Dynamic Sanitary and Phytosanitary Trade Policy

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Abstract

This paper characterizes the optimal use of sanitary and phytosanitary standards to prevent the introduction of harmful pests and diseases through international trade. Because established pest and disease infestations grow and spread over time their introduction has intertemporal consequences. In a dynamic economic model, an efficient trade policy balances the costs of SPS measures against the discounted stream of the costs of control and social damages that are avoided by using SPS measures, where future growth of any established infestation is accounted for. We examine when phytosanitary trade policy makes good economic sense, when it is efficient to provide full protection against pests and diseases, and when restrictive, but not fully protective trade policy is efficient.

Keywords: trade policy, invasive species, pests and disease, intertemporal allocation
1 Introduction

Today, non-native pests and diseases are recognized as one of the leading causes of global environmental change [5]. One of the primary pathways for the introduction of pests and diseases is international trade. As a consequence, international trade agreements recognize that it is important for individual countries to be able to use sanitary and phytosanitary (SPS) standards to protect themselves from the harmful effects of pests and diseases. The World Trade Organization (WTO) Agreement on the Application of Sanitary and Phytosanitary Measures adopts a fairly broad view of SPS standards as any measure applied to protect human, animal or plant health from pests, diseases, toxins and other contaminants. SPS measures may be implemented through product or process standards, testing, inspection, certification, treatment and quarantine (Annex A, Definition 1). SPS measures reduce the import of pests and disease in two ways. First, production and process standards, inspection and treatment reduce the incidence of pests associated with imported goods. Second, SPS measures raise the marginal cost of imported goods, thereby reducing the volume of imports that have the potential to transmit pests and disease. Because SPS measures restrict trade, the WTO Agreement specifies that "when establishing or maintaining sanitary or phytosanitary measures to achieve the appropriate level of sanitary or phytosanitary protection, Members shall ensure that such measures are not more trade-restrictive than required to achieve their appropriate level of sanitary or phytosanitary protection, taking into account technical and economic feasibility."

The key economic question that arises here is, what is the appropriate level of protection? To answer this the WTO Agreement directs countries to "take into account as relevant economic factors: the potential damage in terms of loss of production or sales in the event of the entry, 

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1 The Phytosanitary Agreement of the WTO defines pests as organisms "of economic significance that are either not yet present in an area, or present, but not widely distributed and being officially controlled" [13].
establishment or spread of a pest or disease; the costs of control or eradication in the territory of the importing Member; and the relative cost-effectiveness of alternative approaches to limiting risks." [Article 5 of the WTO Agreement]. This paper examines the economics of SPS measures in this specific context.

Pests and diseases transmitted through trade are specific forms of externalities that cross national borders. A distinctive feature of the negative externalities generated by the invasion of pests and diseases is that, once established, the size of the invasion may grow or spread over time so that the ecological and economic costs to society are dynamic and evolve over time. Further, the true opportunity cost of invasive pests and diseases must account for the future cost of controlling established invasions through domestic public policy measures. Therefore, as recognized in the articles of the WTO agreement quoted above, it is important to analyze the dynamic costs and benefits of SPS trade restrictions. This requires taking into account the natural growth or spread of pests and diseases over time, the corresponding damages, and the future costs of control. This paper examines the economics of SPS measures in an explicit dynamic model that captures these important aspects. To the best of our knowledge, this is the first paper in the literature to do so.

The analysis of SPS trade policy in the existing literature is essentially static and extends the static optimal tariff literature in international trade theory to examine the relation between SPS policy and the allocation of resources within an economy open to trade in partial equilibrium or general equilibrium (see [16] for a survey of this literature). Roberts, Josling and Orden [20] summarize the typical partial equilibrium framework in which SPS policy: (i) acts like a tariff to raise the marginal cost of imported goods, (ii) protects domestic producers from increased costs associated with pest infestations, and (iii) may provide information that affects domestic demand. Beghin and Bereau [3] survey different methods that have been applied to study the impact of

More generally, the analysis of international trade policy in the presence of externalities has received attention in the literature on trade and the environment. However, the focus there is largely on the relationship between trade policy and the comparative advantage of clean and dirty goods production or consumption, and the consequences for the distribution of environmental quality (see [9] for an overview). In the most common setting there is a clean good and a dirty good and pollution is a by-product of production or consumption of the dirty good. Pollution damages may be confined to the country in which production or consumption of the dirty good takes place, as in [6] or [7]. Alternatively, pollution may jointly affect both exporting and importing countries, as in [2], or it may be a global public good as in [23] and [8]. Pests and disease have two features that distinguish them from the kinds of environmental pollution most frequently examined in this literature. First, the geographic distribution of species and the range of habitats suitable for infestation implies that the generation of pest and disease externalities and the mechanism by which they are transmitted through trade differs from the standard models of pollution generated by the production or consumption of dirty goods. Second, as mentioned earlier, pest and disease externalities are not static pollution problems. They evolve over time and the appropriate trade

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2These include the price wedge method, inventory, survey, and gravity based approaches, risk assessment-based cost-benefit analysis, microeconomics based approaches, and multi-market models.

3Case studies that examine the economic impacts of SPS measures typically focus on a single market and pest/disease. Examples include foot-and-mouth disease [19] and [24], Fuji apples [4], US wheat [11], and US citrus [1].
policy must account for this.

As discussed above, the economics SPS policy is linked to the economics of control. In [17] and [18] we examine the optimal domestic control of an established pest and the conditions under which eradication is efficient, and when it is not efficient. The focus is confined to established pest populations. Since it is assumed that there are no introductions, there is no role for trade policy. Important questions relating to prevention of invasive pests and diseases and the possibility of re-introduction after eradication are ignored. The framework in this paper allows us to address these issues. The objective is to characterize how optimal trade policy should be formulated in the presence of trade induced externalities that have intertemporal consequences. Intuitively, the benefits of SPS policy can be measured by the discounted stream of the costs of control and social damages that are avoided by using SPS measures to prevent the introduction of invasive pests and diseases. In this paper we provide a rigorous characterization of how an efficient trade policy balances the costs of SPS measures against the benefits. We examine when phytosanitary trade policy makes good economic sense, when it makes sense to use SPS policies that provide maximal protection against pests and diseases, and when restrictive, but not fully protective trade policy is efficient.

2 Preliminaries

Let $y_t \geq 0$ represent the size of the domestic pest or disease infestation at the beginning of time $t$. Depending on the context this could be an existing pest population, the biomass of an invasive species or the area infested by a pest or disease. If $y_t = 0$ then there is no established domestic infestation. Management occurs on two fronts: prevention and/or control. In the absence of preventive SPS measures, the amount of a pest or disease introduced through imports is given by $i$. Let $r_t$ be the quantity of pest introductions prevented by the SPS measures chosen in period $t$,
where $0 \leq r_t \leq i$. Prevention, or $r_t$, can be viewed as an output produced by the various different SPS policies that are used to prevent the entry of a pest or disease in period $t$. These may include treatment of commodities to reduce infestation rates, inspection of commodities, process standards, import restrictions or tariffs. When $r_t = i$, SPS trade policy is fully protective. There are two ways that SPS measures can offer full protection. First, it may be possible to completely eliminate the risk of pest or disease introductions through the imposition of production and transportation standards as well as product treatment requirements. An example would be fumigation, radiation or other treatment that is 100% effective at killing a target pest. In this case, fully protective SPS trade policy may be consistent with positive imports. In situations where other means do not suffice to eliminate the risk of pest or disease introduction, full protection can be achieved by imposing a ban on imports.

Because it is costly to prevent the introduction of pests and disease, SPS measures raise the price of imported goods. This imposes a domestic welfare cost in each period denoted by $L(r) \geq 0$. The marginal welfare loss is denoted $L_r$ where the subscript signifies the derivative. In the typical case $L$ captures the change in consumer and producer surplus induced by an increase in the price of imported goods, and $L_r$ can be interpreted as the tariff rate equivalent of SPS policy. In some cases $L$ may include domestic administrative costs less any transfer payments from foreign producers. The domestic cost of SPS policy is assumed to satisfy the following assumption:

$A1. \quad L : \mathbb{R}_+ \to \mathbb{R}_+ \text{ is continuously differentiable, strictly increasing, strictly convex, and } L(0) = 0.$

This assumption implies that more stringent SPS trade measures impose a higher domestic welfare cost absolutely as well as at the margin.

In addition to prevention, a variety of control methods can be used to reduce the size of a pest or disease infestation, if and when it becomes established. Let $a_t$ be the reduction in the pest
infestation achieved through manual, chemical, biological or other control methods, where \( a_t \geq 0 \). In this paper, our main interest is optimal trade policy so it is assumed that there is a constant marginal cost of control, \( c \). We shall examine the implications of relaxing this assumption at the end of the paper.

The pest infestation at the end of period \( t \) is given by \( x_t = y_t + i - r_t - a_t \). Any pest or disease infestation that remains has two consequences. First, it causes damages, \( D(x) \). The damage function satisfies:

\[ A2. D : \mathbb{R}_+ \to \mathbb{R}_+ \text{ is continuously differentiable, strictly increasing, strictly convex, and } D(0) = 0. \]

Second, the infestation grows or spreads over time according to a natural growth function \( y_{t+1} = f(x_t) \). The growth function is assumed to satisfy the following properties:

\[ A3. \ f(0) = 0, f(x) > 0 \text{ for all } x > 0. \]
\[ A4. \ f \text{ is continuous on } \mathbb{R}_+ \text{ and continuously differentiable on } \mathbb{R}_{++}. \]
\[ A5. \ f_x(x) > 0 \]
\[ A6. \ f_x(0) > 1. \]
\[ A7. \text{ There exists a } k > 0 \text{ such that } f(k) + i = k \text{ and } f(x) + i < x \text{ for all } x > k. \text{ Further, the initial pest infestation } y_0 \in [0, k]. \]

These assumptions place minimal restrictions on the set of possible growth functions. Assumptions \( A3 - A5 \) require no explanation. Assumption \( A6 \) rules out situations where an infestation is not biologically sustainable even if it is not controlled. Assumption \( A7 \) captures the fact that the growth of any infestation is bounded by climatic, geographical or ecological factors even if nothing is done to manage it.
The intertemporal cost minimization problem is:

\[
\text{Min } \sum_{t=0}^{\infty} \delta^t [ca_t + L(r_t) + D(x_t)]
\]  

subject to: \(i \geq r_t \geq 0, a_t \geq 0, y_t + i - r_t - a_t = x_t, y_{t+1} = f(x_t), y_0 \) given.  

The analysis of efficient trade policy is facilitated by decomposing the intertemporal problem into two parts: a static optimization problem over prevention and control where the pest infestation at the end of the period is taken as given, and a dynamic optimization problem over the pest infestation level. We consider these in turn. The static, minimum cost of using trade policy and control to move from an initial infestation of size \(y \geq 0\) and an uncontrolled introduction of size \(i\), to an end of period infestation of size \(x \in [0, y + i]\) is given by:

\[
F(y + i - x) = \min_{a, r} [ca + L(r)]
\]  

s.t. \(i \geq r \geq 0, a \geq 0, r + a = y + i - x.\)

The convexity of \(L\) implies that this optimization problem is one that minimizes a convex objective over a set of linear constraints. This implies that \(F\) is convex and that the first order conditions are necessary and sufficient. The static cost function depends only on the difference between \(y\) and \(x\) and not their individual magnitudes (\(i\) is considered a parameter). That is, the static cost function depends on the amount by which an infestation is to be reduced through a combination of prevention and control. Efficiency is determined by the least cost means of reducing the pest infestation. If one policy always has lower cost then that policy dominates. Otherwise both policies are utilized and the efficient allocation equalizes their marginal cost. The constraints imply \(r \leq \min\{i, i - (x - y)\}\). This reflects the fact that fully protective trade policy is precluded if the targeted pest infestation at the end of period is larger than the initial infestation.
Proposition 1 Assume that \( x < y + i \). Then (i) phytosanitary trade restrictions are not optimal, \( r = 0 \), if and only if \( c \leq L_r(0) \), (ii) optimal phytosanitary trade policy is fully protective, \( r = i \), if and only if \( x \leq y \) and \( c \geq L_r(i) \), (iii) phytosanitary trade restrictions are optimal but less than fully protective, \( 0 < r < i \), if and only if \( c > L_r(0) \) and either \( c < L_r(i) \) or \( x > y \). In addition, the static minimum cost function, \( F \), is convex.

The above proposition follows directly from the first order (necessary and sufficient) conditions for the minimization problem in (3).

Figure 1 provides a graphical representation of the optimal allocation between prevention and control. The solution depends on the slope of the isocost contours for \( ca + L(r) \). When \( L_r(i) \geq c \), as in Figure 1(a), full protection is optimal, except in the case where \( x > y \) (not shown in Figure 1). At an interior solution, the isocost contour is tangent to the straight line \( r = y + i - x - a \) and \( -c/L_r(r^*) = -1 \), as shown in Figure 1(b). If \( c \leq L_r(0) \), as in Figure 1(c), then it is always more efficient to manage an infestation through control than SPS trade policy. This is not an interesting case for the study of optimal trade policy; hence, the remainder of the paper assumes:

A8. \( c > L_r(0) \).

We now turn to the dynamic optimization problem that balances the costs of phytosanitary policy and control against the intertemporal benefits. A sequence \( \{y_t, x_t, a_t, r_t\}_{t=0}^{\infty} \) that solves (1) is defined to be an optimal program from \( y_0 \). It is straightforward to verify that if \( (y_t, x_t, a_t, r_t) \) solves the dynamic optimization problem then \( a_t \) and \( r_t \) are solutions to (3). This allows a convenient representation of the dynamic optimization problem in (1) using the functional equation of dynamic programming:

\[
V(y) = \min_{0 \leq x \leq y+i} F(y + i - x) + D(x) + \delta V(f(x)),
\]

where \( V(y) \) is the value function.
It is important to recognize that, in spite of the fact that $F$ and $D$ are convex, this is potentially a nonconvex dynamic optimization problem. Every growth function that satisfies assumptions $A6$ and $A7$ is necessarily nonconvex. When $f$ is nonconvex, it is easy to see that the composition of $V$ and $f$ on the right hand side of (4) may be nonconvex. We use techniques from variational analysis to deal with the potential effect of nonconvexities on the solution. One possible complication that arises is that the optimal policy may not be uniquely defined. That is, there may exist more than one selection from the optimal policy that minimizes social cost. In such circumstances we assume:

$A9$. The smallest optimal end of period pest infestation is always chosen.\(^4\)

The technical details and proofs of all propositions in the subsequent sections are contained in the appendix. Rockafellar and Wets [21] provide a rigorous treatment of variational analysis and its application in optimization.

3 Optimal SPS Policy

The benefits of phytosanitary policy and control in period $t$ are the reduction in the discounted stream of current and future social costs that can be attributed to the SPS and control policies implemented in period $t$. Since these policies affect the growth of the infestation in all future periods, the benefits of preventing or reducing a unit of the infestation today include the changes in future damages compounded by the marginal impact of prevention and control on future rates of growth in the pest or disease. The following proposition provides necessary conditions for SPS trade restrictions to be optimal.

Proposition 2 First, for SPS restrictions to be optimal it must be the case that the marginal welfare cost of trade policy, $L_r(r_t)$, is (weakly) smaller than both (a) the sum of present and

\(^4\)The main role of this assumption is that it preserves the monotonicity of the selection from the optimal policy.
discounted future damages caused by a unit of the pest and its growth into the indefinite future,  
\[ D_x(x_t) + \sum_{i=1}^{\infty} \delta^i D_x(x_{t+i}) \prod_{j=0}^{i-1} f_x(x_{t+j}) \],  
and (b) the sum of present and discounted future marginal damages caused by the pest and its growth up to time \( t + T \), plus the discounted cost of removing at time \( t + T \) the accumulated growth resulting from a unit introduction of the pest at time \( t \),  
\[ D_x(x_t) + \sum_{i=1}^{T-1} \delta^i D_x(x_{t+i}) \prod_{j=0}^{i-1} f_x(x_{t+j}) + \delta^T e^{\sum_{j=0}^{T} f_x(x_{t+j})} \],  
and this must hold for all \( T \).

Second, if optimal SPS prevention is positive in period \( t \) and less than fully protective in period \( t + 1 \), then

\[ L_r(r_t) \leq D_x(x_t) + \delta L_r(r_{t+1}) f_x(x_t) \]

i.e., the marginal cost of using SPS restrictions to prevent a unit of the pest today must be (weakly) smaller than the damages from a marginal introduction plus the discounted welfare loss from preventing an equivalent introduction and its growth next period. The inequality arises when it is desirable, but not possible, to shift prevention from the future to the present, as may occur when trade policy is fully protective in period \( t \).

Third, if \( x_t > 0 \) and if optimal SPS prevention is in the interior of \([0, i]\) for periods \( t \) and \( t + 1 \), then

\[ L_r(r_t) = D_x(x_t) + \delta L_r(r_{t+1}) f_x(x_t) \]

In this case the marginal cost of using SPS restrictions to prevent a unit of the pest today must equal the damages from a marginal introduction plus the discounted welfare loss from preventing an equivalent introduction and its growth next period.

In Proposition 2, the marginal welfare cost of trade policy is balanced against its current and future marginal benefits. These conditions are the equivalent of the Euler condition for this model, adjusted to allow for the possibility that the optimal trade policy in the current and future periods is on the boundary of the feasible set. Proper accounting of the benefits of phytosanitary policy
requires that marginal damages in future periods be multiplied by the compound growth over
the intervening periods that results from the marginal infestation that is prevented in the current
period. All else equal, the benefits of phytosanitary policy are greater for pests with higher growth
rates as these pests have more significant negative consequences in the future.

In many instances SPS policy is designed to prevent the introduction of pests or diseases that are
not present domestically. The following corollary provides necessary conditions for fully protective
SPS policy to be efficient when there is no existing infestation.

**Corollary 1** Suppose \( y_t = 0 \). If optimal trade policy is fully protective in period \( t \), then \( L_r(i) \leq \frac{D_x(0)}{1-\delta f_x(0)} \).

For full protection to be optimal it must be better to prevent the pest from being introduced than
to incur the damages and discounted costs of removing it in the future. Note that high discounting
can make it more attractive to postpone control until some future date. Further, the welfare cost
of full protection must be less than perpituity value of intrinsic marginal damages compounded at
the discounted intrinsic growth rate. If \( \delta f_x(0) < 1 \) this will always be true as the perpituity value
of intrinsic marginal damages is infinite.

A characterization of sufficient conditions for SPS policy is complicated by two factors. First,
the dynamic optimization problem is potentially nonconvex. Second, the alternatives to prevention
are to control today or at any future date. To be efficient SPS policy must dominate this infinite
set of alternatives.

Define the minimum rate of growth of infestations larger than \( y \) by

\[
g(y) = \min_{x \in [y, k]} f_x(x),
\]
where $k$ is defined in $A7$. Note that $g(y) < 1$. A sufficient condition for the use of phytosanitary trade policy is:

**Proposition 3** If $D_x(y_t + i) \left[ 1 + \delta f_x(y_t + i) \sum_{j=0}^{\infty} (\delta g(y_t + i))^j \right] > L_r(0)$ then $r_t > 0$ and it is optimal to use phytosanitary trade policy to reduce the introduction of pests.

The left hand side of the inequality is a lower bound on the discounted current and future marginal damages if no preventive trade measures are taken. Future marginal damages are compounded by the growth that results from allowing the pest or disease to enter through trade. The marginal damage terms increase with the size of the existing infestation, $y$, and this increases the likelihood that some trade restriction is optimal. On the other hand, nonconvexity in the growth function implies that the natural growth rate, $f_x(y + i)$, eventually declines in $y$ and this reduces the incentive to impose some trade restriction as the size of the pest infestation increases. In particular, if the growth function is S-shaped, then an increase in $y$ raises the value of SPS trade policy when $y$ is small, but the effect may be in the opposite direction when $y$ is sufficiently large. Proposition 3 applies even in situations where it is not optimal to use control. When control is optimal in some future period, say $T$, then the left hand side can be expressed in terms of the discounted stream of marginal damages incurred up to period $T$, plus the discounted cost of removal through control in period $T$, all adjusted by the appropriate compounded growth rate of the infestation.

Next, we examine sufficient conditions for the use of fully protective phytosanitary trade policy.

**Proposition 4** Suppose that $c \geq L_r(i)$ and

$$D_x(y) \left[ 1 + \delta \min_{y \leq z \leq y + i} f_x(z) \sum_{t=0}^{\infty} (\delta g(y))^t \right] \geq L_r(i). \quad (5)$$

Then fully protective SPS policy is optimal from an existing infestation of size $y$. 

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The term on the left hand side of (5) is a lower bound on the current and future marginal damages from allowing a phytosanitary pest or disease to be introduced through trade. If this exceeds the marginal welfare cost of prohibiting the introduction of such pests then complete prevention of pest or disease introductions through trade is optimal. Note that in many situations a prohibitive level of trade restrictions is effectively a ban on imports.

We now turn our attention to the conditions under which trade policy is more efficient than control but optimal trade restrictions are less than fully protective. Under such conditions the optimal policy allows for positive entry of pests or diseases.

**Proposition 5** Assume \( c \geq L_r(i) \) and that for some \( \hat{y} > 0 \)

\[
D_x(\hat{y}) + \delta \left\{ \max_{0 \leq z \leq \hat{y}} f_z(z) \right\} c < L_r(i).
\]

(6)

Then, if \( y_t \leq \hat{y} \), optimal trade policy is less than fully protective and \( r_t < i \). Further, if trade policy is less than fully protective in period \( t + 1 \) and if

\[
D_x(\hat{y}) + \delta \left\{ \max_{0 \leq z \leq \hat{y}} f_z(z) \right\} L_r(i) < L_r(i)
\]

(7)

Then, if \( y_t \leq \hat{y} \), optimal trade policy is less than fully protective in period \( t \) and \( r_t < i \).

Proposition 5 formalizes the notion that full protection may not be optimal when damages are small, the growth rate is low and the discount rate is high, because then it may be less costly to let some pests in and deal with the problem later. Subsequent remedies may involve either control or prevention. The first condition implies that the cost of controlling the pest after introduction and growth is less than the welfare cost of full protection. Hence, full protection is not efficient and it is cheaper to allow pests to enter and control them later. The second condition involves
the trade-off between eliminating the last unit of introductions today and preventing an equivalent introduction in the future, appropriately adjusted for growth in the infestation. It requires the invasion to be slow growing when it is small, and the discount rate to be high enough that the maximum discounted growth rate, \( \delta \{ \max_{0 \leq z \leq \hat{y}} f_x(z) \} \), is less than one for invasion sizes smaller than \( \hat{y} \). If the discounted growth rate of an infestation is high, then an imperfectly protective trade policy is less likely to be optimal as such a policy will result in higher future social costs. In essence, appropriately adjusted marginal damages must be less than the marginal welfare cost of full protection, in order for fully protective trade policy to be inefficient.

The final result examines the conditions under which it does not make sense to restrict trade in any manner.

**Proposition 6** Assume that the efficient trade policy is always less than fully protective and suppose that for some \( \hat{y} \geq 0 \),

\[
D_x(\hat{y} + i) + \delta \{ \max_{0 \leq z \leq \hat{y} + i} f_x(z) \} \max[c, L_r(i)] < L_r(0).
\]

Then, then if \( y_t \leq \hat{y} \) it is optimal to impose no trade restriction and \( r_t = 0 \).

The economic interpretation of this is that preventive trade policy is inefficient if the marginal cost of the initial unit of prevention, \( L_r(0) \), exceeds the marginal damages associated with the maximum pest introduction plus the marginal cost of reducing the future pest infestation that results from an introduction by an amount equivalent to an upper bound on its growth. That is, trade restrictions are inefficient if it is less costly to incur the damages from an introduction and deal with the pest infestation in the future. All else equal, it is more likely that no trade restrictions are optimal when the pest infestation grows slowly, when the discount rate is high, or when the marginal costs of preventive trade are high.
In addressing alien species that threaten ecosystems, habitats or species the Convention on Biological Diversity argues that "prevention is generally far more cost-effective and environmentally desirable than measures taken following introduction and establishment of an invasive alien species" [5]. Propositions 5 and 6 speak to the circumstances under which this prescription may not always be justified.

4 Conclusion

Our analysis highlights that the post-establishment growth rate and the cost of control play important roles in determining the optimal degree of SPS trade restrictions. Some entry of pests may be optimal if damages are low, the pest growth rate is low and the discount rate is high enough. Further, independent of the ecological and economic damage caused by a pests or disease, optimal trade policy does not need to be restrictive as long as the dynamic domestic cost of control in the post-establishment phase is small enough. In the context of our model, international transfer of technology that makes domestic control of invasive pests and disease more efficient can facilitate a reduction in SPS trade barriers.

The relation between control costs and optimal prevention deserves additional attention. Our assumption of a constant marginal control cost can be generalized in two directions. First, marginal control costs may increase with the amount of control, as in the model of [17] that does not incorporate trade policy. The basic principles of Proposition 1 still apply. The least cost combination of prevention and control is used to achieve the desired reduction in the pest infestation. The question of whether fully protective trade policy is optimal will depend on the relation between the marginal cost of prevention and the intrinsic marginal cost of control, or \( C'(0) \). Second, control costs may depend on the size of the established pest infestation as in [18], again in a model with no role for trade policy. In this case, the marginal cost of control may be higher when the pest
infestation is small. All else equal this will increase the incentive to use SPS trade policy when there is no pest infestation or if the established infestation is small. One complication that arises in this case is that the optimal time path for the pest infestation may exhibit cyclic or complex dynamics. As a consequence, it is possible that the optimal level of SPS trade restriction may vary in a non-monotonic way over time.

5 Technical Appendix

5.1 Subsidiary Lemmas

Lemma 1 Assume $A1 - A7$. If $\{x_t, y_t, a_t, r_t\}$ is an optimal path and $x_t \geq (\leq) x_{t-1}$, then $x_{t+1} \geq (\leq) x_t$. Further, if $x_t$ is optimal from $y_t$ and $x'_t$ is optimal from $y'_t$ where $y_t > y'_t$, then $x_t \geq x'_t$.

Proof. Suppose to the contrary that $x_t \geq x_{t-1}$ and $x_{t+1} < x_t$. From the principle of optimality $V(f(x_{t-1})) = F(f(x_{t-1}) + i - x_t) + D(x_t) + \delta V(f(x_t)) < F(f(x_{t-1}) + i - x_{t+1}) + D(x_{t+1}) + \delta V(f(x_{t+1}))$ and $V(f(x_t)) = F(f(x_t) + i - x_{t+1}) + D(x_{t+1}) + \delta V(f(x_{t+1})) \leq F(f(x_t) + i - x_t) + D(x_t) + \delta V(f(x_t))$. The first inequality is strict since otherwise $x_{t+1}$ yields the same social cost from $f(x_{t-1})$ as $x_t$ and $A9$ implies that the home country always chooses the smallest infestation among those that are optimal. Adding the left and right hand sides together and rearranging yields $F(f(x_t) + i - x_{t+1}) - F(f(x_t) + i - x_t) < F(f(x_{t-1}) + i - x_{t+1}) - F(f(x_{t-1}) + i - x_t)$. Define $z = f(x_t) + i - x_{t+1}$, $z' = f(x_{t-1}) + i - x_{t+1}$ and $\varepsilon = x_t - x_{t+1} > 0$. This yields $F(z) - F(z - \varepsilon) < F(z') - F(z' - \varepsilon)$ which contradicts the convexity of $F$ since $z > z'$. The mathematical intuition behind the proof is that $F$ is convex so that $G(x_{t-1}, x_t) = F(f(x_{t-1}) + i - x_t)$ is a submodular function of $x_{t-1}$ and $x_t$, while $D(x_t)$ and $V(f(x_t))$ are independent of $x_{t-1}$. Minimizing a submodular function yields a set of minimizers whose greatest lower bound is monotone [22] so that, under $A9$, the optimal transition function for $x_t$ is monotone. The second part of the Lemma follows using exactly the
same arguments. To see this simply replace \( f(x_{t-1}) \) with \( y_t \), \( f(x_t) \) with \( y'_t \), and \( x_{t+1} \) with \( x'_t \).

In the presence of possible non-convexities and corner solutions the value function, \( V \), may not be differentiable. Indeed, the one-sided derivatives of \( V \) may not exist. This necessitates the use of subderivatives to characterize the marginal optimality conditions. Since we are dealing with a one state variable problem it is convenient to use the Dini derivatives of \( V \), which exist everywhere. Define the lower, right and left Dini derivatives of \( V \) at \( y > 0 \) by

\[
D_{-}V(y) = \liminf_{\epsilon \to 0} \frac{V(y+\epsilon)-V(y)}{\epsilon}
\]

and

\[
D_{-}V(y) = \limsup_{\epsilon \to 0} \frac{V(y)-V(y-\epsilon)}{\epsilon}
\]

and the upper, right and left Dini derivatives of \( V \) by

\[
D_{+}V(y) = \liminf_{\epsilon \to 0} \frac{V(y)-V(y+\epsilon)}{\epsilon}
\]

and

\[
D_{-}V(y) = \limsup_{\epsilon \to 0} \frac{V(y+\epsilon)-V(y)}{\epsilon}
\]

Note that the the Dini derivatives can assume the value \(+\infty\). The following subsidiary Lemmas characterize the relationship between the Dini derivatives of \( V \) and the underlying elements of the model. These lemmas are used in the proofs of the main propositions.

**Lemma 2** \( D_{+}V(y) \leq D^{+}V(y) \leq c. \)

**Proof.** Let \((a, r, x)\) be optimal from \( y \). Then \((a + \epsilon, r, x)\) is feasible from \( y + \epsilon \) for \( \epsilon > 0 \). The principle of optimality implies \( V(y + \epsilon) - V(y) \leq c(a + \epsilon) + L(r) + D(x) + \delta V(f(x)) \) - \([ca + L(r) + D(x) + \delta V(f(x))] \). Dividing by \( \epsilon > 0 \) and taking the \( \limsup \) on both sides completes the proof. \( \blacksquare \)

**Lemma 3** \( D_{-}V(y) \leq D^{+}V(y) \leq D_{x}(x) + \delta D^{+}V(f(x))f_{x}(x). \)

**Proof.** Let \((a, r, x + \epsilon)\) be optimal from \( y + \epsilon \). The principle of optimality implies \( V(y + \epsilon) - V(y) \leq ca + L(r) + D(x + \epsilon) + \delta V(f(x + \epsilon)) - [ca + L(r) + D(x) + \delta V(f(x))] \).
Using the properties of the \( \limsup \), A5, and the fact that \( D \) and \( f \) are differentiable, we have

\[
\limsup_{\varepsilon \downarrow 0} \frac{V(y + \varepsilon) - V(y)}{\varepsilon} \leq \limsup_{\varepsilon \downarrow 0} \frac{D(x + \varepsilon) - D(x)}{\varepsilon} + \delta \left( \frac{V(f(x + \varepsilon)) - V(f(x))}{f(x + \varepsilon) - f(x)} \right) \left( \frac{f(x + \varepsilon) - f(x)}{\varepsilon} \right) \\
\leq D_x(x) + \delta \left[ \limsup_{\varepsilon \downarrow 0} \frac{V(f(x + \varepsilon)) - V(f(x))}{f(x + \varepsilon) - f(x)} \right] f_x(x) \\
= D_x(x) + \delta D^+ V(f(x)) f_x(x).
\]

Lemma 4 If \( 0 < r \leq i \) then \( L_r(r) \leq D_x(x) + \delta D^+ V(f(x)) f_x(x) \).

Proof. Let \((a, r, x)\) be optimal from \( y \). Then \((a, r - \varepsilon, x + \varepsilon)\) is feasible from \( y \) for sufficiently small \( \varepsilon > 0 \). By the principle of optimality \( c_a + L(r) + D(x) + \delta V(f(x)) - [c_a + L(r - \varepsilon) + D(x + \varepsilon) + \delta V(f(x + \varepsilon))] \leq 0 \). Using A1, A5, the differentiability of \( L, D \) and \( f \), and the properties of the \( \limsup \) we obtain

\[
L_r(r) = \limsup_{\varepsilon \downarrow 0} \frac{L(r) - L(r - \varepsilon)}{\varepsilon} \\
\leq \limsup_{\varepsilon \downarrow 0} \left( \frac{D(x + \varepsilon) - D(x)}{\varepsilon} \right) + \delta \left( \frac{V(f(x + \varepsilon)) - V(f(x))}{f(x + \varepsilon) - f(x)} \right) \left( \frac{f(x + \varepsilon) - f(x)}{\varepsilon} \right) \\
\leq D_x(x) + \delta \left[ \limsup_{\varepsilon \downarrow 0} \frac{V(f(x + \varepsilon)) - V(f(x))}{f(x + \varepsilon) - f(x)} \right] f_x(x) \\
= \left[ D_x(x) + \delta D^+ V(f(x)) f_x(x) \right].
\]

Lemma 5 If \( y > 0 \) and \( r > 0 \) then \( D^- V(y) \geq D_- V(y) \geq L_r(r) \).

Proof. Let \((a, r, x)\) be optimal from \( y \). Since \( r > 0 \), \((a, r - \varepsilon, x)\) is feasible from \( y - \varepsilon \) for \( \varepsilon \) sufficiently
small. The principle of optimality implies

\[
D_- V(y) = \liminf_{\varepsilon \downarrow 0} \frac{V(y) - V(y - \varepsilon)}{\varepsilon} \\
\geq \liminf_{\varepsilon \downarrow 0} \frac{ca + L(r) + D(x) + \delta V(f(x)) - [ca + L(r - \varepsilon) + D(x) + \delta V(f(x))]}{\varepsilon} \\
= \liminf_{\varepsilon \downarrow 0} \left( \frac{L(r) - L(r - \varepsilon)}{\varepsilon} \right) = L_r(r).
\]

Lemma 6 If \( r < i \) then \( D_+ V(y) \leq D^+ V(y) \leq L_r(r) \).

Proof. Let \((a, r, x)\) be optimal from \( y \). Since \( r < i \), \((a, r + \varepsilon, x)\) is feasible from \( y + \varepsilon \) for \( \varepsilon \) sufficiently small. The proof then proceeds in a similar fashion to the proof of Lemma 5.

Lemma 7 If \( y > 0 \) and \( x > 0 \) then \( D^- V(y) \geq D_- V(y) \geq D_x(x) + \delta D_- V(f(x)) f_x(x) \).

Proof. Let \((a, r, x)\) be optimal from \( y > 0 \). Since \( x > 0 \), \((a, r, x - \varepsilon)\) is feasible from \( y - \varepsilon \) for sufficiently small \( \varepsilon > 0 \). The principle of optimality implies

\[
D_- V(y) = \liminf_{\varepsilon \downarrow 0} \frac{V(y) - V(y - \varepsilon)}{\varepsilon} \\
\geq \liminf_{\varepsilon \downarrow 0} \frac{ca + L(r) + D(x) + \delta V(f(x)) - [ca + L(r) + D(x - \varepsilon) + \delta V(f(x - \varepsilon))]}{\varepsilon} \\
= \liminf_{\varepsilon \downarrow 0} \left( \frac{D(x) - D(x - \varepsilon)}{\varepsilon} + \delta \left( \frac{V(f(x)) - V(f(x - \varepsilon))}{f(x) - f(x - \varepsilon)} \right) \left( \frac{f(x) - f(x - \varepsilon)}{\varepsilon} \right) \right) \\
\geq D_x(x) + \delta D_- V(f(x)) f_x(x),
\]

where the last inequality follows using the properties of the \( \liminf \) and the fact that \( D \) and \( f \) are differentiable.

Lemma 8 If \( x > 0 \) then \( c \geq D_x(x) + \delta D_- V(f(x)) f_x(x) \).
Proof. Let \((a, r, x)\) be optimal from \(y\). Since \(x > 0\), \((a + \varepsilon, r, x - \varepsilon)\) is feasible from \(y\) for sufficiently small \(\varepsilon > 0\). The principle of optimality implies \(ca + L(r) + D(x) + \delta V(f(x)) - [c(a + \varepsilon) + L(r) + D(x - \varepsilon) + \delta V(f(x - \varepsilon))] \leq 0\). Rearranging and dividing by \(\varepsilon\) yields

\[
\liminf_{\varepsilon \downarrow 0} \frac{D(x) - D(x - \varepsilon)}{\varepsilon} + \delta \left( \frac{V(f(x)) - V(f(x - \varepsilon))}{f(x) - f(x - \varepsilon)} \right) \left( \frac{f(x) - f(x - \varepsilon)}{\varepsilon} \right) \leq c.
\]

The result then follows using the properties of the \(\liminf\) and the fact that \(D\) and \(f\) are differentiable.

\[\blacksquare\]

Lemma 9 If \(x > 0\) and \(r < i\) then \(L_r(r) \geq D_x(x) + \delta D^- V(f(x)) f_x(x)\).

Proof. Let \((a, r, x)\) be optimal from \(y\). Since \(x > 0\), \((a, r + \varepsilon, x - \varepsilon)\) is feasible from \(y\) for sufficiently small \(\varepsilon > 0\). The principle of optimality implies \(ca + L(r) + D(x) + \delta V(f(x)) - [ca + L(r + \varepsilon) + D(x - \varepsilon) + \delta V(f(x - \varepsilon))] \leq 0\). Rearranging and dividing by \(\varepsilon\) yields

\[
L_r(r) = \liminf_{\varepsilon \downarrow 0} \frac{L(r + \varepsilon) - L(r)}{\varepsilon} \\
\geq \liminf_{\varepsilon \downarrow 0} \left( \frac{D(x) - D(x - \varepsilon)}{\varepsilon} \right) + \delta \left( \frac{V(f(x)) - V(f(x - \varepsilon))}{f(x) - f(x - \varepsilon)} \right) \left( \frac{f(x) - f(x - \varepsilon)}{\varepsilon} \right) \\
\geq D_x(x) + \delta D^- V(f(x)) f_x(x).
\]

The last inequality comes from the properties of the \(\liminf\) and the differentiability of \(D\) and \(f\). \(\blacksquare\)

Lemma 10 If \(a > 0\) then \(c \leq D_x(x) + \delta D^+ V(f(x)) f_x(x)\).

Proof. Let \((a, r, x)\) be optimal from \(y\) and suppose that \(a > 0\). Then \((a - \varepsilon, r, x + \varepsilon)\) is feasible from \(y\) for sufficiently small \(\varepsilon > 0\). By the principle of optimality \(ca + L(r) + D(x) + \delta V(f(x)) -\)
\[ c(a - \varepsilon) + L(r) + D(x + \varepsilon) + \delta V(f(x + \varepsilon)) \leq 0. \]  This implies

\[
c \leq \limsup_{\varepsilon \downarrow 0} \frac{D(x + \varepsilon) - D(x)}{\varepsilon} + \delta \left( \frac{V(f(x + \varepsilon)) - V(f(x))}{f(x + \varepsilon) - f(x)} \right) \left( \frac{f(x + \varepsilon) - f(x)}{\varepsilon} \right)
\]

\[
\leq D_x(x) + \delta D^+ V(f(x)) f_x(x),
\]

where the last inequality follows from the properties of the \( \limsup \) and the differentiability of \( D \) and \( f \).

**Lemma 11** If \( y > 0 \) and \( a > 0 \) then \( D^- V(y) \geq D^- V(y) \geq c. \)

**Proof.** Let \((a, r, x)\) be optimal from \( y > 0 \) and suppose that \( a > 0 \). Then \((a - \varepsilon, r, x)\) is feasible from \( y - \varepsilon \) for sufficiently small \( \varepsilon > 0 \). By the principle of optimality

\[
D^- V(y) = \liminf_{\varepsilon \downarrow 0} \frac{V(y) - V(y - \varepsilon)}{\varepsilon} \\
\geq \liminf_{\varepsilon \downarrow 0} \frac{ca + L(r) + D(x) + \delta V(f(x)) - [c(a - \varepsilon) + L(r) + D(x) + \delta V(f(x))]}{\varepsilon} \\
= c.
\]

**5.2 Proofs of Propositions**

**Proof of Proposition 2.** Assume \( r_t > 0 \). Lemma 3 implies that \( D^+ V(y_{t+1}) \leq D_x(x_{t+1}) + \delta D^+ V(y_{t+2}) f_x(x_{t+1}) \). Shifting time forward and substituting for \( D^+ V(y_{t+2}) \) on the right yields

\[
D^+ V(y_{t+1}) \leq D_x(x_{t+1}) + \delta D_x(x_{t+2}) f_x(x_{t+1}) + \delta^2 D^+ V(y_{t+3}) f_x(x_{t+1}) f_x(x_{t+2}).
\]

Iterating forward and continuing to substitute for \( D^+ V(y_{t+i}) \) in a similar fashion implies \( D^+ V(y_{t+1}) = D_x(x_{t+1}) + \sum_{i=1}^{\infty} \delta^i D_x(x_{t+i+1}) \prod_{j=1}^{i} f_x(x_{t+j}) \). Since \( r_t > 0 \) Lemma 4 implies \( L_r(r_t) \leq D_x(x_t) + \delta D^+ V(f(x_t)) f_x(x_t) \).

Substituting for \( D^+ V(f(x_t)) = D^+ V(y_{t+1}) \) on the right hand side of this expression gives the first
Next assume that \( i > r_{t+1} \) and \( r_t > 0 \). Then Lemmas 4 and 6 imply \( L_r(r_t) \leq D_x(x_t) + \delta f_x(x_t)L_r(r_{t+1}) \). Finally, in addition to the previous assumptions, if \( i > r_t, r_{t+1} > 0 \) and \( x_t > 0 \) then Lemmas 5 and 9 imply \( L_r(r_t) \geq D_x(x_t) + \delta L_r(r_{t+1})f_x(x_t) \). Combining this with the previous inequality completes the proof. \( \blacksquare \)

**Proof of Proposition 3.** Let \( (a, r, x) \) be optimal from \( y \). We first show that \( D_x(y+i) + \delta D_x(y+i)f_x(y+i) \sum_{t=0}^{\infty} (\delta g(y+i))^t > L_r(0) \) implies \( x < y+i \). Suppose to the contrary that \( x = y+i \), which implies \( a = r = 0 \). Then \( y_1 = f(y+i) > y \). The second part of Lemma 1 then implies that \( x \geq x \). From the first part of Lemma 1 it then follows that \( x_{t+1} \geq x_t \geq x = y+i > 0 \) for all \( t \). Since \( x > 0 \), Lemma 9 implies \( L_r(0) \geq D_x(y+i) + \delta D_x(y+i)D_V(f(y+i))f_x(y+i) \). Further, Lemma 7 implies that \( D_V(f(x_{t-1})) \geq D_x(x_t) + \delta D_V(f(x_t))f_x(x_t) \geq D_x(y+i) + \delta g(y+i)D_V(f(x_t)) \), for all \( t \), where the last inequality follows from the convexity of \( D \) and the definition of \( g(y) \). Iterating forward and substituting for \( D_V(f(x_t)) \) in the last inequality one obtains \( L_r(0) \geq D_x(y+i) + \delta D_x(y+i)f_x(y+i) \sum_{t=0}^{\infty} (\delta g(y+i))^t \), which violates the inequality in the statement of the proposition. Hence, it must be that \( x < y+i \), which can only occur if \( r > 0 \) (using Proposition 1 and A8). \( \blacksquare \)

**Proof of Proposition 4.** It follows from Proposition 1 that a necessary condition for trade policy to be fully protective is \( c \geq L_r(i) \). We show that if \( (x, a, r) \) are optimal from \( y \), then \( r = i \). Suppose to the contrary that \( r < i \). Then \( c \geq L_r(i) > L_r(r) \) so that \( a = 0 \) by Proposition 1. Therefore, as in the proof of Proposition 3, \( x_t \geq y+i-r > 0 \) for all \( t \) where \( \{x_t\} \) is the optimal sequence of end of period infestation from \( y \). Since \( x > 0 \), Lemma 9 implies \( L_r(r) \geq D_x(y+i-r) + \delta D_V(f(y+i-r))f_x(y+i-r) \geq D_x(y) + \delta \min_{y \leq z \leq y+i} f_x(z) D_V(f(y+i-r)) \). Further Lemma 7 implies that \( D_V(f(x_{t-1})) \geq D_x(x_t) + \delta D_V(f(x_t))f_x(x_t) \geq D_x(y) + \delta D_V(f(x_t))g(y) \).
for all $t$, where the last inequality follows from the convexity of $D$, the definition of $g(y)$, and the fact that $x_t > y$. Iterating forward, substituting for $D_-(f(x_t))$ in the inequality above one obtains $L_r(i) > L_r(r) \geq D_x(y) + \delta D_x(y)[\min_{y \leq z \leq y+i} f_x(z)] \sum_{t=0}^{\infty} (\delta g(y))^t$, which violates (5).

**Proof of Proposition 5.** Suppose to the contrary that $r_t = i$. Then, $x_t \leq y_t \leq \hat{y}$. Since $r_t = i > 0$, lemma 4 holds and $D_x(x_t) + \delta D^+(f(x_t))f_x(x_t) \geq L_r(i)$. By lemma 2, $c \geq D^+(f(x_t))$. Then, using the convexity of $D$ and the fact that $x_t \leq \hat{y}$, these two inequalities imply $D_x(\hat{y}) + \delta c\{\max_{0 \leq z \leq \hat{y}} f_x(z)\} \geq D_x(x_t) + \delta cf_x(x_t) \geq L_r(i)$, which contradicts (6). Next, suppose $r_t = i$ and $r_{t+1} < i$. Then Proposition 2 implies $L_r(i) \leq D_x(x_t) + \delta L_r(r_{t+1})f_x(x_t) \leq D_x(\hat{y}) + \delta L_r(i)\{\max_{0 \leq z \leq \hat{y}} f_x(z)\}$, which contradicts (7).

**Proof of Proposition 6.** Suppose to the contrary that $r_t > 0$. Then Proposition 2 implies $L_r(0) \leq L_r(r_t) \leq D_x(x_t) + \delta L_r(r_{t+1})f_x(x_t) \leq D_x(\hat{y} + i) + \delta [\max_{0 \leq z \leq \hat{y}+i} f_x(z)] \max[c, L_r(i)]$. This contradicts (8).
References


Figure 1
Static allocation of SPS prevention and pest control, three possible outcomes

(a) full protection
$L_r(i) < c$

(b) partial protection
$L_r(i) > c, L_r(0) < c$

(c) no protection
$L_r(0) > c$