ECO 3355-001: Money and Banking  
Southern Methodist University  
Spring 2008  

Exam 1  

Answer Key  

Total Points 100  

1. (50 points) Consider the following overlapping generations economy where individual lives for two periods:  

1. Each person is endowed with $y_1$ when young and $y_2$ when old. Note that $y_2$ is sufficiently small that everyone wants to consume more than $y_2$ in the second period of life.  

2. Population is constant, $n = 1$;  

3. Money stock is constant, $z = 1$;  

4. The government finances the entire amount of government purchases ($G_t = N_t g_t$) through lump-sum taxation. A fixed tax of $\tau$ goods is collected from each young person such that $\tau = g$.  

(a.) (10 points) Find the feasible constraint for central planners.  

The number of young population at time $t$ is $N_t$, each of them has endowment $y_1$. The number of old population at time $t$ is $N_{t-1}$, and each of them has endowment $y_2$. There exist the government purchases of $g$ goods per young person in every period.  

Total amount of consumption good at time $t = N_t y_1 + N_{t-1} y_2$.  
Total young consumption at time $t = N_t c_{1,t}$.  
Total old consumption at time $t = N_{t-1} c_{2,t}$.  
Total government purchases at time $t = N_t g$.  

$$N_t c_{1,t} + N_{t-1} c_{2,t} + N_t g \leq N_t y_1 + N_{t-1} y_2.$$  

Dividing through both sides of this equation by $N_t$,  

$$c_{1,t} + \left[ \frac{N_{t-1}}{N_t} \right] c_{2,t} + g \leq y_1 + \left[ \frac{N_{t-1}}{N_t} \right] y_2.$$  


Note that the population is constant, \( N_t = N_{t-1} \), the feasible set under stationarity is
\[
c_1 + c_2 + g \leq y_1 + y_2.
\]

(b.) (10 points) Set up the money-market clearing condition in period \( t \) and \( t+1 \) (supply of fiat money equals to demand for fiat money). Find the rate of return on fiat money, \( v_{t+1}/v_t \).

Money stock at any particular time is \( M \). \( M \) units of money can acquire \( \psi M \) units of consumption good. Hence, \( \psi M \) is the supply of money in terms of consumption good.

A group of people who wants to obtain money at any particular time is young people at that given time. Old people no longer want to have money; instead, they want to have consumption of goods. Therefore, the consumption good available for trade is young people’s endowment minus their first-period consumptions and tax expenditure.

At time period \( t \):
\[
v_t M_t = N_t (y_1 - \tau - c_{1,t}).
\]

At time period \( t+1 \):
\[
v_{t+1} M_{t+1} = N_{t+1} (y_1 - \tau - c_{1,t+1}).
\]

\[
\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1} (y_1 - \tau - c_{1,t})}{M_{t+1}}}{\frac{N_t (y_1 - \tau - c_{1,t+1})}{M_t}} = \frac{n N_t (y_1 - \tau - c_1)}{z M_t} = \frac{n}{z} = \frac{1}{1} = 1,
\]
due to stationarity, constant money stock and population growth.

(c.) (10 points) Find the budget constraint for individuals when young and old, and then obtain the lifetime budget constraint. Use the solution in (b.) for your equation.

Besides consumption and money holding young people have to pay part of their endowment for taxes in the first period. They also have endowment \( y_2 \) in their second period of life in addition to money they have been holding from the first period.

First – period constraint:
\[
c_{1,t} + v_t m_t \leq y_1 - \tau.
\]

Second – period constraint:
\[
c_{2,t+1} \leq v_{t+1} m_t + y_2.
\]
Solving for \( m_t \) and substituting it back into the first period constraint, we will get the lifetime budget constraint:

\[
C_{1,t} + \left( \frac{v_t}{v_{t+1}} \right) C_{2,t+1} \leq y_1 + \left( \frac{v_t}{v_{t+1}} \right) y_2 - \tau.
\]

Using the solution we found in part (b) together with the fact that \( \tau = g \) in this problem, the lifetime budget constraint becomes

\[
C_{1,t} + C_{2,t+1} \leq y_1 + y_2 - g.
\]

(d.) (10 points) Draw a graph represents both the feasible set and the budget set with \( c_1 \) on the x-axis and \( c_2 \) on the y-axis. Indicate intercepts on both axes. Show arbitrary indifferent curves for the future generations, and label your equilibria.

![Graph showing feasible set and budget set with intercepts and indifference curves]

(e.) (10 points) Does the monetary equilibrium attain the golden rule allocation? Explain.

The monetary equilibrium attains the golden rule allocation because the budget set is identical to the feasible set. This implies that individuals in the monetary equilibrium will choose the same \((c_1^*, c_2^*)\) combination as the one which maximizes the utility of all future generations.
2. (50 points) Consider an overlapping generations economy where individual lives for two periods. Each person is endowed with $y_1$ when young and nothing when old. Endowment cannot be stored. Population is growing at the rate $n > 1$. The government raises revenue by printing more money at the rate $z > n$.

(a.) (10 points) What is the role of money in the economy?

Money is a medium of exchange. It provides a means for people to acquire goods that they do not possess. People trade part of their endowment for money when young, and spend it for consumption when old.

(b.) (10 points) Is the rate of return on fiat money $\frac{n}{z}$ constant, increasing or decreasing? Give an intuitive interpretation of your answer.

The rate of return on fiat money is equal to $\frac{n}{z}$ (see the calculation in question 1, part b). We are given that $n > 1$, and $z > n$. Therefore, $\frac{n}{z} < 1$; the rate of return on fiat money is decreasing. It tells us that if one unit of good is sold for money in period $t$, that amount of money will get us less than a unit of good in period $t+1$.

(c.) (10 points) We know that the budget set does not coincide with the feasible set in this case. Give an intuitive explanation why the budget set line is flatter than the feasible set line.

In this case, the fiat money stock is growing at the rate $z > n$, and the rate of return on fiat money is decreasing. Inflation discourages the consumption of the $c_2$, which can be acquired by money, in favor of the consumption of $c_1$, which can be acquired without money. Therefore, inflation alters the graph of the budget set and makes it flatter than the feasible set line.

(d.) (10 points) Is monetary equilibrium efficient in this problem? Why? Or why not?

The monetary equilibrium is not efficient when there is inflation. Since the budget set does not coincide with the feasible set, there exists the other equilibrium (golden rule allocation) which is preferred by all to the monetary equilibrium.
(e.) (10 points) Are there any other policies for the government to raise the revenue without distorting the budget set?

Yes. The government can raise the revenue through lump-sum taxation. Under this policy, the money supply can be held constant, and the budget set is not distorted.

Extra Credits (5 points)

What is the objective of this course? How do we approach our study?

The objective of this course is to understand the economy with the presence of money; the behavior of people who hold money and consequences of their actions on economic variables. We approach the monetary economies through construction of a series of model economies that replicate essential features of actual monetary economies.