Risk Sharing, Risk Shifting and Optimality of Convertible Debt in Venture Capital

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Abstract

This paper adopts an optimal contracting framework to explain the predominant use of convertible debt in venture capital financing. We study a contracting problem between an entrepreneur and a venture capitalist where there is a tradeoff between optimal risk sharing and preventing the entrepreneur from increasing risk ex post (risk-shifting). The analysis extends the well known risk shifting result associated with debt financing to any arbitrary combination of debt and equity and shows that in a class with pure debt, pure equity, mixed debt-equity and convertible debt as special cases, convertible debt is optimal.

JEL CLASSIFICATION NUMBERS: D23, G24, G32

KEYWORDS: Convertible Securities, Venture Capital, Agency on Choice of Risk, Financial Contracting

1 Introduction

The venture capital industry in the U.S. has grown dramatically over the last two decades and has become the major channel of creating public companies. Many visible firms including high technology icons like Microsoft, Intel, Ciscosystems and Genentech all received venture capital funding in the early stages of their development. Financing a young entrepreneurial firm with a risky business plan is subject to important informational and incentive problems. Venture capitalists have developed unique contractual practices to overcome such problems. One such aspect quite specific to venture capital financing is the choice of the financial instrument. Rather than traditional instruments like debt or equity, venture capitalists almost exclusively hold a claim called 'convertible preferred security'. In a recent empirical study on venture capital contracting by Kaplan and Stromberg (2001), convertible securities account for over 90% of all financing agreements in their sample. Previous work by Sahlman (1990) and Gompers (1997) also report the extensive use of convertible securities in venture capital context.

Convertible preferred security is a hybrid claim that combines the properties of debt and equity. Preferred claim has a face value, it is redeemable and it has liquidation preference over other claims. This absolute priority makes the cash flow of the preferred analogous to a debt claim. The conversion option attached to the preferred claim gives the claimholder the right to convert the preferred into venture's equity. In that respect, convertible preferred security is analogous to a debt claim convertible into equity.

This paper offers a contract theoretic explanation for the extensive use of convertible securities in venture capital financing. The framework we provide focuses on risk sharing and incentive properties of convertible securities. We argue that the convertible security uniquely provides a cash flow structure that makes the entrepreneur bear the downside risk of the venture and at the same time prevents him from increasing risk ex post. Interestingly, no other combination of debt and equity can achieve this dual function. Our general framework also extends the classic risk shifting result that has been associated with debt security to arbitrary combinations of debt and equity.

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We describe a contract problem where a risk neutral entrepreneur finances his venture by funds provided by a risk averse venture capitalist. Upon receiving the required funds, the entrepreneur adopts a business strategy, which cannot be specified by the contract, and determines the riskiness of the venture. In that context, an optimal financial contract is a sharing rule over the stochastic payoff of the venture which must; (i) make the entrepreneur bear the downside risk of the venture, (ii) induce the entrepreneur to adopt the low risk strategy and prevent him from increasing risk ex post. There is a trade-off between risk sharing and preventing the ex post increase of risk (risk shifting). In the absence of any agency problem on choice of risk, a debt security is optimal and achieves optimal risk sharing. However, due to the well known risk shifting result, debt induces the entrepeneur ex post preference for high risk. We show that the risk shifting problem is not specific to debt but it also extends to any arbitrary combinations of debt and equity. In this trade-off, convertible debt emerges as the optimal sharing rule that achieves both risk sharing and no 'risk shifting' and it strictly dominates pure debt, pure equity and any arbitrary combination of debt and equity.

Previous explanations of convertible securities in venture capital context have mainly focused on the efficient allocation of control rights paradigm. [1]Berglof (1994) provides a model where control refers to the right to bargain with an outside party bidding for the venture and shows that convertible security allocates the control to the party who maximizes the joint surplus of the entrepreneur and the venture capitalist. Another control based explanation is Marx (1998) where a mixture of debt and equity dominates pure debt and pure equity in giving the venture capitalist the efficient liquidation incentives. Finally, Bascha and Waltz (2001) argue that convertible security implements the ex ante agreed optimal exit policy in a venture investment. The issue in that paper is the conflict of interest between going public through an initial public offering and accepting an acquisition bid from a third party. We acknowledge that control considerations are very important in venture capital

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[1]An exception is Cornelli and Yosha (1999). They argue that conversion into wquity option can be desirable because it prevents the entrpreneur from engaging in inefficient window dressing (short-termism) which does not contribute to the long term success of the venture.

investments. However, as Gompers (1997) and Hellman (1998) convincingly argue, allocation of cash flow rights and allocation of control rights can be separated by use of covenants and explicit contractual clauses. Indeed, Gompers (1997) documents the frequent use of covenants that give venture capitalist control rights[2]. We take the view that such control rights are somewhat independent from the design of financial instrument and the primary purpose of convertible security is more likely to be risk sharing and agency considerations, which is the focus of this paper.

In its focus on controlling ex post risk incentives, this paper is close to Green (1984) and Biais and Casamata (1999). However, both of these papers solely focus on agency on choice of risk and the same incentive effect they describe can be achieved by a combination of debt and equity. This is not the case in our optimal contracting framework where risk sharing is also a consideration and convertible security strictly dominates any such debt and equity combination. In his seminal work, Green (1984) takes on the classic risk shifting problem when a company issues risky debt. He shows that the ex post high risk incentives can be mitigated by combining the debt claim with warrants on company's equity. Risk sharing is not a concern in Green (1984) and he does not consider issuing equity as another financing alternative. In contrast, we allow for any combination of debt and equity, including pure equity. In that sense, ours is an optimal security design framework that complements these two papers.

The plan of the paper is as follows: In Section 2, we present the model. Section 3.1. describes the contracting problem and characterizes the optimal contract when there is no agency on choice of risk. Section 3.2. shows that there is a trade-off in risk sharing and preventing the entrepreneur from increasing risk ex post (risk shifting) and extends the risk shifting result to any debt-equity combination. Section 3.3. introduces the conversion feature and shows that in a setting with both risk sharing and risk shifting considerations present, convertible debt is the optimal financial instrument. Section 4 concludes. All proofs that are not presented in the text are in the Appendix.

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[2]Kaplan and Stromberg (2001) also report that venture capitalists hold seats in the board of directors and have explicitly defined control rights. In some cases, they even replace the founder entrepreneur with a professional outside manager. See also Hellman (1998).

2 The Model

There are three dates, $t = 0, 1, 2$. There is an entrepreneur (henceforth EN) who owns a venture idea. The venture requires a fixed investment of \$$I$ at date 0^3. EN has no wealth of his own and relies on a venture capitalist (VC) to provide the investment capital. At date 2, the venture generates a stochastic payoff $y \geq 0$. EN is risk neutral and maximizes his expected wealth. VC maximizes a strictly concave VNM utility function $v(.)$ defined over final wealth with $\lim_{y \downarrow 0} v'(y) = \infty$.

The riskiness of the venture's payoff depends on the action EN chooses at date 1. This action can be thought as the business strategy EN employs once he receives the required funds. For simplicity, there are two mutually exclusive strategies that can be employed, a_L and a_H. The *payoff distribution generated by action a_H is more risky than the one generated by a_L*. Formally, for $i \in \{H, L\}$, let $F_i(y)$ denote the distribution function from action a_i with a continuously differentiable density f_i. Following Rothschild and Stiglitz (1970), we employ the following standard definition of risk:

$$\int_0^\infty y f_H(y) dy = \int_0^\infty y f_L(y) dy \quad (\mathbf{A1})$$

$$\int_0^x F_H(y) dy > \int_0^x F_L(y) dy \text{ for all } x > 0 \quad (\mathbf{A2})$$

Assumptions $(\mathbf{A1})$ and $(\mathbf{A2})$ together imply that F_H is a mean preserving spread of F_L and the action a_H results in a more risky payoff distribution with more weight in the tails.
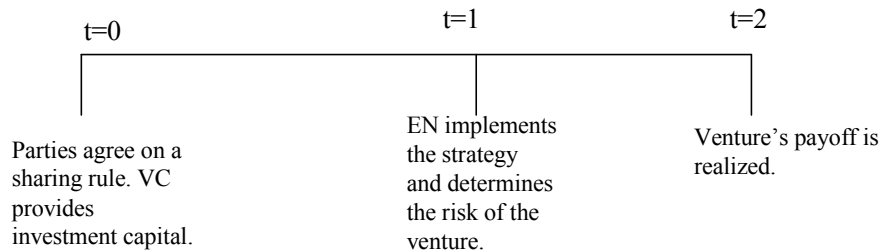
The action a_i is not contractible, i.e., parties cannot write an enforceable contract clause at date 0 that makes EN choose a particular action. EN chooses the action that maximizes his payoff given the agreed upon sharing rule at date 0. A sharing rule (a financial contract) is a function $s(y) : \mathbb{R}_+ \to \mathbb{R}_+$ which specifies the payment to VC for each payoff outcome y. If the venture's realized payoff is y, then VC receives

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[3] Typically, venture capitalists provide funding in multiple stages. Neher (1999) and Kockesen and Ozerturk (2002) provide a formal analysis of the staging decision. Admati and Pfleiderer (1994) and Bergeman and Hege (1998) analyze the optimal continuation decision when financing is staged. Gompers (1995) builds an empirical link between asset specificity and resulting agency problems and the number and duration of financing stages. In this paper, we do not focus on how financing is structured over time periods, but rather study the choice of the financial instrument.

$s(y)$ and EN as the residual claimant receives $y - s(y)$. We also assume that the contracts must exhibit limited liability so that

$$0 \leq s(y) \leq y \text{ for all } y. \tag{1}$$

For reader's convenience, we illustrate the sequence of events with the following time-line.

The basic trade off in the above specification is the interplay between *risk sharing* consideration under limited liability and controlling EN's incentives to increase risk (*risk shifting*) ex post. As we shall show in the next section, an optimal contract must induce EN the choice of low risk strategy and at the same time it has to insure the risk averse VC as much as possible against the uncertainty of the venture's payoff. Notice that this set-up is different from the standard principal-agent framework in two important ways. First, the agency problem analyzed is not related to effort incentives but it is related to risk incentives. Second, we assume that the party that designs the contract (the capitalist) is risk averse, whereas the agent (the entrepreneur) who takes the action is risk neutral. Before laying out the contract problem formally, we discuss the rationale behind these assumptions in the venture capital context.

VCs typically raise their capital through periodic funds. Their ability to raise new funds depends on the track record from the performance of the earlier funds managed. Sahlman (1990) accounts that if the fund suffers huge losses or even in cases of moderate failures, the chances for raising the next fund are odd. In the specific industries VCs invest, the failure rate is very high[4]. Chemmanur and Fulghieri (1999) argue that this high failure rate and the monetary/reputational consequences

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[4]Sahlman (1990) analyzes a sample of 383 venture capital investments. In that sample, about 35% of all projects yielded a net loss and another 50% were only moderately succesful.

of failure induce a great deal of risk aversion to VC's behaviour. On the other hand, we assume that EN is risk neutral. It may be suggested that EN is holding an undiversified portfolio, she also must be risk averse. Another point of view might be that ENs who quit their well paying jobs to pursue 'a once in a lifetime fortune' are risk lovers. Typically, what an EN loses from a failing start-up is considerably less than what a venture capitalist loses[5]. Therefore, at least at the low end of payoff realizations, EN is likely to be more risk tolerant than VC. Indeed, all the results of this paper continue to hold when VC is only more risk averse than EN at the very low payoff realizations (which is a quite natural assumption given that VC loses much more than EN in case of disasters). In that sense, our assumption of a risk averse VC and a risk neutral EN should be seen as a convenience that helps to characterize a clear benchmark case when risk sharing is the only consideration.

The agency problem we analyze involves the choice of the risk of the venture's operations. In the context of a start-up company operating in an innovative industry, the riskiness of venture's business plan is clearly an important concern. Among the many possible ways to increase risk in start-up environments, the most common ones are: rushing the product to the market although further testing is warranted, changing the scope of venture's operations and drifting into uncharted territory, insisting on a very ambitious design feature and thus increasing technical risk. Gompers (1997) argues that although VCs take an active role in advising and monitoring companies, they are not usually involved in the day-to-day activities and so they can not observe or verify changes in strategy. We incorporate this feature by assuming that the riskiness of the venture depends on EN's choice of business strategy and this strategy cannot be dictated by the venture capitalist by an enforceable contract. Rather, as we shall see, the choice of low risk strategy will be achieved by an appropriate design of the financial instrument.

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[5]The following remarks of the founder of Kozmo.com, an online company with a business plan to deliver movies, CDs, magazines, food, etc. within an hour, illustrates this point: 'Let's say I failed in 6 months after launching the company. I mean, I completely failed and lost the whole $28 million...So what? I will have an impressive resume to apply to a business school, having touched to this amount of money...failure might be a badge of success in this business (The New York Times Magazine, Nov. 1999). Obviously, this is not what a venture capitalist would want to say to her fund contributors.

In the next section, we analyze the contract problem.

3   Contract Problem

The contract is determined by some bargaining process that results in an efficient contract subject to participation contraints and any relevant self-selection constraints. Formally, if investment strategy a_i is adopted in equilibrium, the equilibrium contract is the solution to a programming problem in which the expected utility of VC,

$$V_i(s) \equiv \int_0^\infty v(s(y)) f_i(y) dy \tag{2}$$

is maximized subject to a "participation" contraint for EN,

$$U_i(s) \equiv \int_0^\infty (y - s(y)) f_i(y) dy \geq w > 0, \tag{3}$$

the limited liability constraint (1), and any required self-selection constraint. Note that the value w of this "participation" constraint is actually endogenous and reflects the solution to the bargaining problem. For our purposes, however, it is simply a device to characterize the equilibrium contract. Also note that we do not impose a self selection constraint right from the start in our formulation. We will actually show that an optimal contract has to satisfy a self selection constraint and has to induce EN the choice of low risk strategy. In what follows, as a benchmark case, we first characterize the optimal contract in the absence of any agency on choice of risk.

3.1   Benchmark Case

In this section, we assume that the strategy choice is enforceable. Therefore, there is no agency problem and the optimal contract is a simple risk sharing contract. We now show that regardless of the strategy to be adopted at date 1, the risk sharing consideration and the limited liability clause call for a pure debt contract.

Suppose VC and EN agree to adopt strategy a_i, $i \in \{H, L\}$. Then the equilibrium contract is a solution to the following problem:

$$\text{Choose } s_i(y) \text{ to maximize } V_i(s_i(y)) \equiv \int_0^\infty v(s_i(y)) f_i(y) dy$$

$$\text{subject to } U_i(s_i(y)) \equiv \int_0^\infty (y - s_i(y))\, f_i(y)dy \geq w > 0. \quad (4)$$

$$0 \leq s_i(y) \leq y. \quad (5)$$

The first order condition imply a $\lambda > 0$ such that

$$(v'(s_i(y)) - \lambda)\, f_i(y) \leq 0 \quad \text{if } s_i(y) \geq 0$$

$$(v'(s_i(y)) - \lambda)\, f_i(y) \geq 0 \quad \text{if } s_i(y) < y.$$

for all $y \in [0, \infty)$. From the strict concavity of v, it then follows that the optimal sharing rule s_i is a pure debt contract

$$s_i(y) = \begin{cases} y & \text{if } y < m_i \\ m_i & \text{if } y \geq m_i \end{cases} \quad (6)$$

with the repayment obligation m_i determined by the equation

$$v'(m_i) = \lambda \quad (7)$$

Notice that in the absence of the limited liability clause, the optimal contract would be a fixed payment for VC in all payoff realizations (a riskless debt claim) and this would achieve first best risk sharing with the risk neutral EN bearing all the risk. Due to limited liability, however, a fixed payment is not feasible at low payoff realizations. When this is the case, second best risk sharing calls for a risky debt claim where VC receives the whole payoff up to some m_i (when strategy a_i is adopted) and receives the fixed amount m_i for payoff realizations higher than m_i.

Now, we complete the analysis of the benchmark case by showing that VC would strictly prefer to adopt the low risk strategy, if it were enforceable. This follows from the strict concavity of VC's preferences and that the payoff distribution generated by the pure debt contract $s_L(y)$ second order stochastically dominates the one generated by $s_H(y)$. The following Proposition presents this result.

Proposition 1 *When there is no agency on choice of risk, the optimal contract is the pure debt contract $s_L(y)$ and the low risk strategy is adopted in equilibrium.*

Proof: See the Appendix.

The above analysis implies that, in the benchmark case with no agency on choice of risk, the optimal contract requires EN choose the low risk strategy. Moreover, the optimal contract is a pure debt contract that insures the risk averse VC at the downside to some extent (as much as the limited liability allows). Now, we turn to the case where strategy choice is not enforceable and analyze EN's incentives to adopt the low risk strategy under the above optimal risk sharing arrangement.

3.2 Agency on Choice of Risk

Suppose parties cannot write a contract that dictates EN to adopt a particular strategy at date 1. When this is the case, EN chooses the strategy that maximizes her expected payoff, given the sharing rule specified at date 0. It is a well known result in financial contracting literature that with risky debt outstanding, EN has a preference for high risk (See Jensen and Meckling (1976), Stiglitz and Weiss (1980), Green (1984)). Therefore, when VC holds a pure debt claim which is consistent with optimal risk sharing, EN will choose the high risk strategy. This follows because with risky debt outstanding, EN has a call option-like payoff which is convex and benefits from high risk. This is the well known 'risk shifting' problem for debtholders. In this section, we extend the risk shifting result to a broader class of sharing rules, which combines the debt claim with an equity claim. We show that no combination of debt and equity, except pure equity, can induce EN the choice of low risk strategy.

The hybrid class of contracts that combines debt and equity is defined as

$$s(y) \equiv Min(y, m + \pi(y - m)) \tag{8}$$

where $\pi \in [0, 1]$ is VC's equity share and m is the face value of her debt claim. Notice that pure debt and pure equity are special cases in this class: $m = 0$ and $\pi > 0$ corresponds to a pure equity claim, whereas $m > 0$ and $\pi = 0$ corresponds to a pure debt claim. Now, consider any such combination of debt and equity with $m > 0$ and $\pi \geqslant 0$. The following proposition is an extension of the well known 'risk shifting' result in this broader class, which allows for arbitrary combinations of debt and equity.
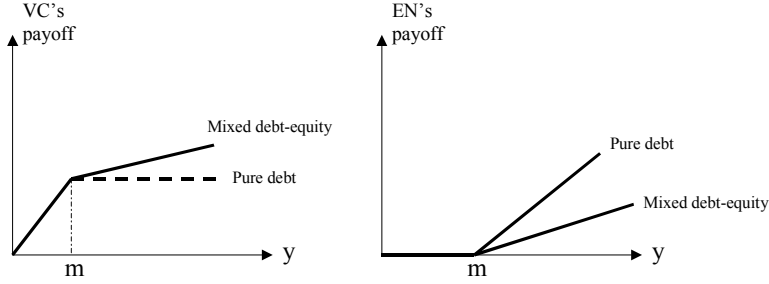
Proposition 2 *When VC holds any arbitrary mixed debt-equity claim $s(y) \equiv Min(y, m+$ $\pi(y-m))$ with $m > 0$ and $\pi \geqslant 0$, EN chooses the high risk strategy. Only with $m = 0$ (pure equity), EN is indifferent between high and low risk choices.*

Proof: Consider the difference between EN's expected payoff from choosing strategy a_H over a_L under any mixture of debt and equity. We now show that this difference is always positive for $m > 0$ and $\pi \geqslant 0$.

$$\begin{aligned}
U_H(s) - U_L(s) &= \int_0^\infty (y - s(y))\,(f_H(y) - f_L(y))\,dy \\
&= (1 - \pi) \left(\int_m^\infty (y - m)\,(f_H(y) - f_L(y))\,dy \right) \\
&= (1 - \pi) \left(\int_m^\infty (F_L(y) - F_H(y))\,dy \right) \quad (9)
\end{aligned}$$

where the last equality follows from integration by parts. But by $(A2)$, this last expression is always positive for any $m > 0$ and $\pi \in [0, 1)$. Therefore, EN strictly prefers to adopt the high risk strategy under any mixed debt-equity contract as defined above. Now consider the case when $m = 0$ and $\pi > 0$, which corresponds to pure equity. In this case, the expression in (9) for $U_H(s) - U_L(s)$ is zero and EN is indifferent between the two strategy choices. Q.E.D.

The intuition for this proposition is similar to the familiar risk shifting result for pure debt. Interestingly, even if VC shares part of the upside of the venture with some equity claim combined with debt, the risk shifting result still holds. This is because VC's claim structure is still concave with mixed debt-equity as it is with pure debt. Equivalently, EN's payoff is convex, has the shape of a call option and benefits from high risk. Choice of high risk strategy increases the likelihood of payoff realizations at the lower and upper tail. With limited liability, EN is protected at the downside and the debtholder (VC) bears the the whole cost of the higher default probability. This result shows that giving an equity claim to VC and making him share the upside of the venture does not eliminate the high risk incentives of EN when VC holds a debt claim that provides her some downside protection. The figure below illustrates the payoff structures for VC and EN under pure debt and mixed debt-equity contracts.

The analysis up to this point established that in the absence of agency on choice of risk, risk sharing considerations call for a pure debt contract and in equilibrium low risk strategy must be adopted (Proposition 1). However, we see from Proposition 2 that a pure debt contract *and* any arbitrary combination of debt and equity makes EN choose the high risk strategy. Only in case of pure equity, EN has no strict preference for high risk. The interesting point here is that debt is desirable for better risk sharing and equity is desirable for implementing the low risk choice; however simple mixtures of debt and equity cannot achieve the dual function of 'risk sharing' and 'no risk shifting'. The question is whether VC can still hold a debt claim to make EN bear the downside risk and also prevent him from increasing risk ex post. In the next section, we show that this is possible with convertible debt.
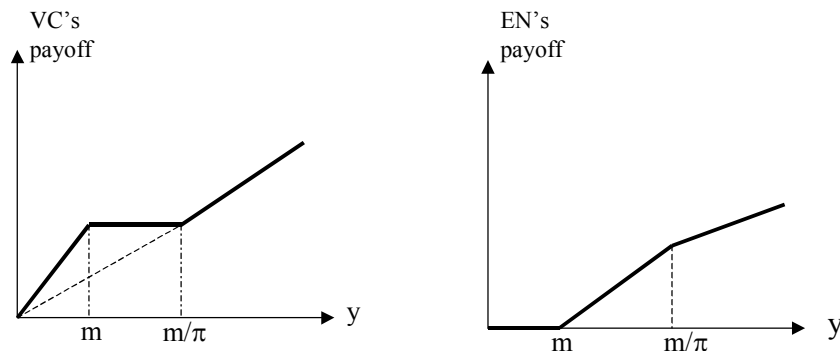
3.3 Convertible Debt

In this section, we introduce a *conversion into equity option* to the debt claim and analyze the optimality of this security in our setting. A debt claim with *a conversion option into equity* is defined as;

$$s_{m\pi}(y) = \max\left\{\min\left\{y, m\right\}, \pi y\right\} \tag{10}$$

This sharing rule specifies a payoff realization m below which VC receives everything. In that sense, m again is the face value of VC's debt claim. The conversion into equity option is specified by the share π of venture's equity. Upon realization of venture's payoff, the debtholder has the right to exchange her debt claim with π share of venture's equity. The conversion option is 'in the money' and thus is exercised for payoff realizations $y \geq m/\pi$ and consequently VC gives up her debt claim m for a

payoff πy. For payoff realizations $y < m/\pi$, the conversion option is not exercised and VC retains the debt claim. It is important to note that pure equity and pure debt are again special cases in this family: $m = 0$ corresponds to a pure equity contract, whereas $\pi = 0$ corresponds to a pure debt contract. The figure below illustrates VC's payoff schedule from a convertible debt claim.

The specification above is consistent with the actual venture capital contracting practice. VCs invest money in exchange for preferred claim that is convertible into venture's common stock (equity) under specified terms (Bartlett (1995), Smith (2001)). According to Gompers (1997), the preferred claim has a face value and liquidation preference (which entitles the claim absolute priority) over other claims. He argues that these features make the cash flow from a preferred claim analogous to a debt claim as long as it is not converted[6]. The face value of the preferred claim and the liquidation preference amount corresponds to m in our formulation. Conversion terms specify a conversion price which determines the equity share VC's preferred claim converts into. This is the equity share π. Conversion typically takes place following an initial public offering or an acquisition or merger, through which venture's equity attains market value and liquidity (Sahlman (1990), Gompers (1997), Kaplan and Stromberg (2001)). In our setting, this corresponds to realization of payoff y[7].

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[6]Kaplan and Stromberg (2001) report that 'Optional redemption and put provisions are commonly used to strengthen the liquidation rights of VC's claim. These provisions give the VC the right to demand that the firm redeems the VC's claim, typically at liquidation preference amount. *This is very similar to the right of repayment of principal at the maturity of a debt claim*' (page 22).

[7]Liquidity is of particular importance to VCs who have to pay back their own fund contributors (limited partners) within a finite time horizon. If the company remains private rather than going public, VCs seem to face large illiquidity costs for their claims. Hellman (2000) examines the liquidity benefit of going public for venture capitalists.

We now characterize the optimal contract in this two parameter family. First, we set up the programming problem. The VC's expected utility from a sharing rule $s_{m\pi}$ is given by;

$$V_L(s_{m\pi}) = \int_0^m v(y)f_L dy + \int_m^{\frac{m}{\pi}} v(m)f_L dy + \int_{\frac{m}{\pi}}^\infty v(\pi y)f_L dy \qquad (11)$$

The participation constraint for EN takes the following form:

$$U_L(s_{m\pi}) = \int_m^{\frac{m}{\pi}} (y - m)\, f_L dy + \int_{\frac{m}{\pi}}^\infty (1 - \pi)y f_L dy \geq w \qquad (12)$$

Finally, define $W(s_{m\pi}) \equiv U_L(s_{m\pi}) - U_H(s_{m\pi})$. Then the self selection constraint requires;

$$W(s_{m\pi}) \equiv \int_m^{\frac{m}{\pi}} (y - m)(f_L - f_H)dy + \int_{\frac{m}{\pi}}^\infty (1 - \pi)y(f_L - f_H)dy \geq 0 \qquad (13)$$

i.e., the equilibrium contract implements the low risk strategy. The optimal contract problem is to choose $m \geq 0$ and $\pi \in [0, 1)$ to maximize $V_L(s_{m\pi})$ subject to (11) and (12). The proposition below formalizes that in our setting, equilibrium contract is a convertible debt contract. In other words, when both risk sharing and risk shifting (agency on choice of risk) are important, convertible debt strictly outperforms pure debt, pure equity and any other arbitrary combination of debt and equity.

Proposition 3 *The equilibrium contract requires $m > 0$ and $\pi > 0$, i.e., it is a convertible debt contract.*

Proof: See the Appendix

How does convertible debt differ from an arbitrary mixture of debt and equity in implementing the low risk choice? The difference is that now VC holds a claim (the conversion into equity option) that benefits from high risk. This is because choice of high risk increases the possibility that the option will be exercised. An option becomes more valuable as the riskiness of the underlying asset (in our case the payoff of the venture) increases. In contrast, with mixed debt-equity, VC merely shares the

upside of the venture, however she does not hold a claim that makes it costly for EN to increase risk ex post.

The idea behind risk shifting is that EN only benefits (but not loses) from increasing risk, while the debtholder (VC) suffers alone from the higher default probability due to high risk. With the conversion option, the loss of value for the debt claim at the downside due to high risk is offset by the gain in the upside due to the higher exercise possibility of the option. The same logic does not apply to arbitrary combinations of debt and equity, since under these sharing rules EN continues to hold a 'call option' on venture's equity, which is a convex claim. VC's conversion into equity option, on the other hand, transforms this convex shape by creating a concave payoff structure for EN (and a convex one for the VC) at the upside of the payoff realizations (see the figure). With convertible debt, EN now has a trade-off in terms of her ex post preference for high risk. He is still *long* a call option on venture's equity because of VC's debt claim at the downside. However, he is also *short* a call option which will be exercised if venture's payoff is higher than m/π. In that respect, m/π is equivalent to the strike price of the option. The binding self selection constraint requires that in equilibrium EN is indifferent between the two risk choices. This is the case when the pair (m, π) is chosen such that the value of the call option EN is long is offset by the value of the call option he is short[8].

We illustrate this intuition with the following simple example:

Example: Suppose if EN chooses a_L, the payoff of the venture can be 0, 1 or 2 with an equal probability of 1/3. If he chooses a_H, the payoff can be either 0 or 2 with an equal probability of 1/2. Now consider a case where VC holds a mixed debt-equity claim with $m = 0.5$ and some equity share $\pi \in [0, 1)$. Clearly, for any such π, EN always chooses the high risk action: a_H yields an expected payoff of $\frac{3}{4}(1 - \pi)$ whereas low risk strategy a_L gives an expected payoff of $\frac{2}{3}(1 - \pi)$. This follows because with high risk, the default probability is higher and EN increases risk at the expense of VC. Now, consider the following convertible debt claim: If payoff is 1, VC retains the

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[8]In options literature, the combination of a long call position and a short call position with a higher strike price is known as a 'bull spread'.

debt claim $m = 0.5$. However, if payoff turns out to be 2, then she converts it into $\pi = 0.5$ share of venture's equity. In this case, EN's expected payoff from choice of low risk is

$$\frac{1}{3}(1 - 0.5) + \frac{1}{3}(1 - 0.5)2 = 0.5$$

and his expected payoff from choosing high risk strategy a_H is also

$$(1 - 0.5)\frac{1}{2}2 = 0.5$$

Therefore, conversion option completely offsets EN's preference for high risk. This follows because with high risk, the default probability is higher, therefore the fixed debt claim m is less valuable. But at the same time, it is also more likely that this debt claim will be converted into equity offsetting this loss at the downside.

As noted earlier, in Green (1984), warrants on company's equity play essentially the same role as the conversion into equity option here in correcting the high risk incentives. However, in Green (1984), a pure equity contract would also serve the same purpose and implement the correct risk choice, but he does not consider this possibility. He conjectures that moral hazard or taxes would preclude equity, although he does not explicitly model them. In contrast, in an optimal contracting setting, we allow for equity and any combination of debt and equity. Furthermore, we simultaneously consider optimal risk sharing and agency on choice of risk and derive the superiority of convertible debt within this tradeoff. The above analysis, in that respect, complements Green (1984) with its focus on optimal financial security design. Our focus on risk sharing also formalizes the point made by Sahlman (1990). In his comprehensive survey on actual venture capital practice, he argues; 'A key feature of contracts and operating procedures is that risk is shifted from the venture capitalists to the entrepreneur...the convertible security shifts some of the risk of poor performance to the entrepreneurial team. Given the liquidation preference embodied in the security, the venture capitalists will be entitled to a larger share of total value, if total value is low (page 510)'.

4 Conclusion

In this paper, we provide a moral hazard-risk sharing based explanation for the use of convertible debt in venture capital financing agreements. Unlike previous work, which are based on allocation of control rights arguments, we focus on cash flow allocation feature of convertible securities. Our analysis suggests that the debt claim arises due to risk sharing considerations and provides downside protection for the venture capitalist. The conversion into equity option attached to the debt claim, on the other hand, prevents EN from increasing risk ex post. Simple combinations of debt and equity are not effective in our setting, where the optimal contract has to achieve risk sharing and 'no risk shifting' simultaneously. In a fairly simple principal-agent framework, convertible security strictly dominates any arbitrary combination of debt and equity.

Although, the analysis is motivated by the empirical observations that report the widespread use of convertible debt in venture capital, some aspects of our optimal contracting framework has general implications for financial contracting literature. Notice that the contract problem we describe is applicable to many situations where the investor (principal) demands prorection against the downside risk of her investment and the borrower (agent) has access to an action that determines the risk of the investment. In this simple setting, we generalize the well known risk shifting problem associated with pure debt to any arbitrary combination of debt and equity and establish the superiority of convertible debt.

In order to focus on optimal design of financial instrument, we abstracted from another contractual practice unique to venture capital financing, namely staged financing. Venture capitalists commit only a portion of the required capital up-front and tie the continuation funds to the progress of the venture. One clear motivation for staged financing is learning over the prospects of the venture. It would be interesting to extend the optimal security design framework of this paper to multiple investment rounds and analyze how the terms of each financing round (the face value of debt claim and the conversion price (the equity share debt converts into) is determined by arrival of new information on venture's progress. We leave this to future research.

5 Appendix

Proof of Proposition 1:

First we establish that the repayment obligation m is higher when the high risk strategy is adopted.

Lemma 4 *The repayment obligation m is higher when the risky investmet strategy is adopted, i.e. $m_H > m_L$.*

Proof: Assumption A1 and participation constraint imply that the expected payoff to VC is the same under both contracts Therefore, we have

$$\int_0^{m_L} y f_L(y) dy + m_L(1 - F_L(m_L)) = \int_0^{m_H} y f_H(y) dy + m_H(1 - F_H(m_H))$$

Integrating by parts yields;

$$m_L - \int_0^{m_L} F_L(y) dy = m_H - \int_0^{m_H} F_H(y) dy \tag{14}$$

which, by A2, implies $m_H > m_L$.

We now show that s_L second order stochastically dominates s_H. The desired result then follows from the strict concavity of v.

Since $m_L < m_H$ by the above Lemma, we may write equation (14) as,

$$\int_0^{m_L} \left(F_H(y) - F_L(y) \right) dy = \int_{m_L}^{m_H} \left(1 - F_H(y) \right) dy \tag{15}$$

Let $\hat{F}_i$ denote the distribution function for s_i. Then, $\hat{F}_i(y) = F_i(y)$ for $y < m_i$ and $\hat{F}_i(y) = 1$ for $y \geq m_i$. Then Assumption A2 plus equation (14) implies

$$\int_0^x \left(\hat{F}_L(y) - \hat{F}_H(y) \right) dy < 0$$

for $x < m_H$ and

$$\int_0^x \left(\hat{F}_L(y) - \hat{F}_H(y) \right) dy = 0$$

for $x \geq m_H$, which establishes that s_L second order stochastically dominates s_H.

Proof of Proposition 3:

The preceding analysis already established that $\pi = 0$ cannot be optimal, since it violates the self selection constraint. (A pure debt contract implements the high risk strategy.) Therefore all we need to show is $m > 0$.

Define m^* by $U(m^*, 0) = w$. Then since U is strictly decreasing in m and π, we can define a function $\pi : [0, m^*] \to [0, 1]$ by $U(m, \pi(m)) \equiv w$. Formally, the implicit function $\pi(m)$ satisfies

$$U(m, \pi(m)) \equiv \int_m^{\frac{m}{\pi(m)}} (y - m) \, f_L dy + \int_{\frac{m}{\pi(m)}}^{\infty} (1 - \pi(m)) \, y f_L dy = w \qquad (16)$$

Implicit differentiation then yields:

$$\pi'(m) = - \frac{\displaystyle\int_m^{\frac{m}{\pi(m)}} f_L dy}{\displaystyle\int_{\frac{m}{\pi(m)}}^{\infty} y f_L dy} < 0. \qquad (17)$$

which simply says that for a given reservation payoff w for EN, increasing VC's debt claim implies decreasing her equity claim so that participation constraint continues to hold as an equality.

Note that a pure equity contract also satisfies the self selection constraint, by $A1$, i.e. $W(0, \pi) = 0$. To show that the solution to the above problem requires $m > 0$, it is sufficient to show that for $m > 0$ sufficiently small, we have:

$$\frac{dV(m, \pi(m))}{dm} > 0 \text{ and } \frac{dW(m, \pi(m))}{dm} \geq 0. \qquad (18)$$

In other words, we start with $m = 0$ and then try to find a sufficiently small $m > 0$ that makes the VC better off due to better risk sharing and yet still satisfying the self selection constraint. Equity share $\pi(.)$ defined by the binding participation constraint (16) adjusts accordingly. Now,

$$\begin{aligned} \frac{dV(m, \pi(m))}{dm} &= V_m + V_\pi \frac{d\pi}{dm} \\ &= \int_m^{\frac{m}{\pi}} v'(m) f_L dy - \int_{\frac{m}{\pi}}^{\infty} y v'(\pi y) f_L dy \left(\frac{\displaystyle\int_m^{\frac{m}{\pi}} f_L dy}{\displaystyle\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \right) \end{aligned} \qquad (19)$$

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$$= \int_m^{\frac{m}{\pi}} f_L dy \left(1 - \frac{\int_{\frac{m}{\pi}}^{\infty} \frac{v'(\pi y)}{v'(m)} y f_L dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \right) \tag{20}$$

$$> \int_m^{\frac{m}{\pi}} f_L dy \left(1 - \frac{\int_{\frac{m}{\pi}}^{\infty} y f_L dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \right) = 0 \tag{21}$$

where (19) follows from direct substitution of $\pi'(m)$ from (17). Equation (20) is obtained by rearranging terms. The inequality in (21) follows from the fact that $v'(m)$ converges to a very large number for m sufficiently small and thus $\dfrac{v'(\pi y)}{v'(m)}$ approaches to zero. This completes the proof that $\dfrac{dV(m, \pi(m))}{dm} > 0$.

Similarly, straightforward algebra yields;

$$\frac{dW(m, \pi(m))}{dm} = W_m + W_\pi \frac{d\pi}{dm}$$

$$= -\int_m^{\frac{m}{\pi}} (f_L - f_H)\, dy + \int_{\frac{m}{\pi}}^{\infty} y\, (f_L - f_H)\, dy \left(\frac{\int_m^{\frac{m}{\pi}} f_L dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \right)$$

$$= -\int_m^{\frac{m}{\pi}} f_L dy + \int_m^{\frac{m}{\pi}} f_H dy + \frac{\int_{\frac{m}{\pi}}^{\infty} y\, (f_L - f_H)\, dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \left(\int_m^{\frac{m}{\pi}} f_L dy \right)$$

$$= \int_m^{\frac{m}{\pi}} f_H dy + \int_m^{\frac{m}{\pi}} f_L dy \left[\frac{\int_{\frac{m}{\pi}}^{\infty} y\, (f_L - f_H)\, dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} - 1 \right]$$

$$= \int_m^{\frac{m}{\pi}} f_H dy - \int_m^{\frac{m}{\pi}} f_L dy \left[\frac{\int_{\frac{m}{\pi}}^{\infty} y f_H dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \right] \tag{22}$$

Now, by Assumption $A2$, we know that $\dfrac{\int_{\frac{m}{\pi}}^{\infty} y f_H dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} < 1$ for any $m > 0$. Therefore;

$$\frac{dW(m, \pi(m))}{dm} \equiv \int_{m}^{\frac{m}{\pi}} f_H dy - \left[\frac{\int_{\frac{m}{\pi}}^{\infty} y f_H dy}{\int_{\frac{m}{\pi}}^{\infty} y f_L dy} \right] \int_{m}^{\frac{m}{\pi}} f_L dy > \int_{m}^{\frac{m}{\pi}} (f_H - f_L) dy > 0$$

for sufficiently small m. Note that we use the fact that for sufficiently small m, $(A2)$ implies, $f_H(y) > f_L(y)$ for all $\dfrac{m}{\pi} < y < m$. This completes the proof that the equilibrium contract requires $m > 0$ and $\pi > 0$, i.e. it is a convertible debt contract.

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