Informational Lock-In and Relationship Financing*

Levent Koçkesen†  Saltuk Ozerturk‡

October 2002
(First version: January 2002)

Abstract

We analyze an entrepreneur’s choice between a multi-period financing arrangement (relationship or staged financing) and up-front financing. Relationship financing is desirable because it allows the financier to acquire information and deny further financing depending on the interim progress of the venture. However, relationship financing completely shuts off the entrepreneur’s access to the competitive capital market. Since she is informationally captured by the initial financier, she ends up sharing part of the continuation surplus. We show that the initial financier’s ability to extract rents due to her superior information is essential for the viability of relationship financing. In equilibrium, despite the loss of future rents, the entrepreneur strictly prefers relationship financing and locks herself in to the financier. Moreover, in anticipation of loss of future rents, she raises and invests too much before information revelation.

JEL Classification: C78, D82, G24, G32.
Keywords: relationship financing, informational lock-in, adverse selection, bargaining, endogenous outside options.

*This is a revised version of our paper “Staged Financing and Endogenous Lock-In”. We would like to thank Alberto Bisin, Andrew Chen, Boyan Jovanovic, Hideo Konishi, David Mauer, Efe Ok, Mike Riordan, Charles A. Wilson, and seminar participants at Society of Economic Design 2002 meetings in New York and Southern Methodist University for helpful comments. The usual disclaimer applies.

†Corresponding author. Department of Economics, Columbia University, 420 West 118th Street, 1022 International Affairs Building, New York NY 10027. E-mail: lk290@columbia.edu.

‡Department of Economics, Southern Methodist University, Dallas, TX 75275-0496. E-mail: ozerturk@mail.smu.edu.
1 Introduction

An important feature of venture capital financing is its staged structure. Rather than committing all the required funds up front, venture capitalists invest in companies at distinct stages of their development, preserving the right to deny further financing. In his comprehensive survey of the actual venture capital contracting practice, Sahlman (1990) accounts for the following stylized facts on venture capital investments:¹

(i) Each financing round corresponds to a milestone where some uncertainty on the venture’s progress is resolved.

(ii) The terms of future financing rounds are not negotiated in advance; they reflect the then-current performance and prospects of the firm.

(iii) It is very rare for the entrepreneur to raise subsequent rounds from parties other than the initial venture capitalist. In other words, the entrepreneur is locked in to the venture capitalist who provides initial funding. In certain cases, the entrepreneur agrees to include a clause in the initial agreement that prevents her from seeking finance from other sources completely (no de novo financing).

This financing structure suggests a multi-period relationship between the capitalist and the entrepreneur, where the two distinguishing features are learning about the prospects of the venture and then bargaining again over continuation terms. Although these features have been recognized in academic research, almost all papers in the literature address financial contracting issues that surround staging, taking the multiperiod relationship as given.² In this paper, we formally analyze an entrepreneur’s choice between relationship (staged) financing and up-front financing in a bargaining theoretic framework and reconcile the above stylized facts. We address two questions: First, can relationship financing, which involves a termination threat and possible loss of future rents, be desirable for the entrepreneur, when she has equal access to up-front financing? Second, what makes relationship financing a viable financing structure, in the absence of any ex ante commitments on continuation terms? How does the viability of relationship financing depend on the nature of information generated within the relationship?

We present a model where the entrepreneur strictly prefers relationship financing and the capitalist’s ability to extract surplus in the future financing rounds is essential for the viability of relationship financing. Without ex-post rent extraction (lock-in) by the capitalist, there is no relationship (or staged) financing. What makes ‘lock-in’ possible is the capitalist’s superior information on the prospects of the venture and his resulting bargaining power in the future financing round. As in Rajan (1992) and Sharpe (1990), the borrower (entrepreneur) is informationally captured and the lender (capitalist) extracts part of the surplus by providing continuation funds in non-competitive

¹Gompers (1995) and Kaplan and Stromberg (2001) also provide extensive empirical evidence on the staged financing practice in venture capital investments.

terms. Unlike those papers, however, ‘lock-in’ is a desirable feature for the entrepreneur ex ante, as it makes relationship financing a viable financing option in the first place. Interestingly, if the entrepreneur anticipates that she will not be informationally captured, say because the information is easily accessible to outsiders as well, it is in her interest to include a clause in the initial contract that locks her in to the initial capitalist, so that financing can be staged. This explains why entrepreneurs are willing to include contractual clauses, like ‘no de novo financing’ or ‘right of first refusal’, that restrict their future access to outside financing sources, once the venture is started.

Our basic model is as follows: There is an entrepreneur who needs to raise financing for her venture. Initially, she has access to a competitive capital market and if she decides to do so, she can attempt to raise all the required capital at once in competitive terms (up-front financing). This mode of financing does not involve any future interaction between the capitalist and the entrepreneur and it corresponds to the so-called arm’s length financing. Alternatively, the entrepreneur may raise and invest only a portion of the required funds initially (staged or relationship financing). In this case, the entrepreneur and the capitalist who provides the initial financing observe an information signal on the venture’s prospects. Upon observing the signal, the two parties bargain over the continuation terms to complete the project. The capitalist may agree to provide the continuation funds or he may deny further financing. Similarly, the entrepreneur can opt out from the bargaining table and seek the continuation financing from competitive outside financiers. However, since the outsiders do not observe the signal, the terms they offer (i.e., the value of the entrepreneur’s outside option) is determined endogenously by their beliefs on the signal realization. The basic trade-off for the entrepreneur in the above formulation is incorporating benefits from learning by relationship financing at the cost of possibly losing her access to the competitive capital market, once the venture is started. We characterize the equilibrium financing mode (up-front versus relationship) within this trade-off.

The main contributions of the paper are threefold: First, we emphasize the importance of ex-post rent extraction by the capitalist on the ex-ante viability of relationship financing. In that sense, our analysis formalizes the general notion that competition and relationships are not compatible. Second, we illustrate how the nature of the information generated in the relationship determines the rent allocation in the bargaining game over continuation and hence the ex ante viability of relationship financing. If this information is easily accessible by the outside parties and the entrepreneur cannot commit not to seek continuation financing from outsiders, relationship financing is not viable. In contrast, when information is not observable by the outsiders, the informational asymmetry between the entrepreneur and the competitive but uninformed outsiders alone can serve as an entry barrier. This protects the initial capitalist from ex post competition and the entrepreneur becomes fully dependent on him for continuation financing. The resulting ability

---

Footnote:

3 This argument is similar to the point made by Chan et al. (1986). They argue that with more ex post competition, borrowers might be tempted to switch to other lenders. When lenders anticipate a shorter relationship, ex ante they find it less worthwhile to acquire costly proprietary information. Hoshi, Kashyap and Scharfstein (1990) provide empirical evidence from Japanese financial market liberalization that supports this “switching” view. They find that successful firms with high Tobin’s q reduced their bank ties to borrow from the markets.
of the capitalist to share continuation surplus in the future financing round makes relationship financing viable in the first place. Third, relationship financing arises endogenously, even when the entrepreneur anticipates being informationally captured in the future. Furthermore, anticipating the loss of rents in the future, she raises and invests too much initially before information revelation. Due to its effect on the subsequent bargaining game for continuation, the initial investment level becomes the price of being an insider (and extracting rents) for the capitalist.

By deriving an explicit link between the extent of ex-post competition between lenders and their ex ante willingness to invest initially, our model generates a number of empirical implications: For investments where the interim information is easily accessible by the public, we predict that relationship contracts would include clauses (like rights of first refusal, pre-emptive rights, no de novo financing) that restrict the borrower’s future outside financing alternatives. We also predict that the more (less) bargaining power the lender (the borrower) has in the future, the higher (lower) is the initial investment to start the relationship. Finally, relationships with multiple lenders increase ex-post competition and decrease their ex ante willingness to invest in the relationship. Therefore, our analysis implies that multiple relationships worsen the availability of initial funding, especially for investments with higher level of initial uncertainty.

Although our paper is motivated by the stylized facts from venture capital contracting practice, the analysis we present bear similarities with the relationship banking literature. In particular, the financing problem we study meets all the three criteria of relationship banking as identified by Berger (1999) (also see Boot (2000)): The relationship financier receives information beyond the readily available public information, information gathering takes place over time through multiple interactions with the borrower and finally this information remains confidential. Our analysis points out how the proprietary nature of the information allows the financier to extract rents ex post and how this, in turn, makes relationship financing viable ex ante. The closest paper in banking literature is the seminal paper by Rajan (1992). We differ from Rajan (1992) in the following aspects: In our paper, the borrower always chooses to lock herself in, since this makes relationship financing viable in the first place. Therefore, lock-in is desirable ex ante from the borrower’s perspective. Second, in Rajan (1992), anticipation of lock-in causes the borrower to underinvest in effort, whereas in our paper, it causes her to borrow and invest too much. Third, in Rajan’s analysis of the bargaining game over continuation, in general the firm is not locked in to the bank with probability one. By contrast, the unique equilibrium bargaining outcome in our analysis is full informational lock-in.

Another related paper is Admati and Pfleiderer (1994) who present a similar setting with a different focus. In their paper, the problem is the truthful revelation of interim information to outsiders and this is achieved by the insider capitalist acting as an information intermediary. Although they do not study it, they mention the possibility of the insider capitalist extracting part

---

4This voluntary restriction the entrepreneur imposes on her future options is somewhat similar to Helmann (1998), where the entrepreneur voluntarily and efficiently gives up control rights to the venture capitalist. Similarly, in Chan et al (1990), the entrepreneur agrees to give the capitalist a buyout option (by which the entrepreneur is relieved from control), in case she fails to demonstrate enough skills.
of the continuation surplus due to his superior information, which is the focus of this paper.\footnote{Boot and Thakor (1994), Neher (1999) and Egli et al (2002) also study relationship financing in different settings. In Boot and Thakor (1994), durable relationships are valuable because they allow banks and firms to subsidize financing intertemporally, reducing the use of costly collateral. Neher (1999) shows that staged financing can be a remedy to a hold-up problem, when the entrepreneur cannot commit to work in the venture. Egli et al (2002) show that relationship financing helps borrowers to build a reputation for repayment when there is a high possibility of strategic default.}

The plan of the paper is as follows. The next section describes the model. Section 3 presents the analysis. Section 4 summarizes and discusses our results and considers some extensions of the basic framework. Section 5 contains the proofs that are not presented in the text.

\section{The Model}

There are three dates, $T = 0, 1,$ and $2$. At date $0$, a risk neutral entrepreneur has a project that requires a fixed investment of $\$1$. If undertaken, the project generates a gross return $Y$, which is a non-negative random variable with distribution function $F$. We denote the expected value of $Y$ with respect to $F$ by $E[Y]$. The entrepreneur does not have any funds to finance the project and hence needs an investor, which we will call a capitalist. The capital market is composed of a continuum of capitalists and requires a competitive rate of return normalized to zero. Therefore, at date $0$, the entrepreneur has monopoly over the particular project in question when she takes it to a capitalist. The entrepreneur has a discount factor $\beta_e \in (0, 1)$ and the capitalist has a discount factor $\beta_c \in (0, 1)$ across dates 0, 1, and 2.

\textit{Financing Possibilities.} The investment in the venture can be undertaken in a single period if all of $\$1$ is raised and invested at date 0. We refer to this mode of financing as \textit{up-front} financing. When this is the case, the payoff of the venture is realized at date 1. We assume that the initial uncertainty is such that

\begin{assumption}
\label{assumption:1}
\beta_e E[Y] - 1 \leq 0.
\end{assumption}

In other words, up-front financing generates a non-positive net expected surplus from the capitalist’s point of view. This implies that if the entrepreneur were to adopt up-front financing, the most the two parties could get is zero.

Alternatively, the entrepreneur can choose to adopt \textit{relationship (staged) financing}. In this case, only a fraction $x \in (0, 1)$ of the required capital is raised and invested initially and the completion of the project is spread over two periods. With relationship financing, no payoff is realized following the initial investment of $x$. However, at date 1, the two parties involved in the venture’s operations during the first period (both the entrepreneur and the capitalist who provides the initial financing) observe a payoff relevant information signal. We denote the information signal with $S$, which is a non-negative random variable jointly distributed with $Y$. Let $G$ denote its marginal distribution and $F(y|s)$ the conditional distribution of $Y$ given $S = s$, with $E[Y|s]$ denoting the conditional expectation of $Y$ given $S = s$. We assume that

\begin{equation}
\beta_c E[Y] - 1 \leq 0.
\end{equation}
Assumption 2. For any strictly increasing function \( u : \mathbb{R} \to \mathbb{R} \),
\[
\int_{\mathbb{R}} u(y) \, dF(y|s') > \int_{\mathbb{R}} u(y) \, dF(y|s) \quad \text{whenever } s' > s.
\]

This assumption is a first order stochastic dominance condition which, among other things, implies that \( E[Y|s] \) is strictly increasing in \( s \). In that sense, all we impose is that a higher realization of \( s \) implies better news on the prospects of the venture.

In order to obtain the payoff \( Y \) from the venture, which is to be realized at date 2 in case of relationship financing, the remaining \( 1 - x \) has to be invested in the second period. This implies that following an initial investment of \( x \) and a signal realization \( s \), the expected continuation surplus, from the point of view of the capitalist, is
\[
R(s, x) \equiv \beta_c E[Y|s] - (1 - x)
\]

Notice that the expected continuation surplus is strictly increasing in the signal realization \( s \) and the initial investment \( x \). A necessary condition for relationship financing to be a feasible financing alternative is

Assumption 3. For all \( x \in (0, 1) \), there exists an \( s^*(x) > 0 \) such that \( G(s^*(x)) < 1 \) and \( R(s^*(x), x) = 0 \).

This assumption together with Assumption 2 means that there exist high enough signal values \( s > s^*(x) \) that makes continuation profitable. If this was not the case, then continuation would never be profitable and hence relationship financing would not be a feasible financing option at all. Also notice that the distribution of the signal is completely independent from the initial investment or any action possibly taken by the entrepreneur and the capitalist. In that sense, we abstract away from any moral hazard and asymmetric information issues between the entrepreneur and the capitalist.\(^6\)

We also assume that the information signal at date 1 is not verifiable. Therefore, we intend to describe a situation where the two parties obtain some soft information on the venture’s profitability. This might be the revelation of the technical and managerial ability of the entrepreneur as in Chan et al (1990) or it might represent an information on the viability and marketability of the product being developed.\(^7\) For our purposes, an important implication of the non-verifiability of the signal is that parties cannot contract upon it at date 0. In other words, parties cannot write an enforceable clause in the date 0 contract that specifies a continuation or termination decision at date 1, contingent on signal realization. In case of relationship financing, the initial contract can only specify the

\[^{6}\text{For a model where the entrepreneur manipulates the signal (window dressing) to secure continuation funds, see Cornelli and Yosha (2001). In Bergemann and Hege (1998) only the entrepreneur observes the true interim state. In Rajan (1992), the likelihood of the good state depends on the entrepreneur’s effort.}\]

\[^{7}\text{Rajan (1992) also makes the same assumption that the intermediate state which is informative on company’s prospects is not contractible. He argues that it is hard for firms to present hard data to the creditors before the project is completed. This difficulty to provide hard data on performance is all the more relevant in case the company is a young entrepreneurial venture or start-up, which our model best fits into.}\]
initial investment $x$ and payoffs to parties if venture is abandoned without further investment at date 1. The capitalist cannot make any ex ante signal contingent commitments to provide further financing or to abandon the venture. Similarly, the entrepreneur cannot make any commitments to raise further financing from the initial capitalist. She can seek continuation financing from outside parties. The continuation decision and the source and the terms of continuation funds are determined by the bargaining between the entrepreneur and the capitalist, following the signal realization. We describe the bargaining game at date 1 next.

**Bargaining.** Upon observing the signal realization, the entrepreneur and the capitalist are in a bilateral bargaining situation as two symmetrically informed parties. In case they choose to continue the project together, the capitalist provides $1 - x$ and the parties specify a sharing rule over the final payoff. Alternatively, the capitalist may choose to deny further financing and liquidate the venture. We assume that in this case all the liquidation value accrues to the capitalist and the entrepreneur gets zero.\(^8\) For simplicity, we set the liquidation value equal to zero as well. However, relaxing this does not change our results in any important way. This assumption is made simply to avoid the uninteresting cases in which the capitalist starts the venture just to liquidate it after one period. As we show later, all we need is that when continuation is socially optimal, liquidation can never yield a higher payoff than continuation. Similarly, if the entrepreneur opts out and continues with another financier, we assume that the initial capitalist receives a zero payoff.\(^9\) The outside capital market is assumed to be competitive and require a zero rate of net return. In case the entrepreneur opts out, the outsiders make offers which specify a sharing rule for the final payoff in exchange for providing continuation capital $1 - x$. To summarize, in the date 1 bargaining game over continuation, the outside option of the capitalist is liquidation, whereas the outside option of the entrepreneur is seeking continuation finance from competitive outside capitalists.

The bargaining game we analyze is a standard Rubinstein alternating offers bargaining game with outside options. However, an important twist is that the value of the entrepreneur’s outside option is not given, but it is determined endogenously in equilibrium. The formal description of the date 1 bargaining game over continuation is as follows:

**Histories, Actions of Players and Payoffs.** At date 1, a history is specified by the initial investment $x$ and payoffs to parties if venture is abandoned without further investment at date 1. The capitalist cannot make any ex ante signal contingent commitments to provide further financing or to abandon the venture. Similarly, the entrepreneur cannot make any commitments to raise further financing from the initial capitalist. She can seek continuation financing from outside parties. The continuation decision and the source and the terms of continuation funds are determined by the bargaining between the entrepreneur and the capitalist, following the signal realization. We describe the bargaining game at date 1 next.

\(^8\)In case of venture capital investments, our assumption that the liquidation rights belong to the capitalist seems accurate. Venture capitalists almost exclusively hold claims, like the so called preferred stock, which entitles them an absolute priority and liquidation preference. (Sahlman (1990), Gompers (1997)). If the company is performing poorly and if the venture capitalist wants to liquidate her existing claims on the venture, she may use the priority of her claims to force the company management to accept an acquisition bid or a liquidation decision. According to Sahlman (1990, p.509), ‘[m]any agreements give the venture capitalist the right to force the redemption of a preferred stock or the right to ‘put’ the stock of the company to achieve liquidity.’ Kaplan and Stromberg (2002) argue that these rights are usually exercised to force the entrepreneur to liquidation or acquisition.

\(^9\)An alternative assumption is that the initial capitalist has to be compensated in case the entrepreneur opts out and seeks continuation funds from an outsider capitalist. The only effect of this extension on the present analysis is to make opting out by the entrepreneur more costly, since he will have to raise additional capital to compensate the initial capitalist. Since our main result is that lock-in arises endogenously due to adverse selection, we do not complicate the analysis with this possibility, which clearly facilitates lock-in.
We denote the bargaining game after any such history by $\Gamma(x)$. Upon observation of the signal realization $s$, the game starts with the entrepreneur making the first offer $\pi^0_e(Y) \in [0, Y]$ to which the insider capitalist responds by accepting, opting out, or rejecting.\footnote{There would be no qualitative change in the results of the paper if the capitalist were to be the first to make an offer.} If the capitalist rejects, she makes a counter-offer next period, $\pi^1_c(Y) \in [0, Y]$, to which now the entrepreneur responds by accepting, opting out, or rejecting. If entrepreneur rejects, she makes an offer $\pi^2_e(Y) \in [0, Y]$ at period 2 and the game continues in this fashion \textit{ad infinitum} with entrepreneur making an offer $\pi^t_e(Y)$ in periods $t = 0, 2, \ldots$, and venture capitalist making an offer $\pi^t_c(Y)$ in periods $t = 1, 3, \ldots$. If payoff state at date $T = 2$ is $Y = y$ and the offer $\pi^t_i(Y)$ of $i \in \{e, c\}$ is accepted by $j \neq i$, then player $i$ receives $\pi^t_i(y)$ at date 2. In other words, the offers by each player specify a sharing rule for each realization of $Y$. If neither party ever accepts an offer or ever opts out, both players receive zero payoff.

We assume that the amount of time that passes between each offer is $\Delta > 0$, and the discount rate for player $i$ is given by $r_i > 0$. Define $\delta_i = e^{-r_i\Delta}$ as player $i$’s discount factor in the bargaining game. Therefore, if the insider capitalist accepts an offer $\pi^t_e(Y)$, his expected payoff is

$$\delta^t_e [\beta_c E [Y - \pi^t_e(Y)]|s] - (1 - x),$$

and that of the entrepreneur is

$$\delta^t_e \beta_e E [\pi^t_e(Y)|s].$$

Similarly, if the entrepreneur accepts an offer $\pi^t_c(Y)$, her expected payoff is

$$\delta^t_e \beta_e E [Y - \pi^t_c(Y)|s],$$

and that of the insider capitalist is

$$\delta^t_c [\beta_c E [\pi^t_c(Y)|s] - (1 - x)].$$

\textit{Outside options.} If the insider capitalist opts out and liquidates, both the entrepreneur and the capitalist receive zero payoff. If the entrepreneur opts out, the initial capitalist gets zero payoff and the outside capitalists make offers $b(Y) \in [0, Y]$ to the entrepreneur which specifies the amount that the entrepreneur is to receive if the venture generates a payoff $Y$ at date 2.\footnote{Notice that we impose a limited liability restriction, which requires that the entrepreneur’s payoff $b(Y)$ cannot be smaller than zero or greater than the realized payoff of the venture.} Therefore, the outsider, whose offer has been accepted by the entrepreneur, provides the continuation capital $1 - x$ and receives an expected payoff of

$$\beta_c E[Y - b(Y)|I] - (1 - x),$$

where $I$ denotes the outsiders’ information set, which contains the event that the entrepreneur has opted out. We analyze two different scenarios regarding the observability of the signal by the outsiders. If the signal is observable, then for each signal the outsiders have a different information
set. If, on the other hand, the signal is not observable, then they have only one information set which contain all the histories after which the entrepreneur opts out as well as all the signal realizations. Also notice that, in equilibrium, outsiders’ offer and thus the equilibrium value of the entrepreneur’s outside option depends on the beliefs of outsiders on the type (the signal) of the opting out entrepreneur. From accepting the outsider’s offer \( b(Y) \), the entrepreneur gets an expected payoff of \( \beta_c E[b(Y) | s] \).

In this formulation, relationship financing simply describes an arrangement where the parties agree to invest only a portion of the required funds and leave the decision to complete the project to a future date, once they receive payoff relevant information. This arrangement differs from up-front financing in the following way: The capitalist limits his initial sunk investment and he has the right to deny further financing. In other words, he attains an option to abandon. However, he does not have an explicit option to further invest, since the entrepreneur may opt out and seek continuation financing from outsiders. Therefore, the initial capitalist may end up not benefitting at all from information revelation. On the other hand, the entrepreneur has full access to a competitive capital market at date 0. If financing is undertaken in two stages, she has to bargain with the initial capitalist for continuation funds and she may end up sharing part of the continuation surplus to raise \( 1 - x \). Consequently, the trade-off for the entrepreneur is limiting the initial capitalist’s sunk cost by giving him an option to abandon versus possibly losing her own access to the competitive capital market.

3 Analysis

We start our analysis by characterizing the equilibrium outcome of the bargaining game at date 1 under two alternative assumptions: (i) signal is also observable to outside capitalists, (ii) signal is observable only to the insider capitalist and the entrepreneur. We show that these two cases have very different implications for the equilibrium value of the outside option of the entrepreneur and thus on how the continuation surplus is shared. The outcome of the bargaining game over continuation, in turn, determines the level of initial investment before information revelation and whether relationship financing can arise or not.

3.1 Equilibrium Analysis of the Bargaining Game at Date 1

3.1.1 Observable Signals

Suppose an initial amount \( x \in (0,1) \) is invested in the venture at date 0 and a signal value \( s \) is realized at date 1. Suppose further that the signal is observable by the outside capitalists as well as the initial capitalist and the entrepreneur. In this case, if the entrepreneur opts out from the bargaining table and seeks continuation financing from outsiders, the equilibrium outside offer \( b(Y) \) she will receive must be such that

\[
\beta_c E(Y - b(Y) | s) - (1 - x) = 0,
\]
since the outside capitalists are competitive and require a net expected return of zero. Recalling
that the expected continuation surplus conditional on signal is given by $R(s, x)$, the above condition
implies that the equilibrium outside offer $b(Y)$ must be such that
\[
E[b(Y)|s] = \frac{R(s, x)}{\beta_c}
\]
if $R(s, x) \geq 0$, and $b(y) = 0$, for all $y$ if $R(s, x) < 0$. By Assumptions 2 and 3, there exists a
unique threshold signal realization $s^*(x)$ such that the expected continuation surplus is zero, i.e.,
$R(s^*(x), x) = 0$. For $s > s^*(x)$ we have $R(s, x) > 0$, whereas for $s < s^*(x)$, we have $R(s, x) < 0$. This
implies that for $s < s^*(x)$, the value of the entrepreneur’s outside option is zero, whereas for
$s > s^*(x)$, this outside option is given by $\frac{\beta_c}{\beta_e}(R(s, x))$. For ease of reference, we report the unique
equilibrium payoffs of the date 1 bargaining game with publicly observable signals in the following
proposition.

**Proposition 1.** Suppose the signal is publicly observable. The unique equilibrium payoffs of $\Gamma(x)$
conditional upon signal $s$ being received is given by $V^1_c(x, s) = 0$ for the initial capitalist and
\[
V^1_e(x, s) = \begin{cases} 
\frac{\beta_e}{\beta_c}(R(s, x)), & s > s^*(x) \\
0, & s \leq s^*(x) 
\end{cases}
\]
for the entrepreneur, as the time period between offers $\Delta \rightarrow 0$.

**Proof.** See Appendix.

In other words, if the signal is publicly observable, then the entrepreneur extracts the entire
expected continuation surplus from the project. For any initial investment level $x$, the capitalist
abandons the project for signal realizations $s < s^*(x)$ and limits his sunk investment to $x$. However,
in instances where the project is completed, he provides continuation financing at competitive terms.
Given this date 1 bargaining outcome, the expected payoff of the capitalist from investing $x > 0$ at
date 0 is negative. Therefore, we have

**Proposition 2.** Suppose the information signal at date 1 is publicly observable. Then, the equi-
librium initial investment is zero and hence relationship financing is not viable.

**Proof.** Given that the date 1 value function of the capitalist is $V^1_c(x, s) = 0$ for every signal
realization, his expected payoff from investing $x > 0$ at date 0 is $V^0_c(x) = -x$. Therefore, any
positive initial investment level yields him a negative expected payoff.

Proposition 2 suggests that for relationship financing to arise, the outcome of the bargaining
game at date 1 must allocate some surplus to the initial capitalist. In a setting where the information
signal is observable to outsiders as well and the entrepreneur cannot commit not to seek continuation
financing from outsiders, this is not possible. In the absence of any contractual commitments
on continuation terms, ex-post competition drives the initial capitalist’s future rents to zero and
discourages any ex ante investment to start the relationship.12 Next, we analyze the bargaining
game at date 1, when the signal is observable only to the initial capitalist and the entrepreneur.

---

12 Petersen and Rajan (1995) convincingly argue that it is in general not possible to promote relationships via
3.1.2 Unobservable Signals and Informational Lock-In

Suppose now that the information signal at date 1 is only observable to the capitalist who provides initial funding \( x \) and to the entrepreneur. In particular, if the entrepreneur opts out and seeks continuation financing from the outside financiers, the terms of continuation financing (the outside offers she will receive) depend on the beliefs of the outsiders on the signal realization. This implies that in this case, the value of entrepreneur’s outside option is determined by the equilibrium opting out behavior.

The equilibrium concept that we employ is that of perfect Bayesian equilibrium (PBE). A PBE is a collection of behavioral strategies and a belief system such that the behavioral strategy of each player is optimal given the beliefs and other players’ strategies, and the beliefs are derived from the chance probabilities and equilibrium strategies via Bayes’ law whenever possible. If the signal is unobservable, then the outsiders can only condition on the event that the capitalist or the entrepreneur opts out in equilibrium, as well as the initial contract \( x \), in forming their beliefs on the signal realization and hence their beliefs on the final payoff of the venture. Since the only event in which the outsiders need to take an action, and hence their beliefs matter, is when the entrepreneur opts out, and since they do not observe anything but that the entrepreneur has opted out in this event, there is only one information set of the outsiders, which we denote by \( I \).

For any function \( u : \mathbb{R} \to \mathbb{R} \), we denote the expected value of \( u(Y) \) conditional upon observing the entrepreneur opting out by \( E[u(Y)|I] \).

We first analyze how the value of outside options are determined in equilibrium. The characterization of the opting out behavior of the capitalist is straightforward, since outsiders do not take any action if the capitalist opts out.

**Proposition 3.** In any PBE of \( \Gamma(x) \), the capitalist opts out on the equilibrium path if \( s < s^*(x) \) and does not opt out if \( s > s^*(x) \).

**Proof.** Fix an initial investment level \( x \in (0, 1) \) and suppose that \( s < s^*(x) \). Then, the most that the capitalist can get by continuing bargaining is the whole expected continuation surplus

\[
R(s, x) < R(s^*(x), x) = 0,
\]

whereas he can get zero if he opts out and liquidates. Therefore, all capitalists with \( s < s^*(x) \) opt out on the equilibrium path. Suppose now that \( s > s^*(x) \) but the capitalist opts out on the equilibrium path at time \( t \) after observing \( s \). This implies that the payoff from accepting the equilibrium offer of the entrepreneur, \( \pi_t^*(Y) \) must be smaller than his outside option of zero. Note contracts, by which the borrower commits to share surplus in the future. “..complicated bonding contracts could commit the firm to sharing surplus. For example, the entrepreneur could commit to asking the bank for a loan at date 1 (giving it a ‘right of first refusal’) at a predetermined high rate. There may be practical difficulties in enforcing such a contract, primarily because a successful firm in a competitive credit market can easily find ways of borrowing without violating the contract.” (p. 415). Certain binding clauses are used in venture capital contracting like preemptive rights and anti-dilution rights which make it costly for the entrepreneur to seek outside financing. (Sahlman 1990). Our analysis explains why an entrepreneur might agree to such terms that restrict her ability to obtain financing from other sources in the first place.
that for \( s > s^* (x) \), we have \( R(s, x) > 0 \) by Assumptions 2 and 3. But, then the entrepreneur, upon observing a signal \( s > s^* (x) \), could offer \( \pi^t_e(Y) \) such that

\[
E \left[ \pi^t_e(Y) | s \right] = \frac{R(s, x)}{2 \beta_c} > 0,
\]

at time \( t \), which the capitalist would certainly accept. This would yield the entrepreneur a strictly positive payoff rather than her equilibrium payoff of zero if the capitalist opts out. Therefore, \( \pi^t_e(Y) \) cannot be an equilibrium offer, yielding a contradiction. \( \square \)

**Remark 1.** Proposition 3 and the fact that the equilibrium value of the outside option of the capitalist is equal to zero continue to be true even if we were to assume that the liquidation value, rather than being zero, is a function \( Q(Y, x) \) with \( Q(Y, x) \leq \beta_c Y - (1 - x) \), for all \( x \in (0, 1) \). This condition requires that the liquidation value can never be higher than the true continuation surplus. One can justify such an assumption by the human capital value of the entrepreneur in the project. In case of liquidation, the entrepreneur no longer works for the venture. Then, the condition \( Q(Y, x) \leq \beta_c Y - (1 - x) \) merely means that the new owners of the venture’s assets who pay for the liquidation value cannot generate a payoff higher than the true continuation value of the project with the entrepreneur.

Now, we turn to the equilibrium value of the outside option of the entrepreneur, when the outsiders do not observe the information signal. The question is whether the entrepreneur can raise the continuation financing from the competitive but uninformed outsiders in this asymmetric information setting or she would be ‘informationally locked in’ and continue with the initial capitalist at the cost of losing part of the continuation surplus. Recall that if the entrepreneur opts out, the outsider capitalists make competitive offers \( b(Y) \in [0, Y] \) which specify the entrepreneur’s payoff for each payoff state. In exchange for providing the continuation funds \( 1 - x \), then, the outside capitalist holds a claim which pays her \( y - b(y) \) if the payoff realization at date 2 is \( Y = y \). Our next result shows that, in any equilibrium in which \( y - b(y) \) is strictly increasing in \( y \), the equilibrium value of the entrepreneur’s outside option is zero, regardless of the information she has over the prospects of the venture. In other words, the entrepreneur is informationally captured by the initial capitalist in the bargaining game over continuation.

**Proposition 4. (Full Informational Lock-In)** Suppose the signal is not observable to outsiders.

In any PBE equilibrium of \( \Gamma (x) \) in which \( y - b(y) \) is strictly increasing,

(a) the entrepreneur opts out on the equilibrium path if \( s < s^* (x) \),

(b) the entrepreneur does not opt out on the equilibrium path if \( s > s^* (x) \),

(c) the equilibrium outside offer is \( b(y) = 0 \), for all \( y \). Therefore, the equilibrium value of the entrepreneur’s outside option is zero for every signal realization.

**Proof.** See the Appendix.

\(^{13}\)This would hold, for instance, in case of equity claims, i.e., the offers of the outsiders are of the form \( b(y) = \alpha y \) for some \( \alpha \in [0, 1] \). In case of venture capital financing, this seems plausible, since venture capitalists usually hold equity-like claims.
The main idea is that an entrepreneur with a high signal suffers from adverse selection in the sense that if she were to attempt to seek financing from an outside capitalist, she would be believed by the outsiders to have received a signal low enough to make the expected continuation surplus negative from the perspective for outsiders. For this reason, entrepreneurs with a signal $s > s^* (x)$, i.e., precisely those entrepreneurs for whom continuation is profitable, never opts out on the equilibrium path. The above proposition also reconciles the empirical observation by Sahlman (1990) that it is very rare for an entrepreneur to raise continuation financing from parties other than the initial capitalist.

Remark 2. An alternative bargaining specification would be allowing the entrepreneur to make an offer to the outside capitalists when she opts out, rather than the outsiders making the offers. In this case, the entrepreneur might have a chance to break the lock-in by signaling her information through the offer she makes. In a model similar to the one we present, Admati and Pfleiderer (1994) show that when the entrepreneur offers outsiders a set of securities (a sharing rule over final payoff) to signal her information, fully revealing equilibrium is not robust. In our context, however, although the entrepreneur may not be able to reveal her information fully, she may still try to signal that she has received an information $s > s^* (x)$, which makes continuation profitable. We analyze this possibility in Section 4 and show that our full informational lock-in result goes through under this specification of the bargaining game as well.

Remark 3. It is important to emphasize the difference between the way we model the continuation bargaining game and the way it is modeled in Rajan (1992). In his paper, after the signal is revealed to the entrepreneur and the initial capitalist (the insider bank in Rajan), the entrepreneur asks both the informed insider and an uninformed outsider to simultaneously submit a sealed bid for continuation financing. This results in two differences between his paper and ours. First, in his model, due to the winner’s curse, no equilibrium in pure strategies exists. Rajan concludes that the informational advantage of the insider may result in a lock-in, but not with certainty. In contrast, in our model information lock-in occurs with probability one. Second, Rajan’s model does not allow for the outsider to condition his beliefs regarding the profitability of the firm on any action taken by the insiders. Therefore, his beliefs that determine his bid remain exogenous. In contrast, in our setting, outsiders only act when the entrepreneur opts

---

14 A similar adverse selection idea for endogenizing the value of outside options is explored in a recent paper by Landier (2001). His model endogenizes outside options for entrepreneurs who have to decide whether to fail and restart or continue with a mediocre project. The value of their outside option (failing) is the cost of funds to restart and this is determined endogenously by outsiders’ belief on the type of the failing entrepreneur, a high type entrepreneur who chooses to fail for a fresh start, or a bad type who really fails (and will fail in the future).

15 They conclude that: “...if we rely on the communication of information through financial contracts, then asymmetric information between the entrepreneur and capital providers is likely to persist in equilibrium.” (page 382).

16 When the interim competition takes the form of simultaneous bidding by an informed and an uninformed lender, a generic result is that due to winner’s curse, no equilibrium exist in pure strategies. Von Thadden (2001) presents a rigorous analysis of this class of games and shows that in that bargaining specification, the borrower (entrepreneur) will only be partially locked-in to the insider (initial capitalist) and in equilibrium borrowers may switch lenders.
out from the bargaining with the insider. Therefore, their beliefs on the entrepreneur’s information is determined in equilibrium by Bayes Rule. Outsiders condition on the event that the entrepreneur has opted out and make a competitive offer given their beliefs on the signal realization.

Now, given the equilibrium value of outside options, we can characterize the equilibrium payoffs of the date 1 bargaining game, when the signal is observable only to the capitalist and the entrepreneur. Let $V^1_i(x, s)$ denote the equilibrium payoff of player $i \in \{c, e\}$ of the bargaining game $\Gamma(x)$ conditional upon the signal $s$. The following result shows that the unique equilibrium outcome is such that the venture is liquidated for signal realizations below $s^*(x)$, and above that threshold level, the initial capitalist provides the continuation financing and captures part of the expected continuation surplus.

**Theorem 1.** Suppose the signal is observable only to the entrepreneur and the initial capitalist. The unique equilibrium payoffs of $\Gamma(x)$ conditional upon signal $s$ being received is given by

$$V^1_c(x, s) = \begin{cases} (1 - \eta)(R(s, x)), & s > s^*(x) \\ 0, & s \leq s^*(x) \end{cases}$$

for the initial capitalist and

$$V^1_e(x, s) = \begin{cases} \eta \frac{r_c}{r_c + r_e} (R(s, x)), & s > s^*(x) \\ 0, & s \leq s^*(x) \end{cases}$$

for the entrepreneur, where $\eta = \frac{r_c}{r_c + r_e}$, as the time period between offers $\Delta \to 0$.

**Proof.** See Appendix.

Having characterized the unique equilibrium outcome of the bargaining game at date 1, we now move to date 0 and solve for the date 0 contract that specifies the initial investment level $x$. In particular, we are interested in describing how the anticipation of sharing future rents with the capitalist affects the entrepreneur’s choice of the financing schedule. Does the entrepreneur prefer to adopt relationship financing and if she does, is relationship financing viable? How does the lock-in after information revelation determine the initial investment? From Theorem 1, we know that for signal realizations above $s^*(x)$, the capitalist provides the remaining $1 - x$ and captures $(1 - \eta)$ of expected continuation surplus, whereas the entrepreneur obtains the remaining $\eta$ of continuation surplus. Recalling that $s^*(x)$ is a function of initial investment $x$, one may write down the ex ante (date 0) payoff functions $V^0_c(x)$ (for the capitalist) and $V^0_e(x)$ (for the entrepreneur) as follows:

$$V^0_c(x) = (1 - \eta) \beta_c \int_{s^*(x)}^\infty R(s, x)dG(s) - x,$$

$$V^0_e(x) = \eta \frac{\beta^2_e}{\beta_c} \int_{s^*(x)}^\infty R(s, x)dG(s).$$

The following proposition, which is instrumental in characterizing the equilibrium initial investment, describes how the date 0 payoff functions of the parties depend on the initial investment $x$. 

14
**Proposition 5.** The date 0 value function of the capitalist (entrepreneur), \( V_e^0 (x) \), is strictly decreasing (increasing) in the initial investment level \( x \).

**Proof:** See Appendix.

The intuition behind this proposition is as follows: First, notice that \( s^* (x) \) is strictly decreasing in \( x \in (0, 1) \), as \( E[Y|s] \) is strictly increasing in \( s \) (by Assumption 2) and \( (1 - x) / \beta_c \) is strictly decreasing in \( x \). Therefore, higher initial investment levels shift the threshold level of signal realization below which the venture is terminated to the left and makes continuation more likely. Second, the expected continuation surplus, \( R(s, x) \), increases in \( x \) for a given level of \( s \). Therefore, the ex ante expected continuation surplus \( \int_{s^*(x)}^{\infty} R(s, x)dG(s) \) increases as \( x \) increases. However, the effect of the decrease in \( s^* (x) \) is negligible for small increases in \( x \), since at the threshold signal value, \( s^* (x) \), the continuation value \( R(s^*(x), x) \) is equal to zero. Therefore, a small increase in \( x \) leads to higher expected continuation value only through its effect on \( R(s, x) \). Furthermore, a higher initial investment \( x \) has no cost for the entrepreneur, and therefore her expected value at date 0 is increasing in \( x \). However, an increase in \( x \) increases the cost for the capitalist by an equal amount, whereas the benefit increases by less, because the capitalist can extract only a portion of the date 1 continuation surplus. Therefore, the net expected value of the capitalist at date 0 is decreasing in \( x \).

Proposition 5 implies that, in designing the date 0 equilibrium contract, the entrepreneur will choose the highest possible initial investment level that satisfies the capitalist’s ex ante (date 0) individual rationality constraint. In other words, due to its effect on the date 1 bargaining game, the initial investment level \( x \) becomes a strategic variable for the entrepreneur, as it would for a monopolist. Although the signal is not contractible and explicit signal contingent continuation schemes are not possible, informational lock-in gives the capitalist an implicit option to further invest in the venture and extract surplus, the cost of which is the initial investment he provides. By making this implicit option as costly as possible (through the highest possible level of initial investment), the entrepreneur forces the capitalist down to his reservation level. Our last result characterizes this equilibrium initial investment level \( x^* \), which sets \( V_e^0 (x^*) = 0 \), and establishes that the equilibrium financing mode is relationship financing.

**Theorem 2.** Suppose the signal is observable only to the entrepreneur and the initial capitalist. The equilibrium initial investment level \( x^* \) is unique, lies in the interval \((0, 1)\) and is determined by the following equation:

\[
    x^* = \beta_c (1 - \eta) \int_{s^*(x^*)}^{\infty} R(s, x^*)dG(s). \tag{3}
\]

At this equilibrium initial investment level \( x^* \), the entrepreneur’s payoff is strictly positive and is given by

\[
    V_e^0 (x^*) = \frac{\eta}{1 - \eta} \left( \frac{\beta_e}{\beta_c} \right)^2 x^* > 0
\]

Therefore, relationship financing is viable and it is the equilibrium financing mode.
Proof: See Appendix.

The above result, together with Theorem 1, completely characterizes the equilibrium financing mode, when the information signal is only observable to insiders who start the project at date 0 and shows that it is relationship financing. Notice that the equilibrium initial investment level $x^*$ described in equation (3) is such that the present value of capitalist’s future expected continuation rents (right hand side) is equal to cost of starting the relationship today (initial investment). The more surplus the capitalist extracts in the future, which could be due to an increased bargaining power, the higher is the initial investment before information revelation.

A key point to note is that, the initial capitalist’s ability to extract rents in the continuation game at date 1 is essential for the viability of relationship financing. In our framework, this is possible with the full endogenous informational lock-in. In contrast to the case with publicly observable signals, relationship financing is viable because signal unobservability creates an informational barrier and eliminates the entrepreneur’s outside option of raising continuation financing from competitive outsiders. In the absence of any ex-ante commitments on continuation terms, this proves essential to protect the initial capitalist from competitive pressure at date 1. Although the entrepreneur cannot ex ante commit to leave him any surplus in both cases, when the signal is unobservable, the bilateral bargaining situation at date 1 allocates the initial capitalist part of the expected continuation surplus. The implication of this for the date 0 contract is that now the initial investment level becomes a strategic variable. It is in a sense the price of the capitalist’s future implicit option to further invest and extract rents. The entrepreneur’s choice of the date 0 investment level is then the highest possible $x$ that the capitalist is willing to pay to reach to the lock-in stage where he attains bargaining power due to his superior information compared to uninformed outsiders.

An important question is whether the equilibrium level of the initial investment is socially optimal. To this end, define the expected social surplus at date 0 as

$$V_s^0 (x) = \beta_s \int_{1}^{s^* (x)} (\beta_s E [Y | s] - (1 - x)) dG (s) - x,$$

where $\beta_s \in (0, 1)$ is the social discount factor. In the current setting the socially efficient investment level is not well defined, as $V_s^0$ is strictly decreasing and $x = 0$ can never be socially optimal. Nonetheless, Theorem 2 implies that, from the society’s point of view, there is overinvestment in equilibrium.

**Corollary 1.** Equilibrium level of date 0 investment is not socially optimal.

**Proof:** The social surplus is strictly decreasing in the level of initial investment, and hence there is a positive investment level that is smaller than $x^*$ which yields a higher social surplus than does $x^*$.

It is illustrative to discuss our overinvestment result in reference to the underinvestment result in Rajan (1992). In Rajan’s model, the initial investment level that takes the firm to the information stage is fixed. What determines the ex-ante likelihood of the good state is the entrepreneur’s
effort following the fixed investment. The project is continued at the good state, but the informed financier extracts part of this continuation surplus. Anticipating that she will only capture part of the benefits from her effort, the entrepreneur underinvests in effort. In our setting, the decision variable at date 0 is not the entrepreneur’s effort, but the amount of capital to raise and invest. This initial investment level has no effect on the realization of the information signal, but for every signal realization, it determines the size of the continuation surplus. Since the entrepreneur anticipates the informational lock-in and the loss of continuation surplus at date 1, she makes this rent extraction as costly as possible to the capitalist by making him overinvest. As we discussed, the initial investment is a strategic variable in our setting, not because it affects the ex ante likelihood of different information states, but because it is essentially the price of becoming an insider and extracting rents in the future due to informational lock-in. In that respect, our overinvestment result is similar to Sharpe (1990) where ex ante competition between capital providers for ex post monopoly rents drives interest rates low and causes overinvestment.

Therefore, one empirical implication of our analysis is that the more bargaining power the relationship lender has in the future due to his superior information, the higher is the initial investment before information revelation. This overinvestment before arrival of information result can also formally reconcile the empirical observation by Gompers (1998) who asks whether the venture capital market should diet: “...none of the existing research indicates that current funding levels are too high.” (p. 1103). The period Gompers refers to is characterized by too many capitalists chasing too few deals in a bullish venture capital market during the heydays of .com start-ups. Clearly, during that period entrepreneurs were enjoying a great deal of bargaining power initially when raising their first round of financing. Our result suggests that the anticipation to have less bargaining power in future rounds might have caused the entrepreneurs to raise and invest too much capital initially.

4 Summary and Extensions

4.1 Summary

Our analysis formalizes the following points:

(i) Ex post competition for providing continuation financing hurts the viability of relationship financing ex ante. For relationship financing to be viable, the relationship financier must be immune from ex post competition, in the sense that he must be able to capture rents in the future financing round. When the information generated in the relationship is observable to outsiders as well, relationship financing is not viable, unless the entrepreneur can commit to give up some surplus to the capitalist by other contractual means.

(ii) In a setting that interim information is not observable to outsiders, the informational asymmetry between the entrepreneur and the competitive but uninformed financiers alone can act as an entry barrier and eliminate the entrepreneur’s access to competitive financing terms. This ex-post
informational lock-in, in turn, allocates the initial capitalist some surplus and makes relationship financing viable.

(iii) Although the entrepreneur anticipates being locked-in to the initial capitalist for continuation financing in the future, she strictly prefers relationship financing. In equilibrium, the entrepreneur borrows and invests too much before information revelation.

Our result on the viability of relationship financing implies that, instead of creating ex-post competition, the entrepreneur might have an ex ante incentive to protect the initial financier from competition, so that she can raise initial funds at date 0. To the extent that the information generated in the relationship is accessible by outsiders, contractual clauses that restrict the entrepreneur’s future outside financing options and protect the initial financier from too much ex-post competition are likely to emerge. This explains the use of certain contractual clauses in venture capital financing, like rights of first refusal, preemptive rights and no de novo financing (see Chan et al (1990)) that restrict the entrepreneur’s ability to seek financing from other sources in the future financing rounds.

4.2 Extensions

Multiple Relationships. Our analysis restricts attention to the case where the entrepreneur can start a relationship only with one capitalist at date 0. Suppose instead that the entrepreneur starts a relationship with two different capitalists and raises the initial investment from these two different sources. How does the presence of multiple insider capitalists affects the division of continuation surplus at date 1 when both of these capitalists observe the information signal? If the insider capitalists are engaged in a Bertrand competition at date 1 (e.g., via a first-price sealed bid auction) then it is easy to see that this scenario is equivalent to the case of publicly observable signals (Proposition 1). Since, the bargaining outcome does not allocate any continuation surplus to any of the two capital providers in this case, none of them invests a positive amount at date 0. In that sense, in our framework, only a single relationship can arise, since multiple relationships create ex-post competition that makes the relationship financing not viable in the first place.

Therefore, another empirical implication of our model is that multiple relationships decrease the willingness of the individual financiers to invest initially, since the initial investment by a financier is increasing in the surplus she extracts in the continuation game. In other words, multiple relationships break the ex-post informational lock-in and therefore discourage relationship financing. Ongena and Smith (2000) provide empirical evidence consistent with this insight and show that multiple relationships worsen the availability of initial funding.

\footnote{This possibility has also been addressed by Von Thadden (1994) who shows that when the two financiers engage in Bertrand competition for providing continuation funds after information arrival, all the expected continuation surplus is captured by the entrepreneur.}

\footnote{Detragiache et al (2000) provide a model where multiple relationships can arise to due to the possibility of premature liquidation of a profitable project. This might happen because the unexpected liquidity problems might make the relationship banks unable to roll over their initial loans. Multiple relationships serve to increase the probability that at least one bank will be able to refinance the firm.}
Signaling. In our specification of the bargaining game at date 1, we assumed that if the entrepreneur opts out, it is the outsiders who make offers to her. Alternatively, the entrepreneur could make contract offers \( \pi(Y) \) (interpreted as the share the entrepreneur would get) to the outsiders and this brings about the possibility that she might be able to communicate the signal realization and hence capture the entire expected surplus. In any equilibrium of this modified game that satisfies a similar monotonicity condition, i.e., \( y - \pi(y) \) is strictly increasing in \( y \), the value of the outside option of the entrepreneur is zero, entrepreneurs who receive signals \( s < s^*(x) \) opt out and receive a zero payoff, whereas those with signals \( s > s^*(x) \) do not opt out.

To see this, first notice that if there is an equilibrium in which the entrepreneur opts out with an offer that is accepted by the outsiders after some signal realizations, then each of these offers must be different, i.e., equilibrium must be fully separating. This is the case since for each signal \( s \) after which the entrepreneur opts out with an offer \( \pi_s(Y) \) that is accepted, it must be the case that

\[
E[\pi_s(Y) | s] \geq \frac{R(s,x)}{\beta_c},
\]

for otherwise the insider capitalist could make a more competitive offer, which the entrepreneur would accept rather than opting out. Therefore, if there are two signals \( s' > s'' \) who make the same offer \( \pi(Y) \) we would have

\[
E[Y - \pi(Y) | s \in \{s', s''\}] - \frac{1-x}{\beta_c} < E[Y - \pi(Y) | s'] - \frac{1-x}{\beta_c} \leq 0
\]

by Assumption 2, contradicting that the offer \( \pi(Y) \) is accepted. This implies that there is no equilibrium in which an entrepreneur with a signal \( s > s^*(x) \) opts out with an offer that is accepted. If there was such an equilibrium, then it would have to be fully separating. Together with the optimality of the insider capitalist and the outsider capitalists, this would imply that

\[
E[\pi_s(Y) | s] = \frac{R(s,x)}{\beta_c} > 0.
\]

Such an offer, however, would always be mimicked by entrepreneurs with signals smaller than \( s^*(x) \), who receive zero in equilibrium and hence has an incentive to make an offer which pay positive amounts at some states. This, in turn, would contradict with the hypothesis that \( \pi_s(Y) \) is accepted in equilibrium. Therefore, the equilibrium outcome that is specified in Proposition 4 is robust to signalling opportunities by the entrepreneur.
5 Appendix

Proof of Proposition 1. First suppose \( s > s^*(x) \). Then, the outside options of the capitalist and the entrepreneur are 0 and \( \frac{\beta_e}{\beta_c} (R(s,x)) \), respectively. It is well known that in the unique (stationary and no-delay) equilibrium, the offers \( \pi_i(Y) \) for \( i \in \{c,e\} \) must satisfy (see Muthoo, 1999)

\[
\beta_c E[Y - \pi_c(Y)|s] = \delta_e \beta_c \max \left\{ E[\pi_e(Y)|s], \frac{R(s,x)}{\beta_c} \right\}
\]

(4)

\[
\beta_e E[Y - \pi_e(Y)|s] - (1 - x) = \delta_c [\beta_c E[\pi_c(Y)|s] - (1 - x)].
\]

(5)

It can be easily verified that assuming \( E[\pi_e(Y)|s] \geq \frac{R(s,x)}{\beta_c} \) leads to a contradiction. Therefore, we have

\[
\beta_e E[Y - \pi_e(Y)|s] = \delta_e \beta_e \left( \frac{R(s,x)}{\beta_c} \right)
\]

(6)

Equations (5) and (6) are solved as

\[
E[\pi_e(Y)|s] = \frac{(1 - \delta_e(1 - \delta_e)) (R(s,x))}{\beta_c}
\]

\[
E[\pi_c(Y)|s] = (1 - \delta_c) E[Y|s] + \frac{\delta_e}{\beta_c} (1 - x)
\]

Recalling \( \delta_i = \exp(-\Delta r_i) \) for \( i \in \{c,e\} \) and taking limits, we have

\[
\lim_{\Delta \to 0} \beta_e E[\pi_e(Y)|s] = \frac{\beta_e}{\beta_c} (R(s,x))
\]

as the entrepreneur’s payoff and

\[
\lim_{\Delta \to 0} \left( \beta_c E \left[ Y - \frac{\beta_c Y(1 - x)}{\beta_c} \right] - (1 - x) \right) = 0.
\]

as the capitalist’s payoff.

If, on the other hand, \( s \leq s^*(x) \), then both the entrepreneur and the capitalist have zero outside options. If \( s < s^*(x) \), then these outside options are binding and the project is liquidated by the capitalist in equilibrium. If \( s = s^*(x) \), then the expected continuation surplus is zero and hence whether one of the players opts out or not, equilibrium payoffs are zero.

Proof of Proposition 4. Part (a) is easy to see as in this case the expected surplus, and hence the most that the entrepreneur can get by continuing bargaining, is negative, whereas she can get at least zero by opting out. Therefore, upon observing a signal \( s < s^*(x) \), the entrepreneur opts out.

To prove (c), recall that the outsiders market is competitive and the required rate of return is zero. Therefore, if \( \beta_c E[Y|I] - (1 - x) < 0 \), then the equilibrium offer by the outsiders is \( b(Y) = 0 \), for all \( y \). If, on the other hand, \( \beta_c E[Y|I] - (1 - x) \geq 0 \), then the equilibrium offer \( b(Y) \) by the outsiders satisfies

\[
E[b(Y)|I] = \frac{\beta_c E[Y|I] - (1 - x)}{\beta_c}.
\]

(7)
Now, suppose that there exists a set $A$ which has positive measure under $G$ such that $\min A > s^*(x)$, and the entrepreneur opts out on the equilibrium path if she observes $s \in A$. With a slight abuse of notation let $A$ be the union of all such sets. Then, it must be the case that
\[
\frac{\beta_c}{\beta_c} \left( R(s, x) \right) \leq \beta_c E \left[ b(Y) | s \right] \text{ for all } s \in A,
\]
for otherwise, the insider capitalist, upon observing an $s \in A$, could make an offer that would be certainly accepted by the entrepreneur and would give the capitalist a positive payoff, rather than the equilibrium payoff of zero.$^{19}$ Since $y - b(y)$ is strictly increasing, we have
\[
\frac{\beta_c}{\beta_c} \left( R(s, x) \right) < \beta_c E \left[ b(Y) | s \right] \text{ for all } s < \min A,
\]
by Assumption 2. Together with part (a) this implies that the entrepreneur opts out if, and only if, $s \leq \max A$. We will first show that $\beta_c E [Y|I] - (1 - x) < 0$, i.e., conditional on the event that the entrepreneur opts out, outsiders will always believe that expected continuation surplus is negative. Suppose, for contradiction, that $\beta_c E [Y|I] - (1 - x) \geq 0$. Then, we have
\[
E \left[ Y - b(Y) | I \right] = E \left[ Y - b(Y) | s \leq \max A \right] = \frac{1 - x}{\beta_c} \geq E \left[ Y - b(Y) | s = \max A \right],
\]
by (7) and (8), which contradicts Assumption 2 and that $A$ has positive measure. Therefore, $\beta_c E [Y|I] - (1 - x) < 0$, and hence $b(y) = 0$ for all $y$. If, on the other hand, the measure of the set of signals $s > s^*(x)$ such that the entrepreneur opts out is zero, then we again have
\[
\beta_c E [Y|I] - (1 - x) = \beta_c E [Y | s \leq s^*(x)] - (1 - x) < 0,
\]
by $R(s^*(x), x) = 0$ and Assumption 2, and hence $b(y) = 0$ for all $y$.

Now, part (b) easily follows as
\[
\frac{\beta_c}{\beta_c} \left( R(s, x) \right) > \beta_c E \left[ b(Y) | s \right] = 0 \text{ for all } s > s^*(x),
\]
and hence the entrepreneur never opts out after observing a signal $s > s^*(x)$.

Proof of Theorem 1. Let $s < s^*(x)$. Then, as it was shown by Propositions 3 and 4, the outside options of the parties are binding and the first player who gets the chance (the capitalist in our model) opts out and both players receive a zero payoff. If, on the other hand, $s = s^*(x)$, then whether the equilibrium path is characterized by opting out or not, both players receive an expected payoff of zero.

$^{19}$One such offer is $\bar{\pi}_c(Y)$ such that
\[
E [\bar{\pi}_c(Y) | s] = \frac{R(s, x) - E [b(Y) | s]}{2},
\]

$^{20}$This statement is not precisely correct as we have not shown that the entrepreneur with signal $s^*(x)$ opts out. But the proof goes through even if entrepreneurs with signal $s^*(x)$ do not opt out. Also, the proof can easily be modified so that the claim is still true if $\min A$ or $\max A$ does not exist.
If $s > s^*(x)$, then, since the outside options are zero in equilibrium, the standard Rubinstein bargaining outcome results. In particular, in the unique (stationary and no-delay) equilibrium, the offers $\pi_i(Y)$ must satisfy

$$\beta_c E[Y - \pi_e(Y)|s] - (1 - x) = \delta_c [\beta_c E[\pi_c(Y)|s] - (1 - x)],$$
$$\beta_c E[Y - \pi_c(Y)|s] = \delta_e \beta_c E[\pi_e(Y)|s],$$

so that both players are indifferent between accepting an offer and rejecting (and making a counter-offer). Noting that $R(s, x) = \beta_c E[Y|s] - (1 - x)$, these equations are solved as

$$E[\pi_e(Y)|s] = \frac{(1 - \delta_c) (R(s, x))}{\beta_c (1 - \delta_c \delta_e)},$$
$$E[\pi_c(Y)|s] = \frac{(1 - \delta_e) \beta_c E[Y|s] + \delta_e (1 - \delta_e) (1 - x)}{\beta_c (1 - \delta_c \delta_e)}.$$

Therefore, the equilibrium payoffs are

$$\beta_c E[\pi_e(Y)|s] = \frac{\beta_c (1 - \delta_c) (R(s, x))}{\beta_c (1 - \delta_c \delta_e)},$$
$$\beta_c E[Y - \pi_e(Y)|s] - (1 - x) = \frac{\delta_c (1 - \delta_e) (R(s, x))}{(1 - \delta_c \delta_e)}.$$

Recalling $\delta_c = \exp(-\Delta r_c)$ and $\delta_e = \exp(-\Delta r_e)$, and taking limits, we have

$$\lim_{\Delta \to 0} \beta_c E[\pi_e(Y)|s] = \frac{r_c}{r_c + r_e} \frac{\beta_c}{\beta_c} (R(s, x)) = \eta \frac{\beta_c}{\beta_c} (R(s, x)), $$

and

$$\lim_{\Delta \to 0} (\beta_c E[Y - \pi_e(Y)|s] - (1 - x)) = \frac{r_e}{r_c + r_e} (R(s, x)) = (1 - \eta) (R(s, x)).$$

Proof of Proposition 5. First, notice that $s^*(x)$ is strictly decreasing in $x$, because $E[Y|s]$ is strictly increasing in $s$ and $(1 - x)/\beta_c$ is strictly decreasing in $x$. Since, expected continuation surplus $R(s, x)$ is strictly increasing in $x$, it is easy to see that $V^0_e(x)$ is also strictly increasing. A more precise description of the effect of $x$ on the entrepreneur’s date 0 value function $V^0_e(x)$ can be obtained by applying Leibniz integral rule:

$$\frac{d}{dx} V^0_e(x) = \eta \frac{\beta^2_e}{\beta_c} \left( \int_{s^*(x)}^\infty dG(s) - \frac{ds^*(x)}{dx} (R(s^*(x), x)) dG(s^*(x)) \right)$$

$$= \eta \frac{\beta^2_e}{\beta_c} \int_{s^*(x)}^\infty dG(s)$$

$$= \eta \frac{\beta^2_e}{\beta_c} (1 - G(s^*(x))) > 0,$$

by Assumption 3.
Similarly, applying Leibniz integral rule to the value function of the capitalist yields:

\[
\frac{d}{dx} V_c^0(x) = (1 - \eta) \beta_c (1 - G(s^*(x))) - 1 < 0.
\]

**Proof of Theorem 2.** Since, the entrepreneur has monopoly power over the project at date 0, she solves the following problem:

\[
\max_{x \in (0, 1)} V_c^0(x) \text{ s.t. } V_c^0(x) > 0.
\]

If the value of this problem exists and is strictly positive, then staged financing is the equilibrium outcome. If \(x^*\) is a solution to the above problem, then

\[
V_c^0(x^*) = \beta_c (1 - \eta) \int_{s^*(x^*)}^{\infty} R(s, x^*) dG(s) - x^* = 0,
\]

for \(V_c^0(x)\) is strictly increasing and \(V_c^0(x)\) is strictly decreasing in \(x\). This yields (3).

If equilibrium exists, then it is unique as \(V_c^0(x)\) is strictly decreasing. To prove the existence of an \(x^* \in (0, 1)\) such that \(V_c^0(x^*) = 0\), first notice that \(V_c^0(x)\) is continuous, and that \(\lim_{x \to 1} s^*(x) \geq 0\), and \(\lim_{x \to 0} s^*(x) < \infty\) (by Assumption 3). Therefore, we have

\[
\lim_{x \to 1} V_c^0(x) = (1 - \eta) \beta_c \int_{\lim_{x \to 1} s^*(x)}^{\infty} \beta_c E[Y|s] dG(s) - 1 \\
\leq (1 - \eta) \beta_c^2 \int_{0}^{\infty} E[Y|s] dG(s) - 1 \\
= (1 - \eta) \beta_c^2 E[Y] - 1 < \beta_c E[Y] - 1 < 0,
\]

(9)

and

\[
\lim_{x \to 0} V_c^0(x) = (1 - \eta) \beta_c \int_{\lim_{x \to 0} s^*(x)}^{\infty} (\beta_c E[Y|s] - 1) dG(s) > 0.
\]

(10)

The last inequality follows from the facts that \(\lim_{x \to 0} s^*(x) < \infty\) and \(\beta_c E[Y|s] - 1 > 0\) for all \(s > \lim_{x \to 0} s^*(x)\). Continuity of \(V_c^0\) and equations (9) and (10) imply that there exists an \(x^* \in (0, 1)\) such that \(V_c^0(x^*) = 0\).

Finally, we have

\[
V_c^0(x^*) = \eta \frac{\beta_c^2}{\beta_c} \int_{s^*(x^*)}^{\infty} (R(s, x^*)) dG(s) = \frac{\beta_c^2}{\beta_c} V_c^0(x^*) + x^* = \frac{\eta}{1 - \eta} \left( \frac{\beta_c}{\beta_c} \right)^2 x^* > 0,
\]

and hence relationship financing is the unique equilibrium outcome. \(\Box\)
References


