Markets with Differentiated Products.

Oligopoly: Horizontal Product Differentiation.

The Linear City Model:
This is the basic model of horizontal product differentiation where the products are separated on one (horizontal) dimension or attribute. It was developed as a (spatial) model of location choice by Hotelling (1929) and has been co-opted by several distinct areas in economics.

There is a linear city of length one, the $[0,1]$ interval. A unit mass of consumers are uniformly distributed on this interval. Consumers are identical except for their location. Each consumer has unit demand. If a consumer buys from a firm located at distance $d$ at price $p$, she incurs a transport cost $t(d)$ and her net surplus is

$$S - p - t(d),$$

where $S$ is the gross surplus from consuming the good, $t(0) = 0$ and $t(d)$ is strictly increasing.

There are two firms in the market that produce the good. Firms are located on the unit interval. The unit cost of production is identical across firms and is independent of location.

The products of the two firms are horizontally differentiated by their location. Other things being equal, each consumer prefers the product of the firm located closest to her.

Interpret $[0,1]$ as the product space. The location of a firm on this interval indicates the type of product it supplies. The location of a consumer indicates her most preferred product type. When a consumer buys from a firm whose product is not her most preferred type, she incurs a psychological cost that depends on how far removed the purchased product is from her most preferred type; the transport cost function captures this psychological cost.

Socially optimal solution: Firms locate at $\frac{1}{4}, \frac{3}{4}$ so as to minimize the total transport cost of society (and serve all consumers as long as $S$ is large enough).

1. Product Choice with No Price Competition:
Assume prices are fixed so that there is no price competition in the market:

$$c \leq p_1 = p_2 = \tilde{p} < S + t(1)$$

In this case, all consumers buy, no matter where firms locate. A consumer strictly prefers to buy from the firm closest to it.

Firms decide on location simultaneously.

Note that payoff of each firm is directly proportional to its market share.
Suppose that firm 1 locates at a point $a$ and firm 2 located at a point $1 - b$. Without loss of generality, assume

$$a \leq 1 - b,$$

i.e., firm 1 locates to the left of firm 2.
The market share of firm 1 is
\[ \frac{a + 1 - b}{2} \]
and the market share firm 2 is
\[ \frac{1 - \frac{a + 1 - b}{2}}{2} = \frac{b + 1 - a}{2}. \]

Unique NE: Minimal Differentiation i.e., \( a = b = 0.5 \).

Proof: First, we show that \( a = b = 0.5 \) is a NE. To see this note that if \( a = 0.5 \) and firm 2 sets \( b < 0.5 \), then only consumers to the right of the midpoint of the interval \([0.5, 1 - b]\) buy from firm 2 so that its market share is
\[ \frac{1 - 0.5 + (1 - b)}{2} = \frac{0.25 + \frac{b}{2}}{2} < 0.5. \]

Next, we note that \( a < 1 - b \) cannot be a NE. That it because firm 1 can increase market share by locating at \( 1 - b - \epsilon > a \) instead, where \( \epsilon > 0 \) is small. Finally, note that \( a = 1 - b \neq 0.5 \) cannot be a NE. In such a situation, all consumers are indifferent between the firms and so each firm gets market share equal to 0.5. If, for instance, \( a = 1 - b < 0.5 \), then firm 2 can move slightly to the right and get market share that is higher than 0.5 (everyone to the right of firm 2 will now strictly prefer firm 2).

This was the original result in Hotelling’s paper. He used this conclude that competition will lead to minimal product differentiation.

Note: This is also one of the basic models of electoral competition in political economy where the location of a political party signifies its choice of political platform or its position on public policy and the consumers are replaced by voters who are located at their most preferred outcome on public policy. The above conclusion translates to convergence of political platforms (to the median voter’s preferred outcome).

2. Price Competition with differentiated products.

Next, consider a situation where the type of the product sold by the two firms are given i.e., their location in the linear city are given. As before, let the fixed locations be denoted by \( a \) for firm 1 and \( 1 - b \) for firm 2 where \( a \leq 1 - b, a > 0, b > 0 \). We focus on the outcome of price competition between the two firms.

The two firms determine their prices simultaneously and consumers then choose whether to buy and who to buy from (the latter, by comparing price + transport cost across two firms).
Assume: \( t(d) = td^2 \).

Assume that \( S \) is large enough so that in equilibrium, all consumers buy.

If \( a = 1 - b \), then both firms sell identical products in the market and the model reduces to a standard homogenous good symmetric Bertrand duopoly. All consumers buy from the firm with the lower price and if they charge equal prices, we can suppose that the consumers randomize evenly between the firms so that each firm sells 0.5 (as long as the price charged is such that all consumers earn non-negative net surplus).

Now, consider \( a < 1 - b \). We first figure out the quantity demanded from each firm at a pair of prices \((p_1, p_2)\) assuming all consumers buy. A consumer located at point \( y \) strictly prefers to buy from firm 1 if, and only if,

\[
    p_1 + t(y - a)^2 < p_2 + t(1 - b - y)^2.
\]

If both firms sell positive quantities, then there must exist some consumer located at some point \( x \in [0, 1] \) who is indifferent between buying from firms 1 and 2. For such \( x \) :

\[
    p_1 + t(x - a)^2 = p_2 + t(1 - b - x)^2
\]

so that

\[
    x = \frac{a + 1 - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)}
\]

and every consumer located to the right of \( x \) strictly prefers to buy from firm 2 while consumers to the left of \( x \) strictly prefer to buy from firm 1. If the price difference is large then the expression above lies outside the \([0, 1] \) interval in which case one firm takes the entire market and the other sells zero.

Therefore, the demand for firm 1’s product at prices \((p_1, p_2)\) is given by

\[
    D_1(p_1, p_2) = \begin{cases} 
    \frac{a + 1 - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)}, & \text{if } -t[(1 - b)^2 - a^2] \leq p_1 - p_2 \leq t[(1 - b)^2 - a^2] \\
    0, & \text{if } p_1 - p_2 \geq t[(1 - b)^2 - a^2] \\
    1, & \text{if } p_1 - p_2 \leq -t[(1 - b)^2 - a^2].
\end{cases}
\]

The demand for firm 2’s product at prices \((p_1, p_2)\) is given by

\[
    D_2(p_2, p_1) = 1 - D_1(p_1, p_2).
\]

The payoff of each firm is given by

\[
    \pi_i(p_i, p_j) = (p_i - c)D_i(p_i, p_j).
\]

[REMARK: In the literature one often finds linear models of differentiated good price competition where the demand function for each firm (such as \( D_i(p_i, p_j) \)) is exogenously and directly specified instead of being derived from a more structural model. For example, \( D_i = \alpha - \beta p_i + \gamma p_j, \beta > \gamma > 0, \alpha > (\beta - \gamma)c \). This is
often called the differentiated Bertrand model. Our analysis here generates this model from a more primitive structure.

Confining attention to prices such that both firms sell and prices $\geq c$. We can derive the reaction function of each firm $i$ (taking $p_j$ as given). The reaction function for firm 1 is given by:

$$D_1 + (p_1 - c) \frac{\partial D_1}{\partial p_1} = 0$$

which yields:

$$\frac{a + 1 - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)} - \frac{1}{2t(1 - a - b)}(p_1 - c) = 0$$

i.e.,

$$p_1 = \frac{1}{2}[c + p_2 + t((1 - b)^2 - a^2)].$$

Similarly,

$$p_2 = \frac{1}{2}[c + p_1 + t((1 - a)^2 - b^2)].$$

Observe that the reaction functions are upward sloping (game of strategic complementarity).

Solving the reaction functions, we obtain the Nash equilibrium (unique):

$$\hat{p}_1 = c + t(1 - a - b)(1 + \frac{a - b}{3})$$

$$\hat{p}_2 = c + t(1 - a - b)(1 + \frac{b - a}{3}).$$

Observe following:

1. The above expressions give the correct NE even when $a = 1 - b$ so that we have a homogenous good symmetric Bertrand price competition model.

2. If $a < 1 - b$, then equilibrium prices exceed marginal cost and firms make positive profit. Thus, product differentiation softens price competition, enables firms to make money and increases market power.

3. If $t = 0$ i.e., consumers do not differentiate between the goods, then independent of how differently the products are positioned, the market is effectively a homogenous good market and we get the standard price = MC Bertrand outcome.

4. As $t$ increases i.e., consumers differentiate the goods more, market power increases.

5. If firms are located symmetrically i.e., $a = b = z$ (say), $\hat{p}_1 = \hat{p}_2 = c + t(1 - 2z)$. In that case, $(1 - 2z)$ is the distance between the two firms and as this increases (product differentiation by product positioning increases), market power increases.
REMARK: If one uses a linear transport cost function i.e., $t(d) = td$, then in case $a > 0$, $b > 0$, we have a problem of non-existence of NE in pure strategies in the price competition game.

3. Two Stage Game: Strategic Product Positioning with Price Competition (Hotelling-Bertrand model).

Consider now a two stage game (in the same framework as above) where firms first decide on their location i.e., position their product types and then choose prices in the second stage. In particular, in the first stage, firms 1 and 2 simultaneously choose $a, b$ (where $a \geq 0, b \geq 0, a + b \leq 1$). In stage 2, firms choose prices simultaneously (after observing the product types chosen in the first stage). We wish to solve for the subgame perfect equilibrium.

Earlier, we have solved the NE prices for any given pair of locations $a, 1 - b$.

$$
\hat{p}_1(a, b) = c + t(1 - a - b)(1 + \frac{a - b}{3})
$$
$$
\hat{p}_2(a, b) = c + t(1 - a - b)(1 + \frac{b - a}{3}).
$$

Using that we can define the reduced form game in stage one where the payoffs are the Nash profits defined in terms of $a, b$.

$$
\hat{\pi}_1(a, b) = \frac{t(1 - a - b)}{2}(1 + \frac{a - b}{3})^2
$$
$$
\hat{\pi}_2(a, b) = \frac{t(1 - a - b)}{2}(1 + \frac{b - a}{3})^2.
$$

From above it is easy to see that for any given $b$,

$$
\frac{\partial \hat{\pi}_1}{\partial a} = -\frac{t}{6}(1 + \frac{a - b}{3})[1 + 3a + b] < 0
$$

so that it is a strictly dominant strategy for firm 1 to set $a = 0$. Likewise, one can show that it is a strictly dominant strategy for firm 2 to set $b = 0$. Thus, the unique NE of the reduced form game is one where $a = b = 0$ i.e., firms locate at the extremes of the linear city and we have maximal product differentiation. On the equilibrium path, firms set prices $\hat{p}_1(0, 0) = \hat{p}_2(0, 0) = c + t$. The equilibrium profit of each firm is $\frac{t}{2}$.

There are two effects that are below the surface: as firm 1 increases $a$, it moves closer to the other firm and so at any given pair of prices, it gains market share. This is the effect we saw when we considered the Hotelling game of product positioning or location with fixed prices. If this were the only effect, we would have an equilibrium where firms converged to each other. However, there is a second strategic effect. As firm 1 moves closer to firm 2, it increases the intensity of price competition that depresses the prices at which it eventually sells. This is the effect we saw when we looked at price competition with given location. It makes a firm want to position its product away from its rival. For the particulars we have chosen, the second effect dominates here.
In general, of course, we are more likely to obtain less than maximal differentiation - depending on the distribution of consumers, asymmetry between firms etc.

Thus result was established formally by d’Aspremont, Gabsewicz & Thissse (Econometrica, 1979) & it marked an overturning of the Hotelling postulate that competition will lead to too little differentiation in the market.

Oligopoly: Vertical Product Differentiation.

Unit mass of consumers each with unit demand.
Goods differentiated by quality $s$.
Consumer's net surplus from buying a unit of quality $s$ at price $p$:
$$\theta s - p.$$ 

If a consumer does not buy, her net surplus is zero.
Consumers differ in their preference parameter $\theta$ (marginal willingness to pay for better quality).

In particular, preference parameter of the unit mass of consumers distributed uniformly on $[\theta, \theta + 1]$ where $0 \leq \theta < \theta + 1$.

Assume, further, that
$$\theta > 2\theta$$
which is equivalent to
$$\theta < 1.$$ 

Assumption (1) ensures that there is sufficient heterogeneity among consumers in the market.

Two firms the quality of whose goods are denoted by $s_1, s_2$ where, without loss of generality, assume that $s_1 \leq s_2$.

Further, assume that
$$s_i \in [\underline{s}, \bar{s}]$$
where $\underline{s}, \bar{s}$ are the lowest and highest possible quality in the market &
$$0 \leq \underline{s} < \bar{s} < \infty$$

Assume that production cost does not depend on quality & each firm produces at constant unit cost $c$. This simplification allows us to abstract from quality choice motivated by cost saving and focus on a pure strategic motive for choosing different qualities in the market.

Assume, further that:
$$c + \frac{\bar{s} - 2\theta}{3}(\bar{s} - \underline{s}) \leq \theta s$$

(2) is a technical assumption that ensures that all buyers buy in equilibrium (the market is fully covered).
Two stage game:
Stage 1: Firms 1 & 2 simultaneously determine the qualities $s_1, s_2$ of their products.
Stage 2: After both firms (as well as consumers) observe the quality choices made, they determine prices $p_1, p_2$.
Solve for subgame perfect equilibrium.
Consider the second stage game where firms choose prices, given a fixed choice of $s_1, s_2$.
If $s_1 = s_2$, then we have a homogenous good symmetric Bertrand model and the unique NE is $p_1 = p_2 = c$.
So, consider $s_1 < s_2$. We look for an equilibrium where all consumers buy and both firms sell. At any pair of prices $(p_1, p_2)$, it is easy to see that if a consumer with certain value of $\theta$ prefers to buy from firm 2 (firm 1), then all consumers with higher (lower) values of $\theta$ must strictly prefer to buy from firm 2 (firm 1). So, for any $(p_1, p_2)$ where the market is covered, if both firms sell, there is an indifferent consumer with parameter $\hat{\theta} \equiv (\theta, \bar{\theta})$ such that
$$\hat{\theta} s_2 - p_2 = \hat{\theta} s_1 - p_1$$
i.e.,
$$\hat{\theta}(p_1, p_2) = \frac{p_2 - p_1}{s_2 - s_1}$$
and the demand for firms 1 & 2 are given by
$$D_1(p_1, p_2) = \hat{\theta}(p_1, p_2) - \frac{\theta}{\bar{\theta}}, \text{ if } \hat{\theta}(p_1, p_2) \in (\theta, \bar{\theta}),$$
$$= \bar{\theta} - \theta = 1, \text{ if } \hat{\theta}(p_1, p_2) \geq \bar{\theta},$$
$$= 0, \text{ if } \hat{\theta}(p_1, p_2) \leq \theta.$$  
$$D_2(p_2, p_1) = \bar{\theta} - \hat{\theta}(p_1, p_2), \text{ if } \hat{\theta}(p_1, p_2) \in (\theta, \bar{\theta}),$$
$$= 1, \text{ if } \hat{\theta}(p_1, p_2) \leq \theta,$$
$$= 0, \text{ if } \hat{\theta}(p_1, p_2) \geq \bar{\theta}.$$  
Each firm $i$ maximizes $(p_i - c)D_i(p_i, p_j)$.
Unique NE:
$$\hat{p}_1 = c + \frac{\bar{\theta} - 2\theta}{3}(s_2 - s_1)$$
$$\hat{p}_2 = c + \frac{\theta - \bar{\theta}}{3}(s_2 - s_1)$$
Observe that $\bar{\theta} - 2\theta > 0$ under (1) which shows that $\hat{p}_i > c, i = 1, 2$. Further, under (2), even a consumer with parameter $\hat{\theta}$ gets non-negative surplus by buying from firm 1. So, all consumers buy in equilibrium. Further,
$$\hat{\theta}(\hat{p}_1, \hat{p}_2) = \frac{\hat{p}_2 - \hat{p}_1}{s_2 - s_1} = \frac{\bar{\theta} + \theta}{3} \in (\theta, \bar{\theta})$$
and so both firms sell in equilibrium and the quantities sold are given by

\[ q_1 = \frac{\theta - 2\theta}{3}, \quad q_2 = \frac{2\theta - \theta}{3}. \]

The NE equilibrium profits are given by:

\[ \pi_1(s_1, s_2) = \frac{(\theta - 2\theta)^2}{9} (s_2 - s_1), \quad (3) \]

\[ \pi_2(s_1, s_2) = \frac{(2\theta - \theta)^2}{9} (s_2 - s_1). \quad (4) \]

Note \( \pi_2(s_1, s_2) > \pi_1(s_1, s_2) > 0 \) when \( s_2 > s_1 \). Further, the expressions above also indicate the equilibrium profit when \( s_2 = s_1 \).

Now, consider the reduced form stage 1 game where firms simultaneously set \( s_1, s_2 \) and the payoffs are given by (3) and (4). It is easy to observe from the expressions, that in any equilibrium where \( s_1 \leq s_2 \), it must be the case that

\[ s_1 = \underline{s}, \quad s_2 = \overline{s}. \]

Of course, there is another equilibrium where \( s_1 > s_2 \),

\[ s_2 = \underline{s}, \quad s_1 = \overline{s}. \]

Thus, we have maximal differentiation (even though increasing quality is costless for a producer). Note that firms sell different qualities not because of any intrinsic difference between them or in the cost of producing higher quality - but simply as a consequence of strategic market interaction.

More generally, maximal differentiation may not hold if (2) does not hold or if higher quality involves higher production cost.

Suppose (1) does not hold and in particular,

\[ \overline{\theta} < 2\theta \]

i.e., there is insufficient consumer heterogeneity. Then, one can check that in the price subgame, the low quality firm charges price equal to \( c \) and sells zero. Thus, the market becomes a natural monopoly with one firm selling and making positive profit in the market. If firm 1 sets quality too low, it cannot sell at price as low as \( c \), because even the consumer with parameter \( \underline{\theta} \) prefers to buy from firm 2; if it it increases quality, firm 2 simply reduces its price (more intense price competition) so that firm 1 still can’t sell. Thus, low consumer heterogeneity creates a natural barrier to entry (even though there are no fixed costs).

[This differs from the horizontal linear city model where, with zero fixed cost, a firm can always enter and earn strictly positive profit by locating away from other firms and selling to a neighborhood of its location at a price slightly above \( c \)].

Shaked and Sutton (1983): More general result. Allow unit cost of production to depend on quality: \( c(s) \). Further, suppose that consumer heterogeneity
is "small" in the sense that if all qualities are sold at price equal to unit cost of that quality, all consumers prefer to buy the highest quality. In that case, there exists some finite number $N$, such that at most $N$ firms can operate with positive market share in the market. This is referred to as the "natural oligopoly" result.

**Oligopoly: Product Variety.**

The "Circular City" Model (Salop, 1979).

Unit mass of consumers are distributed uniformly on the boundary of a circle with perimeter =1.

All travel occurs along the boundary of the circle.

Each consumer has unit demand and her net surplus from buying a unit from a firm located at distance $d$ at price $p$ is given by $S - td - p$. We assume $S$ is sufficiently large so that all consumers buy.

There is a very large number of firms waiting to enter this market. All firms are ex ante identical and produce at constant unit cost $c \geq 0$.

Entering firms are located on the boundary of the circle and we assume they choose their location (product type) so as to be maximally differentiated i.e., if $n$ firms enter the market, they are located symmetrically on the boundary of the circle and the distance between any two adjacent firms is $\frac{1}{n}$.

In this model, the volume of entry is also the number of distinct products on the market.

Question: How does the product variety in oligopoly compare to a socially optimal solution?

Socially optimal solution:

Social planner would serve all consumers & choose entry $n$ so as to:

$$\max_n [S - c - nf - t(2n\int_0^{\frac{\pi}{n}} xdx)]$$

i.e.,

$$\min_n [nf + \frac{t}{4n}]$$

and ignoring the integer problem, this yields:

$$n^* = \frac{1}{2} \sqrt{\frac{T}{f}}.$$ 

Two stage game:

Stage 1: Firms simultaneously decide whether or not to enter by incurring a fixed cost $f > 0$.

Stage 2: Entering firms choose prices simultaneously.

Consider stage 2 where $n > 1$ firms have entered the market.

Solve for a symmetric equilibrium of the price game. Suppose all firms $j \neq i$, choose identical price $p_j = p$, then firm $i$, when it charges a price $p_i$, effectively
faces competition from its two immediate neighbors. If all firms sell, then the indifferent consumer (between firm \( i \) and either of its immediate neighbor) is located at distance \( x \) from firm \( i \):

\[
p_i + tx = p + t\left(\frac{1}{n} - x\right)
\]

which yields

\[
x = \frac{p - p_i}{2t} + \frac{1}{2n}
\]

and the demand for firm \( i \):

\[
D_i(p_i, p) = 2x = \frac{p - p_i}{t} + \frac{1}{n}
\]

Profit (ignoring sunk cost) max:

\[
\max_{p_i}(p_i - c)\left(\frac{p - p_i}{t} + \frac{1}{n}\right)
\]

leads to FOC:

\[
\frac{1}{2}(p + c + \frac{t}{n}) = p_i
\]

and imposing symmetry \( p_i = p \), we obtain the symmetric NE price:

\[
\hat{p} = c + \frac{t}{n}
\]

which yields equilibrium profit:

\[
\frac{t}{n^2}.
\]

Now, consider the reduced form stage 1 game. The net profit to each entering firm if \( n \) firms enter is given by:

\[
\hat{\pi}(n) = \frac{t}{n^2} - f
\]

and so the equilibrium level of entry (ignoring integer problem) is given by the zero net profit condition which yields:

\[
\hat{n} = \sqrt{\frac{t}{f}}
\]

and equilibrium price: \( \hat{p} = c + \sqrt{tf} \).

Equilibrium product variety is increasing in \( t \) i.e., the strength of product differentiation in consumer taste. Also, observe that price >MC, even though firms earn zero net profit.

Finally, observe that

\[
\hat{n} = 2n^*
\]

oligopoly generates significantly higher product variety than is socially optimal.
**Monopolistic Competition:**

In the 1930s, Chamberlin and Robinson recognized that the model of perfect competition was not appropriate for markets with differentiated goods because each firm had some degree of market (or monopoly) power as it produced something that was somewhat different from what other firms produced. They thought of a model of imperfect competition where each firm faced a downward sloping demand curve (unlike a horizontal demand curve in the case of perfect competition) and chose its optimal (monopoly) price, taking that demand curve as given. The only interaction between firms entered through the fact that the demand curve faced by an individual firm depended on how many other firms (or products) were around in the market. As entry occurred, the demand curve faced by individual firms shifted downwards. However, unlike the oligopoly models we have nowadays, the demand faced by a firm did not depend on the prices or outputs or any other actions of other firms in the market. Each firm had a positive fixed cost of entering the market and upward sloping MC curve leading to a U-shaped AC curve. In a free entry equilibrium, each firm earned zero profit (i.e., the demand curve facing the firm was tangent to the AC curve at a point when the output is below its minimum efficient scale - the latter was interpreted as wasteful excess capacity created by imperfect competition) - and that there are too many firms (flawed reasoning).

In Industrial Organization, this model is no longer used. The Salop model discussed above is the closest we come to in terms of free entry differentiated good oligopoly. However, other fields such as trade theorists and macroeconomists find it convenient to work with this model. A particular formulation by Dixit and Stiglitz (1977) and Spence (1976) - is the modern version of monopolistic competition.

Representative consumer’s utility:

\[
U = U(q_0, \left(\sum_{i=1}^{n} q_i^\rho\right)^{\frac{1}{\rho}}), \rho < 1.
\]

Good zero (numeraire) is produced perfectly competitively at price equal to 1.

Budget constraint:

\[
q_0 + \sum_{i=1}^{n} p_i q_i = I
\]

FOC with respect to \(q_i\):

\[
U_1 p_i = U_2 \left(\sum_{j=1}^{n} q_j^\rho\right)^{\frac{1}{\rho} - 1} q_i^{\rho - 1}.
\]

If \(n\) is sufficiently large, \(q_i\) has little effect\(^1\) on \(\sum_{j=1}^{n} q_j^\rho\) and, therefore on \(U_1, U_2\) and in that case the demand for firm \(i\) producing good \(i\) can be approximated

\(^1\)This can be formally captured by assuming a continuum of goods or firms and utility
Firms produce at constant marginal cost $c$ and incur fixed cost $f$. Firm $i$'s optimal price:

$$p_i = \frac{c}{\rho}.$$ 

Lower $\rho$, lower the substitutability between products and higher the price. Zero profit condition determines equilibrium $n$. See section 7.5.2.

**Informational Differentiation.**

Products produced by two different firms that are identical in every physical attribute may still be differentiated in the eyes of a consumer because her information set about one product differs from that of the other. This is true even when the good is a search good. Of course, if the good is an experience or credence good, then full information cannot be directly acquired prior to consumption (unless the market conveys it through sorting or signaling). For this section, confine attention to search goods.

For search goods, differences in information include difference in awareness of the existence of products or firms, knowledge of the exact physical attributes of the products prior to actual inspection etc. Further, the information set may differ across consumers. Finally, informational differentiation may be caused by lack of information about prices charged by some firm and the need to indulge in costly price search to find out this information.

There are two principal instruments that can reduce consumer’s lack of information:

1. Informative Advertising.
2. Consumer Search.

Note that informative advertising is relevant mainly to search goods (otherwise, information in the advertisement may be verifiable prior to purchase and therefore, no better than cheap talk).

**Consumer Search:**

We will, like most of the literature, confine attention to price search.

Our first analysis is based on Diamond (1971). Consider a symmetric homogeneous good oligopoly where $n > 1$ firms compete in prices and produce at constant unit cost $c$. There is a unit mass of identical consumers and each consumer has unit demand with valuation $V > c$ for the good. If consumers learn all prices as soon as they are set, then we have a model of Bertrand competition with a unique NE where all firms charge price $= c$. Now let us suppose that consumers do not learn about prices. In particular, if a consumer wants to find out the prices charged by more than one firm, she incurs a strictly positive search cost $\mu > 0$ for each additional firm.

Function of the form

$$U = U(q_0, \int_0^n [(q(i))^{1/\rho}]^{1/\rho} di).$$

by the form:

$$q_i = kq_i^{1-\rho}, k > 0.$$
There is a unique equilibrium in this market where all firms charge the monopoly price $V$. Consumers expect all firms to charge $V$, therefore they see no point in incurring positive search cost and comparing prices - they randomly buy from a firm. A firm has no incentive to undercut because no consumer compares prices before buying and so price cutting is not going to attract more consumers. Thus, with positive search cost, no matter how small, the market attains the monopoly outcome.

This also illustrates how lack of information transmission (market friction) enables firms to make money & creates market power.

If consumers are heterogeneous in their search cost and some consumers are fully informed about prices or have very low search cost, then the market outcome can be rather different. For example, suppose that a fraction $\alpha \in (0, 1)$ of consumers have zero search cost or are fully informed about prices (e.g., internet savvy) while the rest have search cost $\mu > 0$ as described earlier. This immediately creates an incentive for firms to undercut prices to attract informed consumers and both firms charging monopoly price is no longer an equilibrium. Both firms charging the price $= c$ cannot be a NE, because in such an equilibrium, uninformed consumers would not search and randomly buy from a firm; so a firm can always deviate and earn $(V - c)(1 - \frac{\alpha}{2})$ by charging the monopoly price. In such situations, there is no pure strategy NE. The outcome is one where firms play mixed strategy and this is a major explanation of observed price dispersion.

**Informative Advertising:**

   All firms produce identical product at constant unit cost $c$.
   N consumers, each with unit demand & net surplus $S - p$ from buying.
   Consumers do not know the existence of a firm (brand) and its price.
   Assume only way a firm can inform consumers about its existence and price is by sending advertisements (hereafter, ads).

   Ads are not targeted; any ad sent out can reach each consumer with probability $\frac{1}{N}$.

   A consumer may receive many ads (even multiple ones from the same firm) or none.
   If a consumer receives no ads, she does not buy.
   If she receives ads from only one firm, she buys from that firm as long as the price quoted $\leq S$.
   If she receives ads from several firms, she buys from the firm with the lowest price as long as it does not exceed $S$.
   In case of tie, she randomizes evenly.

   Unit cost of sending an ad $= c'$.
   Assume: $c + c' < S$.
   Free entry of firms (no fixed cost).
   Continuum of firms: each firm is negligible with respect to the market, takes its "demand curve" as given & ignores effect on rivals of its own pricing.
No firm will send an ad with price \(< c + c' \) or \(> S \).

Equilibrium: Every price \(\in [c + c', S]\) is advertised by some firm and each such price yield exactly zero expected profit to the firm that advertises it. Ads with higher prices yield higher profit margin, if accepted by consumer, but have lower probability of acceptance.

Let \(x(p) =\)probability that a consumer receiving an ad with price \(p\) will buy at that price (i.e., receives no other ad with lower price). Of course, this will depend on the total number of ads sent out in the industry and the prices quoted on them. In equilibrium,

\[(p - c)x(p) - c' = 0, \forall p \in [c + c', S]\]

Observe that \(x(c + c') = 1\) (otherwise negative profit at that price).

The important conclusion here is price dispersion resulting from informative advertising. Butters also shows that the free entry equilibrium is socially optimal.

2 Oligopoly & Informative Advertising.

This is Tirole’s version of a model by Grossman & Shapiro (1984).

Consider the linear city model where unit mass of consumers are uniformly distributed on \([0,1]\).

Each consumer has unit demand and derives gross surplus \(S\) from the good. Each consumer incurs transport cost \(td\) of travelling distance \(d\).

There are two firms located at the extreme ends of the interval - firm 1 at 0 and firm 2 at 1.

As in the Butters model, consumers are uninformed about the existence of any firm or the price charged by it. Assume only way a firm can inform consumers about its existence and price is by sending advertisements (hereafter, ads). An ad contains information about location of a firm and its price (no misrepresentation). Ads are not targeted; any ad sent out can reach all consumers with equal probability. A consumer may receive many ads (even multiple ones from the same firm) or none. If a consumer receives no ads, she does not buy. If she receives ads from only one firm, she buys from that firm as long as the price quoted \(\leq S\). If she receives ads from several firms, she buys from the firm with the lowest price as long as it does not exceed \(S\). In case of tie, she randomizes evenly.

Let \(\Phi_i =\) proportion of consumers that receive an ad from firm \(i\); can be called "advertising intensity" of firm \(i\).

Cost of reaching fraction \(\Phi_i\) of consumers through advertising:

\[\frac{a}{2} \Phi_i^2, a > 0.\]

Note that maximum advertising expenditure is \(\frac{a}{2}\).

Assume:

\[a > \frac{t}{2}.\]
Game: Both firms simultaneously decide on their prices and advertising intensities \((p_1, \Phi_1, p_2, \Phi_2)\).

Proportion of consumers that only receive ad (or ads) from firm \(i\) : \(\Phi_i(1-\Phi_j)\).

These consumers buy from firm \(i\) or do not buy at all. This is the "captive" segment of the market for firm \(i\).

Proportion of consumers that only receive ads from both firms: \(\Phi_i\Phi_j\).

These consumers compare prices and location of the two firms. This is the competitive segment of the market.

Note that captive and competitive consumers are all distributed uniformly on the line.

In particular, given prices \(p_1, p_j\), let \(x\) denote the location such that a consumer from the competitive segment located at \(x\) is indifferent between the firms

\[ p_1 + tx = p_2 + t(1 - x) \]

i.e.,

\[ x = \frac{p_2 - p_1 + t}{2t} \]

Thus, the demand for firm \(i\), if both firms sell

\[ D_i = \Phi_i(1 - \Phi_j) + \Phi_i\Phi_j \left( \frac{p_j - p_i + t}{2t} \right) \]

Profit max for firm \(i\):

\[ \max_{p_i, \Phi_i}\{(\Phi_i(1 - \Phi_j) + \Phi_i\Phi_j \left( \frac{p_j - p_i + t}{2t} \right))\{p_i - c\} - \frac{a}{2} \Phi_i^2\} \]

Interior FOC:

\[ p_i = \frac{p_j + c + t}{2} + \frac{1 - \Phi_j}{\Phi_j} \]

\[ \Phi_i = \frac{1}{a}(p_i - c)[1 - \Phi_j + \Phi_j \left( \frac{p_j - p_i + t}{2t} \right)] \]

Note that the price reaction is more aggressive, the lower \(\Phi_j\) is. Indeed, the price reaction function is simply a markup over that in the original linear city model with fully informed consumers and the markup depends on the size of the captive segment.

Symmetric NE: \(p_1 = p_2 = \hat{p}, \Phi_1 = \Phi_2 = \hat{\Phi}\):

\[ \hat{p} = c + \sqrt{2at} \]

\[ \hat{\Phi} = \frac{2}{1 + \sqrt{\frac{2a}{t}}} \]

Equilibrium profit of each firm:

\[ \frac{2}{\left( \frac{1}{\sqrt{\pi}} + \sqrt{\frac{t}{2}} \right)^2} \]
Observe that: Under full information, in the linear city model with linear transport cost, the equilibrium price is \( c + t < \hat{p} \). This shows the effect of informational differentiation in softening competition. Even though firms have the option of reducing the model to one of full information through excessive advertising, they do not do so and the existence of captive segments reduces price elasticity of demand for each firm.

Also, note that lower the marginal advertising cost parameter \( a \), the more firms advertise.

Most interestingly, profits increase with increase in the marginal advertising cost parameter \( a \). This is the product of two effects. First, a direct effect which tends to increase advertising cost and thereby reduce profit. Second, a strategic effect, that come through reduction in advertising, higher informational differentiation - larger captive segments - less aggressive price reaction functions - and eventually, higher prices - that tend to increase profit. The strategic effect dominates the direct effect here.

This explains why industries may lobby for restrictions on advertising.

**Targeted Advertising (Direct Marketing).**

Roy (International Journal of Industrial Organization, 2000).

Homogenous good market.

Consumers located on \([0,1]\) interval (uniform distribution, unit demand, gross surplus \( S \))

Zero transport cost.

Location provides an address to which firms can target ads.

Symmetric duopoly with constant unit cost \( c \). Let \( c = 0 \).

Firms located at two ends of the interval.

Consumers unaware of existence of firms. However, once they are informed about existence of firm/product, they can discover the price with zero search cost.

Firms send advertisements that inform consumers about their existence.

Firms can target their ads to specific consumers.

Firm 1 decides on a segment \([0, \alpha]\) of consumers who are uninformed about its existence through ads; \( \alpha \in [0,1] \). Advertisement cost: \( \mu \alpha, \mu > 0 \).

Firm 1 decides on a segment \([1 - \beta, 1]\) of consumers who are uninformed about its existence through ads, \( \beta \in [0,1] \). Advertisement cost: \( \mu \beta, \mu > 0 \).

Two Stage Game:

Stage 1: Firms decide on their target sets of consumers for advertising i.e., decide on \( \alpha, \beta \in [0,1] \) simultaneously.

Stage 2: Firms set prices \( p_1, p_2 \).

Solve for SPE.

Stage 2.

If

\[ \alpha + \beta \leq 1 \]

there there is no overlap between the target sets of two firms and there is no competition (a local monopoly). NE profits are \( (S - c) \alpha \) and \( (S - c) \beta \) for firms 1 and 2, respectively.
So, consider a situation where 

$$\alpha + \beta > 1,$$

then firm $i$ has a captive segment of size $n_i$

$$n_1 = 1 - \beta, n_2 = 1 - \alpha$$

and the size of the competitive segment is

$$m = (\alpha + \beta) - 1 > 0.$$  

If $n_1 = n_2 = 0$, then we have a model of homogenous good symmetric Bertrand competition which leads to zero profit for both firms.

If $m = 0$, then we have a model of local monopoly and both firms set price $= S$.

So, confine attention to a situation where $n_i > 0$ for some $i$ and $m > 0$.

It is easy to check that there is no NE in pure strategies in the price subgame.

Each firm $i$ can guarantee itself at least $n_i S$ (gross profit) by charging a price $= S$.

Lowest price that will be charged by firm $i$ : $p_i$

$$p_i(n_i + m) = n_i S$$

i.e.,

$$p_i = \frac{n_i}{n_i + m} S.$$  

Note

$$n_i \geq n_j \Leftrightarrow p_i \geq p_j.$$  

The firm with smaller captive market is more aggressive in price undercutting.

Let $p$ be defined by

$$p = \max\{p_1, p_2\}$$

It can be shown that there is a unique mixed strategy equilibrium and the support of each firm’s price mixed strategy is $[p, S]$.

Further, if $n_i \geq n_j$, then

$$\text{Prob}\{p_j < S\} = 1.$$  

and therefore the equilibrium profit of firm $i$ is $n_i S$. In other words, in the price subgame, the firm with the larger captive segment always earns the same payoff as if it charged the monopoly price and sold only to the captive segment.

Also, it can be shown that if $n_i \leq n_j$,

$$\text{Prob}\{p_j > p\} = 1$$

so that firm $i$’s equilibrium profit is

$$p_i(n_i + m) = p_j(n_i + m) = \frac{n_j(n_i + m)}{n_j + m} S.$$
Now, consider the stage 1 reduced form game with payoffs (net profit):

\[ \pi_1(\alpha, \beta) = \begin{cases} 
(1 - \beta)S - \alpha \mu, & \text{if } \beta \leq \alpha, \alpha + \beta \geq 1, \\
\frac{(1 - \alpha)\alpha}{\beta}S - \alpha \mu, & \text{if } \beta \geq \alpha, \alpha + \beta \geq 1, \\
\alpha(S - \mu), & \text{if } \alpha + \beta < 1.
\end{cases} \]

and \( \pi_2(\beta, \alpha) \) defined in a symmetric fashion.

Let us look at the reaction function \( \alpha(\beta) \) of firm 1 in this reduced form game. It is easy to see that

\[ \alpha(\beta) \geq 1 - \beta, \forall \beta \in [0, 1], \]

for otherwise, there would be a segment of consumers that are not targeted by both firms. Next, observe that if \( \beta \leq \frac{1}{2} \), then it is optimal for firm 1 to not target consumers targeted by firm 2 and set \( \alpha = 1 - \beta \). This is because if firm 1 does set a value of \( \alpha > 1 - \beta \), then it creates a competitive segment of the market and since \( \beta \leq \frac{1}{2} \), this implies \( \alpha > 1 - \beta \geq \frac{1}{2} \geq \beta \) (firm 1 is the firm with the larger captive segment) in which case the net payoff of firm 1 is \( (1 - \beta)S - \alpha \mu < (1 - \beta)S - (1 - \beta)\mu \). Thus,

\[ \alpha(\beta) = 1 - \beta, \forall \beta \in [0, \frac{1}{2}]. \]

Indeed, using the above payoff function, one can verify that there exists \( \gamma \in (\frac{1}{2}, 1) \) defined by

\[ \gamma = \frac{S}{2S - \mu} \]

such that

\[ \alpha(\beta) = 1 - \beta, \forall \beta \in [0, \gamma]. \]

For \( \beta \in (\gamma, 1] \), it is optimal for firm 1 to intrude into the area covered by firm 2 i.e., set \( \alpha(\beta) > 1 - \beta \). It can be checked that

\[ \alpha(\beta) = \frac{S - \mu \beta}{2S} > 1 - \beta, \forall \beta \in (\gamma, 1]. \]

A symmetric reaction function for firm 2:

\[ \beta(\alpha) = \begin{cases} 
1 - \alpha, & \forall \beta \in [0, \gamma] \\
\frac{S - \mu \alpha}{2S} > 1 - \alpha, & \forall \alpha \in (\gamma, 1].
\end{cases} \]

Main result: The set of NE of the reduced form game is exactly equal to

\[ E = \{(\alpha, \beta) : \alpha, \beta \in [1 - \gamma, \gamma], \alpha + \beta = 1\}. \]

In every equilibrium, the market is perfectly segmented and the monopoly outcome obtains. This is related to the ability to target information precisely. As
the cost of advertising $\mu \downarrow 0, \gamma = \frac{8}{25-\mu} \downarrow \frac{1}{2}$ and the equilibrium set contracts to a singleton point $\{(\frac{1}{2}, \frac{1}{2})\}$.

Thus, unlike problems of "division of a pie" - not all divisions of the market are consistent with perfect segmentation. The equilibrium market shares cannot be too unequal - for otherwise, the firm with the small captive segment always has an incentive to intrude into its rival’s territory and be an aggressive price competitor; recognizing this, firms cede sufficient territory to their rivals.

**Entry & Exit.**

Why do supernormal profits persist in industries? Why doesn’t entry of new firms wipe out such profits?

These are some of the original questions that made the Harvard school of IO economists led by Joe Bain to suggest the existence of barriers to entry. In fact, Bain defined a barrier to entry as being something that allowed incumbents to enjoy strictly positive economic profits without threat of entry. This may include government regulation-permits, licenses, patents etc. - but we shall abstract from these in our study. Bain argued there are four major types of entry barriers:

1. **Economies of Scale** (fixed cost): If minimum efficient scale is large relative to market demand, there can only be few firms in the market (e.g., natural monopoly) & they may earn strictly positive profit without inviting entry.

2. **Absolute Cost Advantages**: of incumbents (superior technology, previous capital accumulation, firm specific learning etc.)

3. **Product Differentiation Advantages**: Incumbents may have patented product innovations & cornered important niches in the product space, dynamic investment in forming consumer loyalty....

4. **Capital requirement**: imperfect capital market, entrants may find it more difficult or costly to raise capital.

The Chicago school led by George Stigler approached entry barriers as simply being cost asymmetries between incumbent firms and outsiders (incumbents could charge price above average cost because outside firms had higher average cost curves).

Traditional model of entry barrier by incumbent firms:

Limit pricing model by Sylos-Labini, Modigliani etc.: Incumbent prices low enough so that outside firms cannot enter. This theory suffers from a credibility problem. Why would potential entrants believe that incumbents would hold on to those low prices if entry occurred? Not subgame perfect.

Modern version:


Milgrom - Roberts (1982): asymmetric information between incumbents and potential entrants, low prices signal private information about low profitability of potential entrants after entry.

*Scale Economies as Entry Barrier: Contestability.*

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The idea that scale economies act as entry barriers is best exemplified by the case of increasing returns to scale leading to a natural monopoly. If the average cost continually declines with output, no more than one firm can produce profitably in the industry (in a homogenous good market, if two firms sell output $q_1, q_2 > 0$ at price $p$ that allows them to earn non-negative profit, then one of the two firms can sell $q_1 + q_2$ at a price slightly lower than $p$ and since average cost will be significantly lowered, earn higher profit). Even if the AC curve is U-shaped and the minimum efficient scale is large, the number of firms that can produce profitably in the industry is small (natural oligopoly).

Baumol, Panzar and Willig (1982) argued that even though scale economies create entry barriers, the threat of being replaced by potential entrants may act as a disciplining device on current incumbents and restrict the degree of market power. They developed the concept of contestable markets to capture this idea.

Consider a homogenous good industry with $n$ symmetric firms each with cost function $C(q), C(0) = 0$. Note that this allows for fixed cost (that is not sunk) as long as $\lim_{q \to 0} C(q) > 0$. Of these $n$ firms, $i = 1, ..., m$ are incumbents and $i = m + 1, ..., n$ are potential entrants, $n > m$. Market demand is given by $D(p)$.

A sustainable industry configuration is a set of output $\{q_1, ..., q_m\}$ and a price $p$ charged by all firms such that:

(i) market clears (demand = supply)

(ii) incumbent firms make non-negative profits

(iii) there does not exist any price $p^c \leq p$ and output $q^c \leq D(p^c)$ such that $p^c q^c > C(q^c)$ (i.e., it is not possible for a potential entrant to make strictly positive profit by charging a lower price and selling some quantity).

A perfectly contestable market is one where every equilibrium generates a sustainable industry configuration.

Consider the case of natural monopoly with increasing returns to scale:

$$C(q) = cq + f, c > 0, f > 0 \text{ for } q > 0$$
$$= 0, \text{ for } q = 0.$$

Let $\pi^m = \max_q [(P(q) - c)q]$ denote the gross monopoly profit and assume $\pi^m > f$.

One can check that there is a unique sustainable industry configuration here where $m = 1$ and the incumbent charges the price $\bar{p} = P(\bar{q})$ where

$$\bar{p} = c + \frac{f}{\bar{q}}$$

This is identical to the solution obtained by average cost pricing regulation and is the constrained socially optimal solution when subsidies are not allowed. It is remarkable that this solution can emerge through market forces and threat of entry. Thus, regulation of industries with high scale economies may not be warranted.
The trouble is that for other demand and cost functions, there may not be a sustainable industry configuration in a natural monopoly so that a constrained efficient industry structure may not be sustainable against entry. For example: U-shaped average cost curve where the demand curve intersects the AC curve slightly to the right of minimum efficient scale.

How to think of the strategic foundation of the contestable outcome? Easy to see that the outcome in the natural monopoly case requires that incumbent does not alter price after he learns about the output decisions of other firms.

One game: firms choose prices first and then choose output/entry.
Problem: Prices usually adjust faster than output or entry decisions.

_Sunk Cost/Capacity as Barrier to Entry: Stackelberg-Spence-Dixit Model._

Firms enter market at different points of time - some enter early because of technological leadership - and can be thought of as incumbents. Others enter later. Early incumbents accumulate capacity and other forms of capital (including knowledge) over time that is "sunk" at any point of time. This allows firms to compete aggressively (for example, because marginal costs are low or production capacity is bigger). So, when capacity or capital accumulation is observed by potential entrants, the latter take this into account in their calculation of post-entry profitability. Early entrants can prevent entry by using their first mover advantage and engaging in significant capital accumulation.

Model:
Homogenous good market with demand \( D(p) = 1 - p \).
Production cost =0.
2 firms.
Firm 1: Incumbent
Firm 2: (Potential ) entrant.
Entrant incurs fixed entry cost \( f > 0 \) if it enters the market.
For the incumbent, the entry cost is sunk, does not affect the calculations and hence assumed to be zero.
Two stage game:
Stage 1: Firm 1 sets its capacity \( K_1 \).
Stage 2: Firm 2 (after observing firm 1’s choice) sets its own capacity \( K_2 \) (no entry equivalent to \( K_2 = 0 \))
After this both firms set a price that clears the market when they produce at full capacity viz., \( p = 1 - K_1 - K_2 \) (\( p = 0 \), if \( K_1 + K_2 \geq 1 \)).

Profits:
\[
\pi^1(K_1, K_2) = \begin{cases} 
K_1(1 - K_1 - K_2), & \text{if } K_1 + K_2 \leq 1 \\
0, & \text{if } K_1 + K_2 \geq 1 
\end{cases}
\]
\[
\pi^2(K_1, K_2) = \begin{cases} 
K_2(1 - K_1 - K_2) - f, & \text{if } K_2 > 0 \text{ and } K_1 + K_2 \leq 1 \\
-f, & \text{if } K_2 > 0 \text{ and } K_1 + K_2 \geq 1 \\
0, & \text{if } K_2 = 0.
\end{cases}
\]

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**Definition:** Entry is said to be blockaded if the entrant does not enter even though the incumbent’s action is identical to what would be optimal (for the incumbent) if there was no threat of entry. Entry is said to be deterred if the entrant does not enter even because the incumbent chooses an action that would be suboptimal (for the incumbent) if there was no threat of entry. Entry is said to be accommodated if entry occurs and the incumbent adjusts his behavior reconciling to entry.

**Blockaded entry:**
If there was no threat of entry, incumbent would choose $K_1$ at the monopoly output level i.e., solving
\[
\max_{K_1} K_1 (1 - K_1)
\]
which yields $K_1^m = \frac{1}{2}$. Given this capacity of firm 1, the maximum profit that firm 2 can make by entering is given by:
\[
\max_{K_2} K_2 \left( \frac{1}{2} - K_2 \right) - f
\]
\[
= \frac{1}{16} - f.
\]
Thus: entry is blockaded if $f \geq \frac{1}{16}$.

**Entry Deterrence:**
If $f < \frac{1}{16}$, entry will occur if the incumbent ignores the possibility of entry and sticks to monopoly capacity level. However, if it sets capacity at a sufficiently higher level, the entrant will find it unprofitable to enter. What is the critical level of incumbent’s capacity $K_1^b$ such that entrant is indifferent between entering and not entering:
\[
\max_{K_2} K_2 (1 - K_1^b - K_2) = f
\]
and this implies:
\[
K_1^b = 1 - 2\sqrt{f}.
\]
Note $f < \frac{1}{16}$ implies $K_1^b = 1 - 2\sqrt{f} > \frac{1}{2}$. Obviously, $K_1^b < 1$.

Consider stage 2 subgame for $f < \frac{1}{16}$. If the capacity set in stage 1 is $K_1 \geq K_1^b$, then no entry occurs (i.e., $K_2 = 0$).

If, on the other hand, $K_1 < K_1^b$, then entry occurs and firm 2 sets $K_2$ so as to:
\[
\max_{K_2} K_2 (1 - K_1 - K_2) - f
\]
which yields reaction function:
\[
K_2(K_1) = \frac{1 - K_1}{2}
\]
which yields the following profit for firm 1:
\[
K_1 \left( \frac{1 - K_1}{2} \right).
\]
Now, consider the reduced form game in stage 1. Firm 1’s reduced form payoff:
\[ \pi^1(K_1) = K_1 \left( 1 - \frac{K_1}{2} \right), K_1 < K^b_1 \]
\[ = K_1(1 - K_1), K_1 \geq K^b_1. \]

Observe that firm 1 will never set \( K_1 > K^b_1 \) because \( K^b_1 > \frac{1}{f} \) and
\[ \frac{\partial \pi^1}{\partial K_1} \bigg|_{K_1 > K^b_1} = 1 - 2K_1 < 0. \]
Therefore, the optimal capacity choice for firm 1 on \([K^b_1, 1]\) is \( K^b_1 \) yielding profit:
\[ K^b_1(1 - K^b_1) = 2\sqrt{f}(1 - 2\sqrt{f}) \quad (5) \]
This is the profit from deterring entry.

On \([0, K^b_1)\), the profit maximizing capacity of firm 1 is given by setting:
\[ \frac{\partial \pi^1}{\partial K_1} \bigg|_{K_1 < K^b_1} = 0 \]
which yields:
\[ K_1 = \frac{1}{2} \]
and profit:
\[ \frac{1}{8}. \quad (6) \]
This is the profit from accommodating entry.

One can check that entry deterring profit in (5) ≥ entry accommodating profit in (6) iff
\[ 2\sqrt{f}(1 - 2\sqrt{f}) \geq \frac{1}{8}, \]
Let \( f^* \) be defined by
\[ 2\sqrt{f}(1 - 2\sqrt{f}) = \frac{1}{8}, \]
It can be checked that \( f^* \in (0, \frac{1}{16}) \).

Thus: for \( f \leq f^* \), entry is accommodated.

For \( f \in (f^*, \frac{1}{16}) \), entry is deterred (and the incumbent holds large capacity equal to \( 1 - 2\sqrt{f} \)).

Note that when entry is deterred, the market appears to be a monopoly but market power is lower than in a monopoly. Potential entry restrains the exercise of market power.

Also, verify that when entry is accommodated, the subgame perfect equilibrium is \( K_1 = \frac{1}{2}, K_2 = \frac{1}{4} \) leading to profits \( \pi^1 = \frac{1}{8}, \pi^2 = \frac{1}{16} - f \). Even if \( f = 0 \), firm 1 has a first mover’s advantage.

The version of the above sequential game where \( f = 0 \) is called the Stackelberg game.