Strategic Behavior
Summer II, 2011.
Solution to Problem Set 3.

Exercise 2.11.

Yes. Note that the one shot game has two Nash equilibria (in pure strategies): (T,L) and (M,C).

Consider the following history dependent strategies for the twice repeated game:

Player 1 plays B in period 1. In period 2, player 1 plays T if (B,R) was played in period 1, and plays M otherwise.

Player 2 plays R in period 1. In period 2, player 2 plays L if (B,R) was played in period 1, and plays C otherwise.

Consider the subgame in period 2 reached after (B,R) was played in period 1. The prescribed strategies suggest that player 1 play T and player 2 play L which is a Nash equilibrium in that subgame. Consider any subgame in period 2 reached after (B,R) was not played in period 1. The prescribed strategies suggest that player 1 play M and player 2 play C which is a Nash equilibrium in that subgame.

Consider the reduced form game in period 1 taking into account the Nash equilibrium payoffs in period 2 resulting from the prescribed strategies. The total payoffs are as follows:

	L	C	R
T	$4, 3$	$1, 2$	$6, 2$
M	$3, 3$	$2, 4$	$4, 3$
B	$2, 4$	$1, 3$	$7, 5$

It is easy to check that player 1 will find it optimal to follow the prescribed strategy of choosing B in period 1, if it believes that player 2 will follow its strategy and choose R in period 1, and vice-versa.

Thus, the prescribed strategies constitute a subgame perfect Nash equilibrium and players attain a payoff of (4,4) in period 1 & a payoff of (3,1) in period 2.

2.17

Fix some wage level w^* where

$$c \leq w^*$$

Strategies:

Period 1 worker: Expend effort at cost c (and produce output y).

Period t worker, $t > 1$,

If history is such that in *all* previous periods $\tau = 1, ...t-1$, workers expended effort c and the firm paid a wage $w_\tau \geq c$, then expend effort at cost c. If history is otherwise, expend zero effort in the firm.

Firm:

In period 1, pay wage $=c$ if worker has expended effort and pay zero wage otherwise.

In periods $t > 1$:

If history is such that in *all* previous periods $\tau = 1, ...t-1$, workers expended effort and the firm paid a wage $w_\tau \geq c$, and, further, in the current period t, the worker expended effort, then pay wage $=c$.

Otherwise, pay zero wage.

To see whether and when these constitute a SPNE, observe that subgames can be of only 2 types:

1. A subgame beginning from some period $t > 1$ onwards where in some period $\tau < t$, the worker did not expend effort and/or the firm paid a wage $\neq c$.

2. A subgame beginning from period 1 onwards or from some period $t > 1$ onwards where in *all* previous periods $\tau = 1, ...t-1$, workers expended effort and the firm paid a wage $w_\tau \geq c$.

In a subgame of type 1, we can see that the specified strategies constitute a NE as neither the firm nor the workers have a unilateral incentive to deviate from the prescribed strategies. Given history and the strategy of the firm, workers know they will be paid zero every period in the subgame independent of how they act, and therefore it is optimal to expend zero effort in every period. Similarly, given history and the strategy of workers, the firm knows that there is no future gain from paying positive wage to current worker (the strategy of workers will imply that they will expend zero effort in this subgame regardless of what they are paid) - and so it is optimal to pay zero wage every period.

Consider a subgame of type 2 where the strategies suggest that the workers expend effort and firm the compensate them with wage $= c$.Given history and firm's strategy, worker in every period expects to be paid c if it expends effort (incurring cost c) and zero, otherwise. So, there is no gain to the worker by deviating from the strategy and not expending effort. Given history and the strategy of workers, a firm can either

(a) Stick to the strategy and pay wage $= c$ every period (after observing output y)

or (b) Deviate and pay zero wage (in which case workers revert to zero effort ever after)

Payoff to firm in case (a) in terms of present value in the first period of the subgame:

$$\begin{aligned} & (y-c) + \delta(y-c) + \delta^2(y-c).... \\ = \; & (y-c)(1 + \delta + \delta^2 + ...) \\ = \; & \frac{y-c}{1-\delta}. \end{aligned}$$

Payoff to firm in case (b):

$$\begin{aligned} & y + \delta.0 + \delta^2.0 + ... \\ = \; & y \end{aligned}$$

So, it is optimal for firm to not deviate i.e., stick to the prescribed strategy as long as:

$$\frac{y-c}{1-\delta}. \geq y$$

i.e.,

$$\delta \geq \frac{c}{y}.$$

Thus, if $\delta \geq \frac{c}{y}$, then there is a SPNE where workers expend effort and produce output y in every period. [It can be shown that this is also a necessary condition for there to be a SPNE where workers expend effort every period.]

Problem 1.

Let π^m denote the optimal monopoly profit. If all firms charge the optimal monopoly price (the perfectly collusive outcome) then each firm's profit is $\frac{\pi^m}{n}$. Define the trigger strategies for each firm exactly as done in class i.e., each firm i sets price equal to monopoly price p^m in period 1 and continues to do so as long as history is such that all firms set their prices equal to p^m in all previous periods. If history is such that some firm has set price $\neq p^m$ in some previous period, then set price equal to c.To see whether these trigger strategies constitute a SPNE, observe that subgames can be of only 2 types:

1. A subgame beginning at $t > 1$ where in some period $\tau < t$, some firm set a price $\neq p^m$.

2. A subgame beginning at $t = 1$ or, $t > 1$ where in periods $\tau = 1, ...t - 1$, all firms have set prices equal to p^m.

In a subgame of type 1, we can see that the specified strategies constitute a NE as no firm has a unilateral incentive to deviate from the prescribed strategies. Given the history and the strategy of other firms, firm i knows that all firms $j \neq i$ will set price equal to c in every period of this subgame regardless of action is chosen by firm i within this subgame and therefore it is optimal for firm i to set its price equal to c i.e., to stick to its trigger strategy.

Consider a subgame of type 2 where the strategies suggest that firms charge monopoly price p^m every period. Given history and and the strategies of other firms, firm i can either

(a) Stick to the strategy (and, therefore, charge monopoly price p^m every period)

or (b) Deviate and undercut rivals slightly (earning almost the entire monopoly profit today but facing zero profit next period onwards).

Payoff to firm i in case (a) in terms of present value in the first period of the subgame:

$$\begin{aligned}
& \frac{\pi^m}{n} + \delta\frac{\pi^m}{n} + \delta^2\frac{\pi^m}{n} + ... \\
= \quad & \frac{\pi^m}{n}(1 + \delta + \delta^2 + ...) \\
= \quad & \frac{\pi^m}{n}(\frac{1}{1-\delta}).
\end{aligned}$$

Payoff to firm in case (b) is (just below):

$$\pi^m$$

So, it is optimal for firm i to not deviate as long as:

$$\frac{\pi^m}{n}(\frac{1}{1-\delta}) \geq \pi^m$$

i.e.,

$$\delta \geq 1 - \frac{1}{n}.$$

Thus, if $\delta \geq 1 - \frac{1}{n}$, then there is a SPNE where perfect collusion is sustained every period.

As n increases, the minimum value of δ required to sustain collusion given by $(1 - \frac{1}{n})$, increases indicating the fact that collusion is harder to sustain as the number of firms goes up.

Problem 2.

In the one period game, the unique NE is one where all firm charge price equal to c. Using backward induction (you need to explain this in details), in any finitely repeated game, the unique subgame perfect outcome is identical to the NE outcome of the one period game viz., all firms charge prices equal to c every period.