delta} + 1] > 0.$$

Finally, to prove (iii), we have  $f_\delta(x) = e^{(\frac{1}{\delta}-1)\log(1+x^{-\delta})}$ . Then

$$\frac{df_\delta(x)}{d\delta} = f_\delta(x)\left[-\frac{1}{\delta^2}\log(1+x^{-\delta}) - \frac{(\frac{1}{\delta}-1)x^{-\delta}\log x}{1+x^{-\delta}}\right].$$

This expression is negative when  $x \geq 1$ . Consider now the case where  $0 < x < 1$ . The last expression can be rewritten as

$$\begin{aligned} \frac{df_\delta(x)}{d\delta} &= -\frac{f_\delta(x)x^{-\delta}}{\delta^2(1+x^{-\delta})}[x^\delta \log(1+x^{-\delta}) + \log(1+x^{-\delta}) + \delta \log x - \delta^2 \log x] \\ &= -\frac{f_\delta(x)x^{-\delta}}{\delta^2(1+x^{-\delta})}[x^\delta \log(1+x^{-\delta}) + \log(1+x^\delta) - \delta^2 \log x] < 0. \quad \square \end{aligned}$$

**Proof of Proposition 3.2:** Equation (2) implies that the best response of country  $A$  is given by

$$\frac{dx_A^*}{dx_B} = -\frac{\gamma N_B}{(1-\alpha)x_A^{-\alpha-1}(1+x_A^{-\alpha})^{\frac{1-2\alpha}{\alpha}} + 2\gamma N_A}. \quad (6)$$

Similarly, the best response of country  $B$ , given by (3), is determined by

$$\frac{dx_B^*}{dx_A} = -\frac{\gamma N_A}{(1-\beta)x_B^{-\beta-1}(1+x_B^{-\beta})^{\frac{1-2\beta}{\beta}} + 2\gamma N_B}. \quad (7)$$

The inverse of equation (6) represents the slope of the best response curve of country  $A$  with respect to  $x_A$  axis. This inverse is given by the formula

$$-\frac{(1-\alpha)x_A^{-\alpha-1}(1+x_A^{-\alpha})^{\frac{1-2\alpha}{\alpha}} + 2\gamma N_A}{\gamma N_B}. \quad (8)$$

However, the expression in (7) is greater than  $-\frac{N_A}{2N_B}$ , whereas the expression in (8) is smaller than  $-\frac{2N_A}{N_B}$ .

That is, with respect to  $x_A$  axis, the best response curve of country  $B$  is everywhere flatter than that of country  $A$ . Thus, two best response curves do not intersect more than once and, together with Proposition 3.1, it implies the existence of a unique equilibrium.  $\square$

Before proceeding with the proof of Propositions 4.1-4.3, we provide the proofs of Propositions 4.4-4.8:

**Proof of Proposition 4.4:** Note that the subtraction of (5) from (4) implies that for every  $p \in P$

$$[f_\alpha(x_A^e(p)) - \gamma N_A x_A^e(p)] - [f_\beta(x_B^e(p)) - \gamma N_B x_B^e(p)] = \left[ \frac{b_A}{N_A} - \frac{b_B}{N_B} \right] \quad (9)$$

Let  $N = N_A = N_B$ . (9) yields:

$$[f_\alpha(x_A^e(p)) - \gamma N x_A^e(p)] - [f_\alpha(x_B^e(p)) - \gamma N x_B^e(p)] < 0.$$

Since, by Lemma  $\mathcal{A}$ , the function  $f_\alpha(x) - \gamma N x$  is declining in  $x$ , it follows that  $x_A^e(p) > x_B^e(p)$ .  $\square$

**Proof of Proposition 4.5:** Let  $N = N_A = N_B$  and  $b = b_A = b_B$ . From (9),

$$[f_\alpha(x_A^e(p)) - \gamma N x_A^e(p)] - [f_\beta(x_B^e(p)) - \gamma N x_B^e(p)] = 0.$$

Since, by Lemma  $\mathcal{A}$ ,  $f_\alpha(x_A^e(p)) > f_\beta(x_A^e(p))$ , we have

$$[f_\beta(x_A^e(p)) - \gamma N x_A^e(p)] - [f_\beta(x_B^e(p)) - \gamma N x_B^e(p)] < 0.$$

Invoking Lemma  $\mathcal{A}$  again, we conclude that the function  $f_\beta(x) - \gamma N x$  is declining in  $x$ , yielding  $x_A^e(p) > x_B^e(p)$ .  $\square$

**Proof of Proposition 4.6:** If  $x_A^e(p) \geq x_B^e(p)$ , the statement is straightforward. If  $x_A^e(p) < x_B^e(p)$ , then by (9),

$$[f_\alpha(x_A^e(p)) - f_\alpha(x_B^e(p))] < [\gamma N_A x_A^e(p) - \gamma N_B x_B^e(p)].$$

Since, by Lemma  $\mathcal{A}$ ,  $f_\alpha(x_A^e(p)) > f_\alpha(x_B^e(p))$ , it follows that  $N_A x_A^e(p) > N_B x_B^e(p)$ .  $\square$

**Proof of Proposition 4.7:** If  $x_A^e(p) \geq x_B^e(p)$ , the statement is straightforward. Let  $x_A^e(p) < x_B^e(p)$ , Since  $\frac{b_A}{N_A} < \frac{b_B}{N_B}$ , (9) implies that

$$[f_\alpha(x_A^e(p)) - \gamma N_A x_A^e(p)] - [f_\beta(x_B^e(p)) - \gamma N_B x_B^e(p)] < 0.$$

By Lemma  $\mathcal{A}$ ,  $f_\alpha(x_A^e(p)) > f_\beta(x_A^e(p))$ , and we have

$$[f_\beta(x_A^e(p)) - f_\beta(x_B^e(p))] - [\gamma N_A x_A^e(p) - \gamma N_B x_B^e(p)] < 0.$$



Since, by Lemma  $\mathcal{A}$ ,  $f_\beta(x_A^e(p)) > f_\beta(x_B^e(p))$ , it immediately yields  $N_A x_A^e(p) > N_B x_B^e(p)$ .  $\square$

**Proof of Proposition 4.8:** Let  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P'$ . Suppose that  $x_A^e(p) \geq x_B^e(p)$ . If  $x_A^e(p) = x_B^e(p)$ , (9) implies that  $x_A^e(p) = \frac{b_A}{\gamma N_A N_B}$ . Substituting this expression into (4) leads to

$$f_\alpha(x_A^e(p)) = c + \frac{2b_A}{N_A} + \frac{2b_B}{N_B}.$$

Since  $p \in P'$ , it follows that  $f_\alpha(x_A^e(p)) < 1$ , which is a contradiction to Lemma  $\mathcal{A}$ . Thus,  $x_A^e(p) > x_B^e(p)$ .

Invoking (9), we obtain

$$[f_\alpha(x_A^e(p)) - \gamma N_A x_A^e(p) - \frac{b}{N_A}] - [f_\alpha(x_A^e(p)) - \gamma N_B x_A^e(p) - \frac{b}{N_B}] > 0,$$

or

$$x_B^e(p) < x_A^e(p) < \frac{b}{\gamma N_A N_B} \equiv \bar{x}.$$

Since the optimal response of country  $A$  is a declining function of  $x_B$ , equation (2) implies that

$$f_\alpha(\bar{x}) - 2\gamma N_A \bar{x} < c + \gamma N_B \bar{x} + \frac{b}{N_A} < c + \frac{2b}{N_A} + \frac{2b}{N_B}.$$

Since, by Lemma  $\mathcal{A}$ , the function  $f_\alpha(\cdot)$  is decreasing and, moreover,  $\lim_{x \rightarrow +\infty} f_\alpha(x) = 1$ , the fact that  $p \in P'$  yields a contradiction. Thus,  $x_A^e(p) < x_B^e(p)$ .  $\square$

**Proof of Proposition 4.1:** Equations (2) and (3) imply that the equilibrium welfare levels of two countries  $W_A^e(p)$  and  $W_B^e(p)$ , respectively, are given by:

$$W_A^e(p) = N_A(1 + (x_A^e(p))^\alpha)^{\frac{1}{\alpha}-1} + \gamma N_A^2 (x_A^e(p))^2 = N_A f_\alpha\left(\frac{1}{x_A^e(p)}\right) + \gamma N_A^2 (x_A^e(p))^2 \quad (10)$$

$$W_B^e(p) = N_B(1 + (x_B^e(p))^\beta)^{\frac{1}{\beta}-1} + \gamma N_B^2 (x_B^e(p))^2 = N_B f_\beta\left(\frac{1}{x_B^e(p)}\right) + \gamma N_B^2 (x_B^e(p))^2 \quad (11)$$

By Proposition 4.4,  $x_A^e(p) > x_B^e(p)$ . By Lemma  $\mathcal{A}$ , the function  $f_\alpha(\frac{1}{x})$  is increasing in  $x$ . (10)-(11) then imply that  $W_A^e(p) > W_B^e(p)$ .  $\square$

**Proof of Proposition 4.2:** By Proposition 4.5,  $x_A^e(p) > x_B^e(p)$ . Thus, as in the proof of Proposition 4.1, (10)-(11) guarantee that  $W_A^e(p) > W_B^e(p)$ .  $\square$

**Proof of Proposition 4.3:** If  $x_A^e(p) \geq x_B^e(p)$ , the proof is straightforward. Let  $x_A^e(p) < x_B^e(p)$ . By Lemma  $\mathcal{A}$ , the function  $f_\alpha(x)x$  is increasing in  $x$ . Thus

$$f_\alpha\left(\frac{1}{x_A^e(p)}\right) \frac{1}{x_A^e(p)} > f_\beta\left(\frac{1}{x_B^e(p)}\right) \frac{1}{x_B^e(p)} \quad \text{or} \quad \frac{f_\alpha\left(\frac{1}{x_A^e(p)}\right)}{f_\beta\left(\frac{1}{x_B^e(p)}\right)} > \frac{x_A^e(p)}{x_B^e(p)}.$$

Since, by Proposition 4.6,  $\frac{x_A^e(p)}{x_B^e(p)} > \frac{N_A}{N_B}$ , (10)-(11) imply that  $W_A^e(p) > W_B^e(p)$ .  $\square$

**Proof of Proposition 5.1:** Take a point  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$ . The cooperative outcome,  $(x_A^c(p), x_B^c(p))$  satisfies the following first order conditions:

$$\frac{\partial(W_A + W_B)}{\partial x_A} = 0 \quad \text{and} \quad \frac{\partial(W_A + W_B)}{\partial x_B} = 0.$$

Expressions (2) and (3) imply, therefore, that the cooperative outcome satisfies the following;

$$f_\alpha(x_A) - 2\gamma N_A x_A = c + 2\gamma N_B x_B + \frac{b_A}{N_A}, \quad (12)$$

$$f_\beta(x_B) - 2\gamma N_B x_B = c + 2\gamma N_A x_A + \frac{b_B}{N_B}. \quad (13)$$

Notice that the only difference between the pairs of equations (12)-(13) and (4)-(5) is one of the coefficients on the right side. Consider therefore a more general one-parametrical system of two equations that subsumes both pairs, (4)-(5) and (12)-(13):

$$f_\alpha(x_A) - 2\gamma N_A x_A - c - q N_B x_B - \frac{b_A}{N_A} = 0, \quad (14)$$

$$f_\beta(x_B) - 2\gamma N_B x_B - c - q N_A x_A - \frac{b_B}{N_B} = 0. \quad (15)$$

If  $q = \gamma$  it turns into (4)-(5) and if  $q = 2\gamma$  it turns into (12)-(13). Denote the solutions of (14)-(15) by  $x_A^q$  and  $x_B^q$ , respectively. It suffices to show that the function  $N_A x_A^q + N_B x_B^q$  declines in  $q$  on the interval  $[\gamma, 2\gamma]$ .

By the Implicit Functions Theorem we have:

$$\frac{dx_A^q}{dq} = -\frac{-N_B x_B^q (f'_\beta(x_B^q) - 2\gamma N_B) - q N_A N_B x_A^q}{\Delta},$$

where

$$\Delta = (f'_\alpha(x_A^q) - 2\gamma N_A)(f'_\beta(x_B^q) - 2\gamma N_B) - q^2 N_A N_B.$$

Since  $f'_\alpha$  and  $f'_\beta$  are both negative, and  $q \leq 2\gamma$ , it follows that  $\Delta > 0$ .

Similarly,

$$\frac{dx_B^q}{dq} = -\frac{-N_A x_A^q (f'_\alpha(x_A^q) - 2\gamma N_A) - q N_A N_B x_B^q}{\Delta}.$$

Finally,

$$\frac{d(N_A x_A^q + N_B x_B^q)}{dq} = -\frac{N_A N_B \{[-f'_\alpha(x_A^q) x_B^q - f'_\beta(x_B^q) x_A^q] + (2\gamma - q)[N_A x_A^q + N_B x_B^q]\}}{\Delta}.$$

Since the function  $f_\alpha(x)$  is decreasing and  $2\gamma - q \geq 0$ , the last expression is negative.  $\square$

**Proof of Proposition 5.2:** Let  $p = (\alpha, \beta, N_A, N_B, b_A, b_B) \in P$  be such that  $\alpha = \beta, N_A = N_B, b_A = b_B$ . Proposition 5.1 implies that  $x_A^c(\bar{p}) = x_B^c(\bar{p}) < x_A^e(\bar{p}) = x_B^e(\bar{p})$ . By the continuity argument, these relations will be preserved around the point  $p$ . That is, there is a neighborhood  $U(p) \subset P$  such that  $x_A^c(p) < x_A^e(p)$  and  $x_B^c(p) < x_B^e(p)$  for every  $\bar{p} \in U(p)$ .  $\square$

**Proof of Proposition 5.3:** Consider the function  $W_A(\lambda x_A, \lambda x_B)$  where  $\lambda$  is a positive number. By (2),

$$W_A(\lambda x_A, \lambda x_B) = N_A(1 + x_A^\alpha)^{\frac{1}{\alpha}} - [c + \gamma(N_A x_A + N_B x_B)]N_A x_A - b_A x_A,$$

Let us take the derivative of this welfare function with respect to  $\lambda$ :

$$\frac{dW_A(\lambda x_A, \lambda x_B)}{d\lambda} = N_A(1 + (\lambda x_A)^\alpha)^{\frac{1}{\alpha}-1} x_A^\alpha \lambda^{\alpha-1} - [c + 2\lambda\gamma(N_B x_B + N_A x_A)]\lambda N_A x_A - \lambda b_A x_A.$$

By evaluating the last expression at  $\lambda = 1$  and at the point  $(x_A^e(p), x_B^e(p))$ , we obtain:

$$x_A^e(p)(f_\alpha(x_A^e(p)) - 2\gamma N_A x_A^e(p) - 2\gamma N_B x_B^e(p) - c - \frac{b_A}{N_A}),$$

which, by (4), is negative. Thus, a sufficiently small increase in  $\lambda$  would raise the welfare of country  $A$ . The argument for country  $B$  proceeds along the same lines.  $\square$

**Proof of Proposition 5.4:** Consider the function  $W_A(x_A + \varepsilon, x_B + \varepsilon)$ , where  $\varepsilon$  is a real number. By (2), we have

$$W_A(x_A + \varepsilon, x_B + \varepsilon) = N_A(1 + (x_A + \varepsilon)^\alpha)^{\frac{1}{\alpha}} - [c + \gamma(N_A(x_A + \varepsilon) + N_B(x_B + \varepsilon))]N_A(x_A + \varepsilon) - b_A(x_A + \varepsilon).$$

By taking the derivative of this welfare function with respect to  $\varepsilon$  at  $\varepsilon = 0$ , we obtain:

$$N_A(1 + x_A^\alpha)^{\frac{1}{\alpha}-1} x_A^{\alpha-1} - [c + 2\gamma(N_B x_B + N_A x_A)]N_A x_A - b_A.$$

Equation (4) implies that at the point  $(x_A^e(p), x_B^e(p))$  the last expression is negative. Thus, a sufficiently small increase in  $\varepsilon$  would raise the welfare of country  $A$ . The argument for country  $B$  proceeds along the same lines.  $\square$

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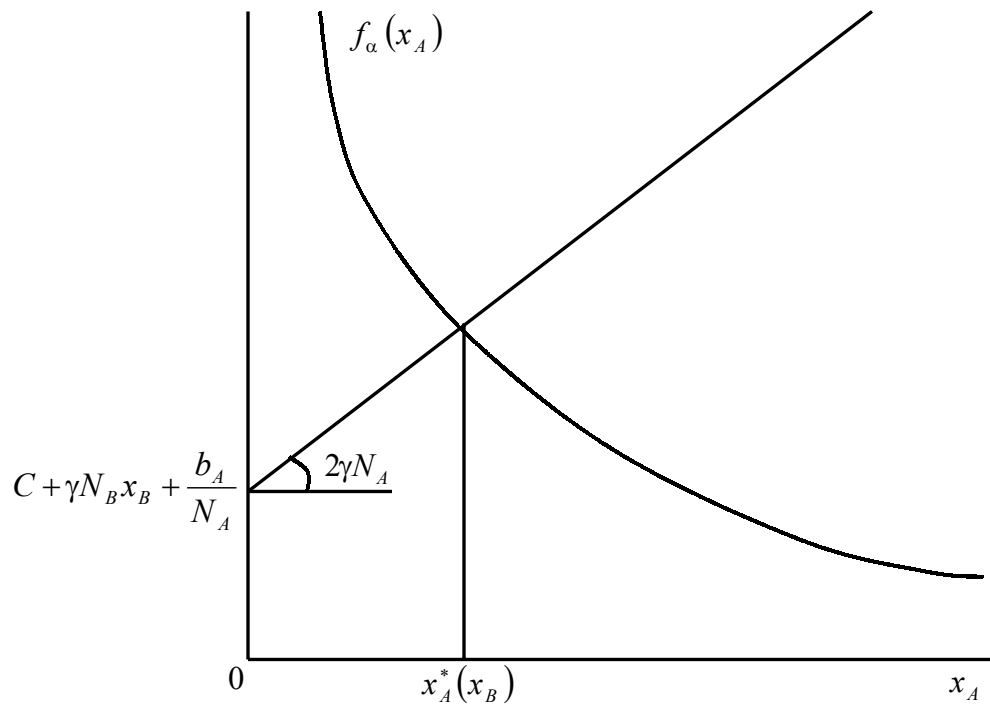


Figure 1. The determination of the best response function  $x_A^*(x_B)$

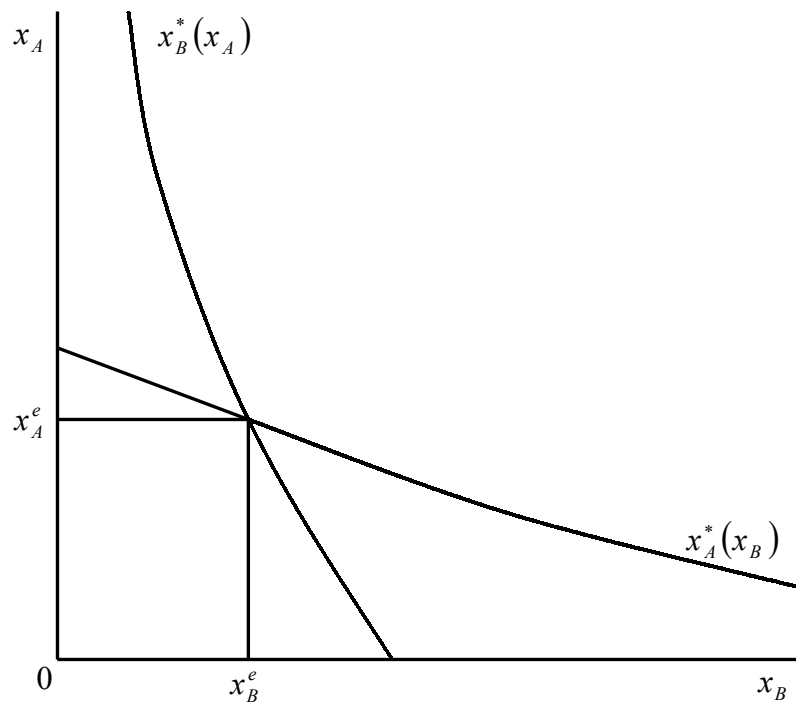


Figure 2. The uniqueness of the equilibrium