Costly Managerial Hedging, Market Liquidity and Firm Value

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Abstract

Empirical evidence suggests that managers hedge the systematic risk in their compensation by trading in the financial markets. This paper analyzes the implications of the manager’s hedging ability on her optimal compensation scheme, incentives and firm value. We allow the manager to adjust her systematic risk exposure by trading the market portfolio. We find that (i) the manager’s optimal hedge depends on the liquidity of the market. With imperfect liquidity, the manager’s optimal hedge is not complete and she bears some systematic risk. (ii) The equilibrium pay-performance sensitivity is decreasing in the company’s beta and increasing in the market liquidity. (iii) Since better hedging ability increases the manager’s equilibrium incentives, the firm value increases in the liquidity of the market where systematic risk is traded. This last result contrasts with previous studies that suggest a negative relationship between stock market liquidity and production efficiency.

JEL Classification: G30, G32

Keywords: Pay-Performance Sensitivity, Hedging, Managerial Compensation, Liquidity, Systematic Risk

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1 Introduction

One central theme in the corporate finance literature is to align manager-shareholder interests by tying the managerial compensation to firm performance. This principal-agent model of executive compensation has illustrated a trade-off between providing incentives to the manager and optimal risk sharing. Tying the manager’s compensation to firm performance increases her incentives to maximize firm value, but such schemes also expose the manager to some uncertainty over which she has no control. Therefore, the theory predicts, executive compensation in riskier firms should be less sensitive to firm performance.¹

An implicit assumption in the literature on risk and incentives is that it is prohibitively costly for managers to trade in the financial markets to privately alter their risk exposure.² Restrictions that prevent the managers to trade in their own firms are commonplace. However, general transactions by managers in the stock market are not commonly restricted. Recent evidence suggests that the managers do use the financial markets: An article in The Economist reports that the use of derivatives to hedge managerial exposure to firm risk has become a business of hundred millions of dollars.³ Bettis et al. (2000) document executives’ increasing use of zero-cost collars and equity swaps for hedging purposes. By using such instruments or simply by trading in a market index, managers seem to be able to adjust the risk in their compensation.

This paper analyzes the implications of the manager’s hedging ability on

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¹Aggarwal and Sanwick (1999), Haubrich (1994) and Garen (1994) find supporting empirical evidence for this prediction. For a very recent critique of the central prediction of the principal-agent theory, see Prendergast (2000). Raith (2001) also challenges the trade-off between risk and incentives in a model of product competition.

²Amihud and Lev (1981) is an early exception that addresses the manager’s risk reduction motives.

her optimal compensation scheme, her incentives and the firm value. To this end, we decompose the total risk into its systematic and firm specific components. We allow the manager to trade in the financial markets (but not in the stock of her own company) to adjust the systematic risk exposure in her compensation. Our manager trades as a rational but uninformed hedger in a Kyle type market setting. This market microstructure framework helps us to endogenize the manager’s diversification costs. In particular, it allows us to relate the manager’s optimal hedging behavior to the liquidity of the market where systematic risk is traded. We show that due to imperfect liquidity, hedging the systematic risk is costly and hence the manager’s optimal hedge is only partial: She does not diversify away all of the systematic risk exposure in her compensation. This result is contrast to a recent work by Jin (2002) where hedging is costless and the manager hedges completely. Our manager hedges more the more liquid is the market. The more systematic risk she hedges, the better incentives she can be given. Accordingly, the optimal pay-performance sensitivity of her compensation is also increasing in the liquidity of the market. Therefore, we build a previously unexplored ‘diversification link’ between market liquidity, managerial incentives and the firm value. Equilibrium incentives and hence the firm value are increasing in the liquidity of the market portfolio (or the systematic risk security).

Three very recent papers also address the managers’ ability to hedge the systematic risk exposure in their compensation (see Acharya and Bisin (2002), Garvey and Milbourn (2002) and Jin (2002)). A common theme of these papers is that the optimal compensation contract should take into account the manager’s ability to diversify away the systematic risk (Garvey and Milbourn (2002) and Jin (2002)) or substitute between systematic and firm-specific risk factors (Acharya and Bisin (2002)). The innovation of our paper is to explicitly link the manager’s optimal hedging behavior to market liquidity and hence derive a novel link between the financial market
microstructure, incentives and the firm value.

In Jin (2002), hedging is costless, therefore the manager diversifies completely. However, he acknowledges that, ‘Although in theory CEOs could fully adjust their market risk exposure through trading, in reality constraints might be placed on doing so.’ In our setting, the hedging costs for the manager arise due to imperfect liquidity and the consequent price risk, which are absent in Jin (2002). Garvey and Milbourn (2002) capture diversification costs by an exogenously specified cost function. They motivate this diversification cost by referring to short selling and wealth constraints for the manager. Their conclusion is that market (systematic) risk is an important determinant of the pay-performance sensitivity for younger managers, since it is the young managers who face diversification costs. Our analysis makes a complementary point by linking diversification cost to market liquidity. If the hedge market has low liquidity, the manager can only partially diversify the systematic exposure in her compensation. Therefore, the liquidity of the systematic risk security is an important determinant of the pay-performance sensitivity and equilibrium incentives.

The theoretical research investigating the link between market liquidity and production efficiency has produced mixed results. On one side, researchers have emphasized that liquidity by definition requires dispersed and temporary ownership of the company’s stock and hence higher liquidity implies less incentives to monitor the management. In other words, this literature asserts a trade-off between liquidity and control. Bhide (1993) and Admati, Pfleiderer and Perry (1994) show how liquid markets undermine effective corporate governance by providing investors with an easy exit. Other theoretical models arrive at the opposite conclusion: liquidity can reduce agency problems. Kahn and Winston (1998) and Maug (1998) show that liquidity reduces the costs that an investor bears in taking a large position to influence management. Holmstrom and Tirole (1993) show how liquidity can
improve incentive contracts by increasing the information content of stock prices.

Our main insight that relates market liquidity to managerial incentives is complementary to Holmstrom and Tirole (1993). They illustrate that stock market provides a monitoring role. In particular, they show that market liquidity improves performance evaluation and hence incentives. In their framework, the effect of market liquidity is to provide incentives to a speculator to gather more precise information and trade more aggressively. This in turn makes the company’s stock price more informative about managerial actions and improves performance evaluation. There are two fundamental differences between this paper and Holmstrom and Tirole (1993). In our analysis, the market liquidity refers to the liquidity of the market where the systematic risk factor (not the company’s stock but the market portfolio) is traded. In that sense, we refer to liquidity of the stock market in general. Another important distinction is that in our setting the manager can trade in the financial market to adjust the systematic risk exposure in her compensation. Increased market liquidity, then, improves the manager’s ability to diversify away the systematic risk. Therefore, liquidity affects the incentives and the firm value through a diversification channel, as opposed to the performance monitoring channel of Holmstrom and Tirole.

The paper is organized as follows. The next section formally describes the model and lays out the contract problem between the shareholder and the manager. Section 3.1 solves the equilibrium of the trading game and characterizes the manager’s optimal hedging decision. Section 3.2 characterizes the equilibrium effort choice of the manager. Section 4 describes the optimal linear incentive contract and derives implications of the manager’s hedging ability on her optimal incentive scheme. Section 4.1 relates market liquidity to firm value. Section 5 considers the implications of a competitive hedge market. Section 6 concludes.
2 The Model

We employ a standard principal-agent setting to analyze the trade-off between giving the manager incentives to maximize the firm value and making her bear risk. The basic ingredients of the model are as follows:

Technology: An agent (the manager) runs a firm owned by a principal (the shareholder). The principal is risk neutral and maximizes the final firm value net of the manager’s compensation. The manager has exponential preferences with a constant absolute risk aversion coefficient \( a > 0 \). The final value of the firm, \( \tilde{X} \), is determined by the following stochastic technology;

\[
\tilde{X} = e + \tilde{\omega}
\]

where \( e \) is the costly and unobservable effort expended by the manager and \( \tilde{\omega} \) is the stochastic component over which the manager has no control. For tractability, we assume that the manager’s cost of effort is given by \( c(e) = ke^2/2 \) with \( k > 0 \), a constant.

Manager’s Compensation: Drawing on the optimality results in Holmstrom and Milgrom (1987), we restrict attention to linear compensation contracts. In particular, the manager’s compensation contract is described by a pair \((F, s)\), where \( F \) is a fixed payment and \( s \) is the manager’s share from final the firm value. Accordingly, the manager’s compensation is given by;

\[
F + s\tilde{X}
\]

In what follows, we refer to \( s \) as the pay-performance sensitivity of the manager’s reward scheme.

Risk Factors: We assume that \( \tilde{\omega} \) can be decomposed into two orthogonal risk factors;

\[
\tilde{\omega} = \beta(\tilde{v} - \bar{v}) + \tilde{e}
\]
where \( \tilde{v} \sim N(\bar{v}, \Sigma) \) is an aggregate (systematic) risk factor and \( \tilde{\varepsilon} \sim N(0, \eta) \) is a firm-specific (nonsystematic) risk factor with \( E(\tilde{v}\tilde{\varepsilon}) = 0 \) (for specifications along the same lines see also Acharya and Bisin (2002), Garvey and Milbourn (2002) and Jin (2002)). Furthermore, \( \beta \), the firm’s beta, is defined as

\[
\beta \equiv \frac{Cov(\tilde{X}, \tilde{v})}{Var(\tilde{v})}.
\]

The motivation behind decomposing the total risk into systematic and nonsystematic (firm specific) components is to investigate whether they affect the manager’s equilibrium incentives differently. This possibility is even the more relevant when one recognizes that the manager might be able to trade a market portfolio to adjust her exposure to systematic risk.

**Trading in Financial Markets:** We depart from the standard principal-agent setting and allow the manager to trade in the financial markets to hedge the systematic risk \( \tilde{v} \) in her compensation. The trading of the systematic risk factor \( \tilde{v} \) can simply be thought of trading the market portfolio. On the other hand, we assume that the manager can not trade a security to hedge the firm-specific risk \( \tilde{\varepsilon} \). This restriction is consistent with the empirical evidence documented by Bettis, Coles and Lemmon (2000). They document the extensive use of firm-specific bans and short selling restrictions on such trading.

We follow Kyle (1985) to model the trading game. In particular, other than the manager trading for hedging reasons, there is (i) a risk neutral informed speculator who knows \( v \) perfectly at the time of the trading, (ii) a risk neutral market maker who sets prices according a zero profit condition and (iii) noise traders who are not strategic and trade a random amount \( \tilde{u} \). The noise trade \( \tilde{u} \) is independent from all the other random variables and it is distributed normally with mean zero and variance \( \phi \). Given the pricing rule of the market maker and the trading rule of the informed speculator, the manager submits an order \( D^* \). It is common knowledge in the market.
that the manager is an uninformed trader. We leave the formal description of this well-known and extensively common framework to Section 3.1.

This completes the description of the model. For the reader’s convenience we summarize the sequence of events with the timeline below.

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principal offers the manager a linear compensation scheme.</td>
<td>Manager trades in the financial market to hedge the aggregate risk in her compensation.</td>
<td>Manager chooses her effort.</td>
<td>Firm value is realized. Consumption takes place</td>
</tr>
</tbody>
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### 2.1 The Contract Problem

The principal optimally sets the compensation rule \((F, s)\) taking into account the subsequent hedging \((D^*)\) and effort \((e^*)\) choices of the manager. For a compensation scheme \((F, s)\), the manager’s final wealth \(\tilde{W}_m^*\) is,

\[
\tilde{W}_m^*(D^*, e^*) = F + s[e^* + \beta(\bar{v} - \bar{v}) + \tilde{\varepsilon}] + D^*(\bar{v} - \bar{p}) - k(e^*)^2/2 \tag{5}
\]

The value of the hedge position is given by \(D^*(\bar{v} - \bar{p})\) where \(\bar{p}\) is the equilibrium price for the systematic risk security (or market portfolio). The contract \((F, s)\) must also satisfy the manager’s participation constraint,

\[
E\left[\tilde{W}_m^*\right] - (a/2)\text{Var}\left[\tilde{W}_m^*\right] \geq 0 \tag{6}
\]

where the manager’s reservation payoff is normalized to zero. The complete formulation of the contract problem is

\[
\begin{align*}
\text{Max}_{(F,s)} & \quad E\left[(1 - s)\bar{X} - F\right] \\
\text{subject to} & \\
e^* & \in \arg\max E\left[\tilde{W}_m(s, F, D^*)\right] - (a/2)\text{Var}\left[\tilde{W}_m(s, F, D^*)\right] \tag{7}
\end{align*}
\]
\[ D^* \in \arg \max E \left[ \tilde{W}_m \right] - (a/2) \text{Var} \left[ \tilde{W}_m \right] \]  
\[ D_s \in \arg \max E [D_s (v - \tilde{p}) | v] \]  
\[ \tilde{p} = E [\tilde{v} | D_T \equiv D^* + D_s + u] \]

and the participation constraint (6). Equation (7) describes the manager’s optimal effort problem. Equations (8)-(10) describe the trading equilibrium. (8) is the manager’s optimal hedge problem. (9) stands for the informed speculator’s optimal trading strategy. Given the equilibrium strategies of the other market participants, informed speculator submits an order flow \( D_s \) to maximize her expected profits. Finally, (10) is the market maker’s pricing rule that sets the price for the security according to a zero profit condition, conditional on the aggregate order flow \( D_T \equiv D^* + D_s + u \) she receives.

### 3 Manager’s Optimal Hedge and Effort

#### 3.1 Trading Equilibrium

In this section, we solve for the trading equilibrium and characterize the manager’s optimal hedge for the systematic risk in her compensation. The trading equilibrium is described by equations (8)-(10). Following Kyle (1985), we restrict attention to a linear equilibrium: The informed speculator’s optimal trading strategy is given by \( D_s = \delta (v - \bar{v}) \). The market maker’s equilibrium pricing rule is linear in the total order flow she receives: It is given by \( p = \alpha + \lambda D_T \). In the original Kyle (1985) framework, there is no rational uninformed hedger like our manager. As we show below, however, a linear equilibrium continues to exist. This follows, because the market maker filters out the manager’s hedge order from the total order flow as uninformative. Anticipating this, in equilibrium, the informed speculator does not respond to the manager’s order flow either. The following proposition describes this equilibrium.
Proposition 1 There is a linear equilibrium in the market where the systematic risk factor is traded. In this equilibrium, the manager’s hedge is given by

\[ D^* = -\left[ \frac{a}{a + (2\lambda/\Sigma)} \right] s\beta, \]  

(11)

the market maker’s pricing rule is

\[ p = \bar{v} + \lambda(D_T - D^*) \]

and the informed speculator’s trading rule is

\[ D_s = \delta(v - \bar{v}) \]

where

\[ \lambda = \frac{1}{2} \left( \frac{\Sigma}{\phi} \right)^{1/2} \quad \text{and} \quad \delta = \left( \frac{\phi}{\Sigma} \right)^{1/2} \]

Proof: See the Appendix.

The above proposition shows that the manager’s hedge consists of a short position in the market portfolio to counter the systematic risk in her compensation scheme. Examination of the optimal hedge described in (11) yields the following two key observations:

Corollary 2 The manager trades merely for hedging purposes.

Note that when \( s = 0 \), i.e., without any systematic risk in her compensation, the manager does not trade the market portfolio. This follows, since in this market microstructure framework with asymmetric information, holding a position in the market portfolio is costly for an uninformed trader due to the volatility in price. In other words, there is a price risk for the uninformed hedger and hence hedging is costly. This is in contrast to Jin (2002) where hedging is costless. In his analysis, the market portfolio is traded in a competitive CAPM framework. In this setting, even without any systematic
risk to hedge the manager trades the market portfolio as long as there is a risk premium. We revisit this difference in Section 6 where we describe the implications of a competitive hedge market.

**Corollary 3** *As long as the market is not perfectly liquid, the manager’s optimal hedge is not complete.*

This is also in contrast to Jin (2002) where the manager hedges completely. The incomplete hedge is another implication of the asymmetric information framework and the consequent price risk. As standard in the market microstructure literature, our measure of liquidity is the inverse of the market maker’s pricing response $\lambda$ to the order flow. For any liquidity parameter $\lambda > 0$, the manager’s hedge position does not eliminate all the diversifiable risk in her compensation. Only when $\lambda = 0$, i.e., when the market is perfectly liquid, the manager’s hedge is given by $D = -s\beta$ and she hedges her systematic risk exposure completely.

How does the market liquidity affect the manager’s hedging demand? The manager’s order flow is identified by the market maker as uninformative. Therefore, it has no effect on the equilibrium price. However, as long as $\lambda > 0$, i.e., the market maker responds to the noisy total order flow to set the price, the manager faces a volatility in price. It is straightforward to verify that the distribution of the equilibrium price is given by $\tilde{p} \sim N(\bar{v}, \Sigma/2)$. This price uncertainty introduces a trading cost to the manager’s hedge portfolio. Consequently her optimal hedge is not complete and she bears some systematic risk.

### 3.2 Effort Choice

The derivation of the manager’s optimal effort decision, described in (7) is straightforward and does not directly depend on her optimal hedge. Given
the compensation scheme \((F, s)\), one can solve the effort problem in (7) and obtain
\[
e^* = \frac{s}{k}
\]  \hspace{1cm} (12)
Note however, that the manager’s equilibrium effort will depend on her hedging ability once we endogenize her optimal pay-performance sensitivity \(s\). This is analyzed next.

4 Optimal Pay-Performance Sensitivity

Now, we go back to the shareholder’s problem of choosing the optimal compensation scheme \((F, s)\). Substituting the manager’s optimal hedge in (11) and optimal effort in (12) into her wealth distribution in (5), one obtains
\[
\tilde{W}_m^* = F + \frac{s^2}{2k} - s\beta\tilde{v} + \left[ \frac{s\beta(2\lambda)}{\Sigma a + 2\lambda} \right] \tilde{v} + s\tilde{z} + \left[ \frac{s\beta\Sigma a}{\Sigma a + 2\lambda} \right] \tilde{p}
\]  \hspace{1cm} (13)
Note once again that there are three sources of risk in the manager’s ex ante wealth distribution: (i) Systematic risk factor \(\tilde{v}\) that she cannot perfectly hedge due to imperfect liquidity, (ii) the firm specific risk factor \(\tilde{z}\) and (iii) the price risk. In equilibrium, the principal sets \((F, s)\) such that the manager’s participation constraint in (6) holds as an equality and hence \(F\) is given by;
\[
F = \left( a/2 \right) \text{Var} [\tilde{W}_m^*] - \frac{s^2}{2k}
\]  \hspace{1cm} (14)
Substituting this into the shareholder’s expected final wealth, the shareholder’s problem of optimal pay-performance sensitivity choice reduces to;
\[
\text{Max}_s \left( 1 - s \right) \left( \frac{s}{k} \right) + \frac{s^2}{2k} - \left( a/2 \right) \text{Var} [\tilde{W}_m^*]
\]  \hspace{1cm} (15)
where
\[
\text{Var} [\tilde{W}_m^*] = s^2\beta^2 \left[ \frac{(a + z)^2 + z^2}{2(a + z)^2} \right] \Sigma + s^2\eta
\]  \hspace{1cm} (16)
and \(z \equiv 2\lambda/\Sigma\). Our next result characterizes the optimal pay-performance sensitivity \(s^*\) and relates it to market liquidity.
Proposition 4  (i) The optimal pay-performance sensitivity $s^*$ is given by

$$s^* = \frac{1}{1 + ak\Gamma}$$  \hspace{1cm} (17)

where

$$\Gamma \equiv \beta^2 \left[ \frac{(a + z)^2 + z^2}{2(a + z)^2} \right] \Sigma + \eta \quad \text{and} \quad z \equiv 2\lambda/\Sigma$$  \hspace{1cm} (18)

(ii) The optimal $s^*$ is decreasing in the systematic risk, firm-specific risk $\eta$ and managerial risk aversion $a$ and it is increasing in the measure of market liquidity (inverse of $\lambda$).

Proof: See the Appendix.

Proposition 4 has a number of empirical implications.

(1) Pay-performance sensitivity $s^*$ decreases in firm specific risk $\eta$ and manager’s risk aversion. This is a central prediction of the principal-agent theory and follows directly from the well known risk-incentive trade-off. Aggarwal and Samwick (1999) find strong empirical evidence that the pay-performance sensitivity for executives at firms with the least volatile stock prices is an order of magnitude greater than the pay-performance sensitivity for executives at firms with the most volatile stock prices. Jin (2002) extends these empirical findings by distinguishing between systematic and firm-specific risk. He finds that firm specific risk is clearly negatively related to $s^*$. Furthermore, he finds that firm specific and systematic risk influence $s^*$ differently.

(2) Pay-performance sensitivity is decreasing in the systematic risk: This follows because, (i) the shareholder is risk neutral and thus it is costless for the shareholder (rather than the manager) to bear this diversifiable risk (ii) due to imperfect liquidity ($\lambda > 0$), the manager cannot diversify away all the systematic risk. This prediction differs from Jin (2002) where the manager diversifies all the systematic risk in her compensation and therefore $s^*$ is independent from systematic risk. In his regressions, however, Jin finds that
systematic risk does matter for incentives when CEO’s are younger and less wealthy, thus more likely constrained in their ability to hedge the systematic exposure in their compensation. This empirical result is also supported by Garvey and Milbourn (2002). The conclusion is, then, as long as the manager faces constraints to diversify away the systematic risk, pay-performance sensitivity is decreasing in that risk. Our next empirical prediction identifies lack of liquidity as one such constraint.

(3) Pay-performance sensitivity is increasing in the liquidity of the market where systematic risk is traded: To the best of our knowledge, this is a new empirical prediction and has not been tested. It relates the principal-agent theory of executive compensation to a financial market fundamental which endogenously determines the extent the manager can diversify.

4.1 Market Liquidity and Firm Value

An interesting feature of the above analysis is the link between the liquidity of the market where the systematic risk factor is traded and the firm value. With higher market liquidity (low $\lambda$), the manager can better hedge the systematic risk exposure in her compensation. This further implies that the equilibrium pay-performance sensitivity is increasing in the liquidity of the market where systematic risk is traded ($\Gamma$ is decreasing, hence $s^*$ is increasing in the liquidity of the hedge market). In other words, since the manager bears less of the systematic risk factor, she can be given high powered incentives. This, in turn, increases the manager’s equilibrium effort and thus the firm value. This testable implication of our analysis is stated in the following proposition.

**Proposition 5** In equilibrium, the firm value is increasing in the liquidity of the market where the aggregate (systematic) risk factor is traded.

Proof: See the Appendix.
Homstrom and Tirole (1993) also build a positive link between market liquidity and production efficiency. It is important to note, however, that in their theory market liquidity refers to the liquidity of the company’s stock, whereas we refer to the liquidity of the market where a systematic risk security, like a market index, is traded. This distinction is important, since the main message of Holmstrom and Tirole is that a dispersed ownership structure which promotes the liquidity of the company’s stock has an implication on managerial incentives. In their theory, stock market plays a monitoring role. Liquidity creates incentives to a speculator to produce better information and trade more aggressively. This, in turn, results in a more informative stock price and improves performance evaluation. In contrast, in our setting liquidity refers to the liquidity of the market portfolio in general and it is completely exogenous to the firm. Furthermore, the way liquidity improves the incentives is through a diversification channel. Liquidity improves the manager’s ability to diversify away the systematic exposure in her compensation. To the extent that she can do so, she can be given high powered incentives.

5 Competitive Hedge Market and Complete Diversification

In this section, we describe the implications of assuming a competitive hedge market for the systematic risk security and contrast our analysis with that of Jin (2002). Suppose that the manager can trade the market portfolio at some competitive price $p_c$. Accordingly, for a compensation contract $(F, s)$ and a position $D_c$ in the market portfolio, the manager’s wealth distribution is

$$\tilde{W}_m = F + s[c + \beta(\tilde{v} - \bar{v}) + \tilde{\varepsilon}] + D_c(\tilde{v} - p_c) - k(e)^2/2$$
It is straightforward to show that the manager’s optimal position is now given by

\[ D_c = \frac{\bar{v} - p_c}{a\Sigma} - s\beta \]  

(19)

The above optimal position is in contrast with the one derived in Proposition 1 in two respects. First, the manager does not merely trade for hedging reasons. Her portfolio in the systematic risk security has two components: 

\( (\bar{v} - p_c) / a\Sigma \) is the optimal systematic risk exposure that would be taken by any investor with a risk aversion coefficient \( a \) and with a prior on \( \tilde{v} \) given by \( \tilde{v} \sim N(\bar{v}, \Sigma) \). This component is completely independent from the manager’s contract. The second component \( -s\beta \) is related to the manager’s hedging response to her compensation contract. Note that when she can trade the systematic risk security at a competitive price, the manager completely diversifies the systematic risk exposure in her compensation. The implication of this hedging behavior on the optimal pay-performance sensitivity has been studied in Jin (2002): When the manager can hedge at a competitive price, she diversifies away the systematic risk exposure in her compensation completely. In this case, the optimal pay-performance sensitivity \( s \) is independent from the systematic risk factor\(^4\) and the company’s beta and it is given by:

\[ s = \frac{1}{1 + ak\eta} \]  

(20)

6 Conclusion

This paper analyzes the implications of the manager’s hedging ability on her optimal compensation scheme. We introduce the manager as a rational but uninformed hedger in a Kyle type market setting and allow her to privately adjust the systematic exposure in her compensation. Our results are (i) due

\(^4\)Note that if the shareholder could write a contract separately on systematic risk and firm specific risk an equivalent result would obtain. In this case, the manager’s reward scheme would compensate her relative performance and it would be given by \( F + sX - s\beta(\bar{v} - \tilde{v}) \) where \( s \) is again determined as in (20).
to imperfect liquidity the manager’s optimal hedge of the systematic risk is not complete. She bears some systematic (market) risk. (ii) The equilibrium pay-performance sensitivity is decreasing in the market risk and increasing in the market liquidity. (iii) Since better hedging ability increases the manager’s equilibrium incentives, the firm value is increasing in the liquidity of the market where systematic risk is traded. The analysis extends the standard principal-agent theory of executive compensation to a market microstructure setting. We emphasize the role of liquidity, a financial market fundamental, on managerial incentives and the firm value through a previously unexplored ‘diversification’ channel.
Proof of Proposition 1

**Market Maker:** Given the trading strategy \( D_s(v) = \delta(v - \bar{v}) \) of the informed trader and hedge order \( D^* \) by the manager, the market maker sets
\[
p = E[\tilde{v}|D_T = D^* + D_s + u]
\] (21)

In order to illustrate the market maker’s signal extraction problem, we note that
\[
\tilde{D}_T = \delta(\tilde{v} - \bar{v}) + u + D^*
\] (22)
and thus we can define
\[
\tilde{Z} \equiv \frac{\tilde{D}_T - D^* + \delta\tilde{v}}{\delta} = \tilde{v} + \frac{1}{\delta}\tilde{u}.
\] (23)

Conditioning on \( Z \) is statistically equivalent to conditioning on \( \tilde{D}_T \). Therefore, we have
\[
E[\tilde{v}|D_T] = E[\tilde{v}|Z] = \frac{\tilde{v}\phi + \delta^2\Sigma Z}{\phi + \delta^2\Sigma}
\] (24)

Now, substitute back \( Z \) and obtain
\[
E[\tilde{v}|D_T] = E[\tilde{v}|D_T] = \tilde{v} - \left( \frac{\delta\Sigma}{\phi + \delta^2\Sigma} \right) D^* + \left( \frac{\delta\Sigma}{\phi + \delta^2\Sigma} \right) D_T
\] (25)

Therefore, \( p = \bar{v} - \lambda D^* + \lambda D_T \) where
\[
\lambda = \frac{\delta\Sigma}{\phi + \delta^2\Sigma}
\] (26)

**Informed Trader:** Given the pricing strategy \( p = \alpha + \lambda D_T \) of the market maker, the informed trader chooses \( D_s \) to maximize her expected profits \( \pi \):
\[
\pi = E[D_s(v - \bar{p})|v] = D_s v - D_s E(\bar{p}) = D_s v - D_s(\alpha + \lambda[D_s + D^*]) = D_s v - D_s\bar{v} - \lambda(D_s)^2
\] (27)
Since the market maker filters out $D^*$ from the total order flow, the speculator takes this into consideration. Therefore, his portfolio choice do not depend on $D^*$. The optimal informed order $D_s$ is given by

$$D_s = \frac{v - \bar{v}}{2\lambda}$$

(28)

which implies that $\delta = 1/(2\lambda)$. Solving this together with $\lambda = \delta\Sigma/\left[\phi + \delta^2\Sigma\right]$ yields the equilibrium coefficients $\delta$ and $\lambda$.

The Manager: Given the pricing rule of the market maker and the trading strategy of the informed speculator, the manager’s wealth is given by

$$\tilde{W}_m = F + se - \frac{k\varepsilon^2}{2} + s[\beta(\tilde{v} - \bar{v}) + \bar{\varepsilon}] + D(\tilde{v} - \tilde{p})$$

where $\tilde{p} = \alpha + \lambda[\delta(\tilde{v} - \bar{v}) + D + \bar{u}]$. Therefore,

$$E[\tilde{W}_m] = F + se - \frac{k\varepsilon^2}{2} + D[\tilde{v} - \alpha - \lambda D]$$

$$Var[\tilde{W}_m] = [s\beta + (1 - \lambda\delta)D]^2\Sigma + (\lambda D)^2\phi + s^2\eta$$

Accordingly, the first order condition that maximizes (8) is given by

$$\tilde{v} - \alpha - 2\lambda D - a\{(1 - \lambda\delta)(s\beta + (1 - \lambda\delta)D)\Sigma + \lambda^2\phi D\} = 0$$

Now note that $1 - \lambda\delta = \frac{1}{2}$, $\alpha = \tilde{v} - \lambda D$ and $\lambda^2\phi = \frac{1}{4}\Sigma$. Substituting these into the above first order condition and solving for $D$ gives;

$$D^* = -\left[\frac{a}{a + (2\lambda/\Sigma)}\right]s\beta$$

Proof of Proposition 4

The optimal pay-performance sensitivity in (17) follows from substituting (16) into (15) and maximizing it with respect to $s$. The comparative static result with respect to market liquidity follows by noticing that $\Gamma$ is increasing.
in $\lambda$ and thus decreasing in the market liquidity (inverse of $\lambda$). The other comparative statics results are straightforward from (17) and (18).

**Proof of Proposition 5**

Directly follows, since expected firm value is given by

$$E(\tilde{X}) = e^* + \beta \tilde{v} = s^* + \beta \tilde{v}$$

and $s^*$ is increasing in market liquidity.
References


