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Risk sharing, risk shifting and the role of convertible debt

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ABSTRACT

This paper considers a financial contracting problem between a risk neutral entrepreneur and a risk averse investor. Once the venture is started, the entrepreneur chooses an action that determines the riskiness of the venture's payoff. When action choice is contractible, the optimal risk sharing consideration under limited liability calls for a pure debt contract and the low risk action is adopted. When the action choice is not contractible, due to the risk shifting problem implementing the low risk action requires a deviation from the optimal risk sharing. I focus on situations where despite this deviation, the risk averse investor prefers to implement the low risk action and show that a convertible debt contract is superior to pure debt, pure equity and any mixture of debt and equity.

1. Introduction

Financing a young entrepreneurial firm with a risky business plan is subject to important informational and incentive problems. Rather than more common instruments like debt or equity, investors who provide financing for entrepreneurial firms typically hold a convertible debt claim. In a recent empirical study on financial contracting by Kaplan and Stromberg (2003), convertible securities account for over 90% of all financing agreements in their sample of start-up firms. Previous work by Sahlman (1990) and Gompers (1997) also report the extensive use of convertible debt in venture capital backed entrepreneurial firms.

A convertible debt contract combines the properties of debt and equity. The conversion option gives the claimholder the right to convert the debt claim into company's equity. This paper offers an explanation on why convertible debt can be superior to pure debt, pure equity and mixed debt–equity in the venture capital context. I describe a financial contracting problem where a risk neutral entrepreneur finances his venture by funds provided by a risk averse financier/venture capitalist. The admissible sharing rules on the final payoff of the venture are constrained by a limited liability condition. Upon receiving the funds, the entrepreneur adopts a business strategy, which cannot be specified ex ante by the contract. This action choice determines the riskiness of the venture's payoff. In particular, I consider a simple model with two possible actions where the high risk action a_H yields a payoff distribution which is second order stochastically dominated by the distribution induced by the low risk action a_L.

When the action choice is enforceable, under limited liability a pure debt contract achieves optimal risk sharing between parties. Furthermore, under enforceability of actions the financier prefers the low risk action. When the action choice is *not*

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enforceable, a debt contract induces the entrepreneur a preference for the high risk action due to its convex residual payoff. Therefore, implementing the low risk action requires a deviation from the optimal risk sharing arrangement provided by the debt contract. I focus on situations where despite this deviation from optimal risk sharing, implementing the low risk action makes the risk averse financier better off compared to opting for the high risk action. In this setting, a convertible debt contract can be superior to pure debt, pure equity and any mixture of debt and equity. This result follows, because a convertible debt contract combines two desirable properties in this problem. Its debt component assigns the whole payoff to the risk averse financier at the low end of payoff realizations and provides better insurance against the downside risk. The conversion into equity option, on the other hand, provides a convex payoff schedule for the financier at the upper end of payoff realizations, and corrects the entrepreneur's high risk incentives arising from the debt portion of the contract. This role cannot be achieved by simple mixtures of debt and equity, since mixed debt–equity contracts also yield a concave payoff for the financier (and hence a convex one for the entrepreneur) and implement the high risk action.

Since the risk sharing consideration between the risk averse financier and the risk neutral entrepreneur is a crucial aspect of the analysis, the assumption on the risk attitudes of the two parties deserves further comment. The financial contracting problem posed in this paper can be best placed in the venture capital industry context. A typical venture capitalist periodically raises what is called a *venture capital fund* from cash rich institutions such as pension funds and insurance companies. A venture capital fund has a lifetime of 5–7 years, at the end of which the returns are distributed to the fund contributors. In that sense, the venture capitalist acts as a fund manager by choosing which ventures to invest. The ability to raise new capital for future funds depends on the performance of the earlier investments. Sahlman (1990) notes that if a venture capitalist's fund suffers huge losses or even in cases of moderate failures, the chances for raising new capital for the next fund are very limited. As pointed out by Chemmanur and Fulghieri (1999), in a given fund cycle the venture capitalist can only invest in a few ventures, all of which are quite risky, and therefore the capitalist's investment portfolio remains poorly diversified.[1] They also consider a risk averse venture capitalist and argue that the compensation scheme of a venture capitalist from managing a fund involves significant penalties for failures, thereby inducing risk averse behavior.[2] Furthermore, the success or failure of a particular project can significantly affect the reputation of the venture capitalist who made the decision to invest in that venture, again leading to risk aversion.[3]

As in Innes (1990) and Chemmanur and Fulghieri (1999), I consider a risk neutral entrepreneur with the following justification. Unlike the 'agent' of a standard principal-agent model (typically interpreted as a risk averse employee of the principal), an entrepreneur who quits a well paying job to pursue a fortune by launching a new company does not seem to be exhibiting risk averse behavior. Furthermore, what an entrepreneur loses from a failing venture is considerably less than what a venture capitalist loses. For a typical entrepreneur, even receiving funds for the venture can work as a badge of success, and having been in charge of a company, even if it eventually fails, is a valuable experience for a future career, especially in a growing young industry.[4]

Related literature. Previous explanations of convertible securities have focused either on the efficient allocation of control rights paradigm or on an effort type moral hazard problem. Berglof (1994) provides a model where control refers to the right to bargain with an outside party bidding for the venture and shows that convertible security allocates the control to the party who maximizes the joint surplus of the entrepreneur and the financier. Another control based explanation is Marx (1998) where a mixture of debt and equity dominates pure debt and pure equity in giving the financier the efficient liquidation incentives. However, as Gompers (1997) convincingly argues, allocation of cash flow rights and allocation of control rights can be separated by use of covenants and explicit contractual clauses. Indeed, Gompers (1997) documents the frequent use of covenants that give investors control rights. We take the view that such control rights are somewhat independent from the design of financial instrument and the primary purpose of convertible security is more likely to be risk sharing and agency considerations, which is the focus of this paper. Cornelli and Yosha (2003) show that conversion into equity option can be desirable, because it may prevent the entrepreneur from window dressing (short-termism) which does not contribute to the long term success of the venture. Trester (1998) shows that a financier's conversion into equity option may prevent the entrepreneur from defaulting strategically and walking away from the venture. However, it seems that what prevents an entrepreneur from walking away from the venture is the fact that her stakes are vested over time and only become liquid after a certain period of time. Furthermore, many agreements give the investor the right to purchase a departing entrepreneur's share at a low price (see Sahlman, 1990). Schmidt (2003) shows that convertible debt gives efficient investment incentives when the entrepreneur and investor move sequentially in a double moral hazard type problem.

[1] Sahlman (1990) analyzes a sample of 383 venture capital investments and reports that 35% of all projects yielded a net loss and another 50% were only moderately successful.

[2] For a similar discussion for the risk averse behavior of banks, see Diamond (1984).

[3] Consistent with this view, Sahlman (1990) notes that when valuing a company, a venture capitalist computes the present value of a company by applying a very high discount rate, usually in the range of 40–60%.

[4] The following remarks of Joseph Park, the founder of Kozmo.com, illustrates this point: 'Let's say I completely failed in 6 months after launching the company and lost all the money I raised. So what? I will have an impressive resume to apply to a business school' (from the documentary film e-Dreams). Q1 Kozmo.com was a venture capital driven online company that promised free 1-h delivery of anything from DVD rentals to Starbucks coffee. After raising about $280 million, the company had to shut down its operations in 2001. Perhaps ironically, it is now a widely studied example of the dot-com excess and made Joseph Park a celebrity in business school case studies.

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Another related paper is Innes (1990) who considers an effort type agency model, where higher levels of costly effort by the entrepreneur induces better payoff distributions in the sense of the monotone likelihood ratio property. Innes first allows for non-monotonic sharing rules and shows that the optimal contract punishes the entrepreneur by giving the investor all the realized payoff below a threshold, whereas the entrepreneur is awarded by receiving all the payoff over this threshold. To rule out this non-monotonic optimal contract, Innes then imposes a monotonicity constraint on the admissible sharing rules and establishes that under this additional constraint a simple debt contract emerges as optimal. Different than Innes who assumes risk neutrality for both parties, the risk sharing consideration plays an important role in my model. Furthermore, the entrepreneur's action choice determines the riskiness of the venture's payoff, but does not affect its expected value.

The plan of the paper is as follows. Section 2 presents the model. Section 3 provides the benchmark case when the action choice of the entrepreneur is enforceable. Section 4 considers the case when action choice is not observable. Section 5 concludes.

2. The model

There are three dates, $t = 0, 1, 2$. There is an entrepreneur (henceforth EN) who owns a venture idea. The venture requires a fixed investment of K at date 0. The EN has no wealth of his own and relies on a financier/investor (henceforth FI) to provide the investment capital. This financier can be thought as a venture capitalist focusing on young entrepreneurial firms. At date 2, the venture generates a random payoff $\tilde{y}$. The realizations of the random variable $\tilde{y}$ (that I denote with y) are drawn from a support $[0, \infty)$. The EN is risk neutral and maximizes expected wealth. The FI maximizes a strictly concave VNM utility function $v(.)$ with $\lim_{y \downarrow 0} v'(y) = \infty$.

The distribution of the venture's payoff $\tilde{y}$ depends on the action that EN chooses at date 1. This action can be thought as the business strategy employed by EN upon receiving the required funds. For simplicity, I consider two mutually exclusive actions denoted by a_{L} and a_{H}. Formally, for $i \in \{H, L\}$, let $F_i(y)$ denote the distribution function from action a_i with a continuously differentiable density f_i. Following Rothschild and Stiglitz (1970), the following assumption ranks the riskiness of the two distributions under two actions.

Assumption 1. The payoff distribution F_{L} second order stochastically dominates F_{H}, i.e.,

$$\int_0^x (F_{\mathrm{H}} - F_{\mathrm{L}})\,\mathrm{d}y \geq 0 \quad \text{for all } x > 0, \text{ and} \tag{1a}$$

$$\int_0^\infty y f_{\mathrm{L}}\,\mathrm{d}y = \int_0^\infty y f_{\mathrm{H}}\,\mathrm{d}y = \mu. \tag{1b}$$

The above assumption says that the action a_{H} yields a riskier payoff distribution than action a_{L}. In the context of a start-up company operating in an innovative industry, the riskiness of the business plan is an important determinant of failure or success. Among the many possible ways to increase risk in start-up environments, the most common ones are: rushing the product to the market although further testing is warranted, changing the scope of venture's operations and drifting into uncharted territory, insisting on a very ambitious design feature and thus increasing technical risk.[5]

While the realization of the venture's payoff is observable and contractible, I assume that the action a_i is not contractible, i.e., the two parties cannot write an enforceable contract clause at date 0 that dictates EN to choose a particular action. A sharing rule (a financial contract) is an integrable function $s : \mathbb{R}_+ \to \mathbb{R}_+$ which specifies the payment to FI for each payoff outcome y. If the realized payoff is y, then the FI receives $s(y)$ and EN as the residual claimant receives $y - s(y)$. As in Innes (1990), I assume that the sharing rule $s(y)$ must exhibit limited liability so that $0 \leq s(y) \leq y$ for all y. This limited liability constraint implies that EN cannot be forced to pay FI more than what the venture generates ($s(y) \leq y$) and FI cannot be forced to make an additional transfer once the venture's payoff is realized ($s(y) \geq 0$).

I consider a setting with many entrepreneurs seeking funds for their ventures, but only a few financiers/venture capitalists who can provide financing for young entrepreneurial firms. The venture capitalists typically specialize in certain industries such as biotechnology and telecommunications and their industry specific expertise also serves as an entry barrier for less specialized financiers (Gompers, 1997). Furthermore, especially at the initial stage of a venture, the business plan and the skills of an entrepreneur are completely untested, giving EN not much bargaining power. Accordingly, I assume that FI has all the bargaining power and chooses the sharing rule that gives her the maximal expected utility subject to a participation constraint for EN, any required incentive compatibility constraint and the limited liability constraint. The assumption that FI extracts all the surplus does not affect the qualitative results.[6]

[5] I do not consider the possibility of learning about the venture's prospects over time. For a model with this feature, see Bergemann and Hege (1998).

[6] The formulation that maximizes FI's expected utility subject to giving EN his reservation utility is merely a tool to describe the Pareto optimal solution for any given reservation utility level for EN.

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3. Action choice enforceable

As a benchmark, this section considers the case when the action choice a_i is enforceable. The optimal contract in this case is determined only by risk sharing considerations. To state the optimal contracting problem formally, define

$$U_i(s) \equiv \int_0^{\infty} (y - s(y)) f_i \, dy \text{ for } i \in \{H, L\} \tag{2}$$

as the expected payoff for EN from action a_i under a sharing rule s. An optimal contract with action a_i is then the solution to a programming problem in which the sharing rule s is chosen to maximize the expected utility of FI,

$$V_i(s) \equiv \int_0^{\infty} v(s(y)) f_i \, dy \tag{3}$$

subject to a participation constraint for EN,

$$U_i(s) \geq w > 0, \tag{4}$$

and the limited liability constraint

$$0 \leq s(y) \leq y \text{ for all } y. \tag{5}$$

I refer to the above problem as (P1). The following proposition establishes that the optimal risk sharing consideration under limited liability calls for a pure debt contract.

Proposition 1a. *If the action choice is enforceable, the optimal risk sharing contract with action a_i for $i \in \{H, L\}$ is a pure debt contract*

$$s_i \equiv Min(y, m_i), \tag{6a}$$

where the face value m_i is uniquely determined by EN's participation constraint and solves

$$m_i = \mu - w + \int_0^{m_i} F_i \, dy. \tag{6b}$$

Proof. In an optimal solution to (P1), EN's participation constraint (4) must hold as an equality, and hence we must have

$$E[s] = \int_0^{\infty} s(y) f_i \, dy = \mu - w. \tag{7}$$

Consider the fixed payment contract $\bar{s}_i \equiv \mu - w$ for all y. For every $y < \mu - w$, the contract $\bar{s}_i$ violates the limited liability constraint (5), and gives EN a negative payoff. Replacing EN's negative payoff by zero at such low values of y and adjusting FI's payoff accordingly so that (7) is still satisfied, one obtains the debt contract $s_i \equiv \text{Min}(y, m_i)$ where the repayment obligation m_i is determined by EN's participation constraint as[7]

$$\int_{m_i}^{\infty} (y - m_i) f_i \, dy = w \Rightarrow m_i = \mu - w + \int_0^{m_i} F_i \, dy.$$

I now show that the debt contract $s_i \equiv \text{Min}(y, m_i)$ second order stochastically dominates *any* arbitrary contract $\hat{s}_i$ that satisfies (7) and the limited liability constraint (5). First, note that the distribution function $\hat{F}_i(.)$ of s_i is given by

$$\hat{F}_i(y) = F_i(y) \text{ for } y < m_i \text{ and } \hat{F}_i(y) = 1 \text{ for } y \geq m_i.$$

Let us denote the distribution function of our arbitrary contract $\hat{s}_i$ with $\hat{G}(y)$. Since $\hat{s}_i(y) \leq s_i(y) = y$ for all $y < m_i$, we have

$$\hat{G}(y) \geq \hat{F}_i(y) \text{ for all } y < m_i.$$

Therefore, it follows that

$$T(x) \equiv \int_0^{x} (\hat{F}_i - \hat{G}) \, dy \leq 0 \text{ for all } x < m_i. \tag{8a}$$

What remains to be shown is that $T(x) < 0$ for all $x \in [m_i, \infty)$ as well. Now note that since both contracts $\hat{s}_i$ and s_i satisfy (7), we have

$$E[\hat{s}_i] = E[s_i] \Rightarrow \int_0^{\infty} (\hat{F}_i - \hat{G}) \, dy = 0, \tag{8b}$$

[7] In this expression, the term $\int_0^{m_i} F_i \, dy$ reflects the adjustment in FI's payoff over $\mu - w$.

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i.e., we have $T(x=\infty)=0$. Since $\hat{F}_i(y)=1$ for all $y\in[m_i,\infty)$, we have

$$\hat{F}_i(y)-\hat{G}(y)=1-\hat{G}(y)\geq 0 \text{ for all } y\in[m_i,\infty), \tag{8c}$$

and therefore $T(x)$ is increasing in x for every $x\in[m_i,\infty)$. But if $T(x=\infty)=0$ and $T(x)$ is increasing for every $x\in[m_i,\infty)$, then we must have $T(x)<0$ for all $x\in[m_i,\infty)$. This argument establishes, together with (8a), that

$$T(x)\equiv\int_0^x\left(\hat{F}_i-\hat{G}\right)\,dy\leq 0 \text{ for all } x>0. \tag{9}$$

Accordingly, the debt contract $s_i\equiv \text{Min}(y,m_i)$ second order stochastically dominates all other contracts that satisfy (7) and the limited liability constraint (5). The optimality of the debt contract s_i in (P1) then follows from the strict concavity of v(.). □

The next result provides a comparison of FI's expected utility under the optimal debt contracts s_L and s_H.

Proposition 1b. *We have $V_L(s_L)>V_H(s_H)$, i.e., the optimal debt contract s_H with the high risk action yields a strictly lower expected utility to the risk averse FI than the optimal debt contract s_L with the low risk action.*

Proof. First I establish that $m_H>m_L$. From (6b), we have

$$m_L-m_H=\int_0^{m_L}F_L\,dy-\int_0^{m_H}F_H\,dy. \tag{10a}$$

For a contradiction, suppose that $m_L\geq m_H$ and rewrite (10a) as

$$m_L-m_H=\int_0^{m_H}(F_L-F_H)\,dy+\int_{m_H}^{m_L}F_L\,dy. \tag{10b}$$

By Assumption 1, the first term $(\int_0^{m_H}(F_L-F_H)\,dy)\leq 0$. Therefore, if $m_L\geq m_H$ from (10b) we would have

$$m_L-m_H\leq\int_{m_H}^{m_L}F_L\,dy. \tag{10c}$$

But this last inequality in (10c) is a contradiction, since $F_L(y)<1$ for $y\in[m_H,m_L]$ and hence $\int_{m_H}^{m_L}F_L\,dy<m_L-m_H$. Therefore, we must have $m_L<m_H$.

Now, given $m_L<m_H$, I show that s_L second order stochastically dominates s_H. The desired result $V_L(s_L)>V_H(s_H)$ then follows from the strict concavity of $v(.)$. Let $\hat{F}_i$ denote the distribution function for s_i. We need to establish that

$$K(x)\equiv\int_0^x(\hat{F}_L-\hat{F}_H)\,dy\leq 0 \text{ for all } x>0.$$

First, note that $\hat{F}_i(y)=F_i(y)$ for $y<m_i$ and $\hat{F}_i(y)=1$ for $y\geq m_i$. Therefore,

$$\int_0^x(\hat{F}_L-\hat{F}_H)\,dy=\int_0^x(F_L-F_H)\,dy\leq 0 \text{ for all } x<m_L,$$

and hence $K(x)\leq 0$ for all $x<m_L$. To see that $K(x)\leq 0$ for all $x\geq m_L$ as well, recall that the expected payoff is the same under both s_L and s_H, and hence we have $\int_0^\infty(\hat{F}_L-\hat{F}_H)\,dy=0$. But since $\hat{F}_i(y)=1$ for $y\geq m_i$, it follows that

$$K(x)=0 \text{ for all } x\geq m_H.$$

Since $\hat{F}_i(y)=1$ for all $y\geq m_i$, we have $\hat{F}_L(y)-\hat{F}_H(y)\geq 0$ for all $y\in[m_L,m_H)$, which implies that $K(x)$ is increasing in x for all $x\in[m_L,m_H)$. Given $K(x)=0$ for all $x\geq m_H$ and $K(x)$ is increasing for all $x\in[m_L,m_H)$, we must have $K(x)<0$ for all $x\in[m_L,m_H)$ which completes our claim that $K(x)\leq 0$ for all $x>0$. □

An immediate corollary of Proposition 1b is that when the action choice is enforceable, FI strictly prefers the low risk action a_L. After characterizing this benchmark case under enforceability, I now analyze the case when the action choice is not enforceable.

4. Action choice not enforceable

4.1. Risk shifting

Suppose now the action choice of EN is not enforceable, and EN chooses the action that maximizes his expected payoff given the sharing rule specified at date 0. A well known result in the financial contracting literature is that under a risky debt contract the risk neutral EN has a preference for high risk (see, for example, Jensen and Meckling (1976), Green (1984)). To illustrate the risk shifting incentives in our framework, consider any pure debt contract with a repayment obligation m. The **Q2** change in EN's payoff from switching to the high risk action a_H under the debt contract is given by

$$\int_0^m (F_H - F_L)\,dy \geq 0 \text{ for all } m > 0. \tag{11}$$

Therefore, a pure debt contract, which provides optimal risk sharing under enforceability, implements the high risk action a_H when action choice is not enforceable.

The reason behind EN's preference for high risk is the convexity of the residual payoff for EN under a pure debt contract. To see this more transparently, consider any concave sharing rule s^c (including the pure debt contract) that yields a convex residual payoff $y - s^c$ for EN. By Assumption 1, we have

$$\int_0^\infty (y - s^c(y))(f_H - f_L)\,dy = \int_0^\infty s^c(y)(f_L - f_H)\,dy \geq 0. \tag{12a}$$

and hence EN's preference for high risk extends to any concave sharing rule s^c. For example, a mixed debt–equity contract of the form $s \equiv Min(y, m + \pi(y - m))$ with $m > 0$ and $\pi \geq 0$, which gives FI a share of the upside of the venture, is also concave, and implements the high risk action a_H. In a mixed debt–equity contract, EN's payoff from switching to a_H is given by

$$(1 - \pi)\int_0^m (F_H - F_L)\,dy \geq 0. \tag{12b}$$

4.2. Implementing the low risk action

Due to the risk shifting problem, implementing a_L requires FI to deviate from optimal risk sharing achieved by the debt contract and satisfy an additional incentive compatibility constraint. Formally, the problem that FI needs to solve to implement a_L optimally can be stated as follows:

$$\underset{s}{\text{Max}} V_L(s) = \int_0^\infty v(s(y)) f_L\,dy, \tag{P2}$$

$$\text{s.t.} \int_0^\infty (y - s(y)) f_L\,dy \geq w > 0, \tag{13a}$$

$$\int_0^\infty (y - s(y))(f_L - f_H)\,dy \geq 0, \tag{13b}$$

and the limited liability constraint $0 \leq s(y) \leq y$ for all y. I refer to the above problem as (P2).

Before proceeding, a discussion is in order.[8] Note that implementing a_L by satisfying (13b) involves an agency cost relative to the first best achieved by $s_L = \text{Min}(y, m_L)$, since it requires a deviation from the optimal risk sharing rule. On the other hand, FI can always opt for offering the debt contract $s_H = \text{Min}(y, m_H)$ to implement a_H and ensure an expected utility

$$V_H(s_H) = v(m_H) - \int_0^{m_H} v'(y) F_H\,dy. \tag{14}$$

In other words, the 'fully' optimal incentive compatible contract in the absence of enforceability is the better of the two contracts: the optimal solution to (P2) that implements a_L and the optimal debt contract s_H discussed in Section 3 that implements a_H. If we have $V_L(s^*) < V_H(s_H)$ at an optimal solution s^* to (P2), then the second best fully optimal contract is s_H and the high risk action a_H is to be implemented.[9] One question is whether FI is better off from satisfying (13b) and solving (P2) compared to implementing a_H and receiving $V_H(s_H)$. Given that the other contract has already been discussed in Section 3, in what follows I consider the case where despite the deviation from optimal risk sharing and associated agency cost the second best involves implementing a_L.[10]

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[8] I am grateful to an anonymous referee for suggesting this discussion.

[9] This outcome would be similar to an agency model with costly hidden effort where the agency cost of inducing the agent the high effort level is higher than the efficiency benefits on expected output and as a result the principal has to settle for the low effort.

[10] The only restriction imposed on F_H and F_L is a stochastic dominance condition, which is too weak to mathematically characterize a simple condition that ensures FI is better off from implementing a_L by solving (P2). However, it should be noted that a sharing rule that satisfies (13b) and implements a_L can yield the risk averse FI a higher expected utility than $V_H(s_H)$, even if it is not an optimal solution to (P2). In particular, FI may be better off from implementing a_L even with a pure equity contract which is *not necessarily an optimal solution* to (P2). In Appendix A, I construct and solve an example to illustrate this point.

4.3. Convertible debt contract

My main objective is to provide an explanation on why convertible debt can be preferred to more standard financial contracts such as pure debt, pure equity and mixed debt–equity. The ‘convertible debt’ contract can be described as a sharing rule

$$s_{m,\pi} \equiv \max\{\min\{y, m\}, \pi y\} \text{ for } m \geq 0 \text{ and } \pi \in [0, 1). \tag{15}$$

The above contract specifies a payoff realization m below which FI receives all the realized payoff. Therefore, the parameter m again serves as the face value of FI’s debt claim. The conversion into equity option is described by the share π of venture’s equity. Upon realization of the venture’s payoff, FI has the option to exchange the debt claim m for a share π of the venture’s equity. This conversion into equity option is exercised for payoff realizations $y \geq m/\pi$, whereas for $y < m/\pi$ FI retains the debt claim. It should be noted that the pure equity and the pure debt contracts are special cases in this family. The contract $s_{m=0,\pi>0}$ corresponds to a pure equity contract, whereas the contract $s_{m>0,\pi=0}$ corresponds to a pure debt contract.

Let us now consider a variation of (P2) by restricting the set of feasible contracts to convertible debt contracts as described in (15). As noted, the class of contracts in (15) also includes all pure debt and pure equity contracts. Formally, within this restricted class the problem becomes choosing $m \geq 0$ and $\pi \in [0, 1)$ to maximize

$$V_L(m, \pi) = \int_0^m v(y) f_L \, dy + \int_m^{m/\pi} v(m) f_L \, dy + \int_{m/\pi}^{\infty} v(\pi y) f_L \, dy. \tag{P3}$$

$$\text{s.t.} \int_m^{m/\pi} (y - m) f_L \, dy + \int_{m/\pi}^{\infty} (1 - \pi) y f_L \, dy \geq w > 0, \tag{16}$$

$$\int_m^{m/\pi} (y - m)(f_L - f_H) \, dy + (1 - \pi) \int_{m/\pi}^{\infty} y (f_L - f_H) \, dy \geq 0. \tag{17}$$

Let us refer to the above problem as (P3). In the above formulation, the constraint in (16) is EN’s participation constraint, and (17) stands for the incentive compatibility constraint that must be satisfied to implement the low risk action. The proposition below establishes that the solution to (P3) is neither debt nor equity, but it is a non-degenerate convertible debt contract.

Proposition 2. *The optimal solution to* (P3) *is a non-degenerate convertible debt contract* $s_{m,\pi}$ *with* $m > 0$ *and* $\pi > 0$.

Proof. The preceding analysis ruled out the optimality of a pure debt contract, since it implements the high risk action, and violates (17). I now show that a non-degenerate convertible debt contract $s_{m,\pi}$ with $m > 0$ and $\pi > 0$ is superior to an equity contract as a solution to (P3). Let us define

$$U_L(m, \pi) \equiv \int_m^{m/\pi} (y - m) f_L \, dy + \int_{m/\pi}^{\infty} (1 - \pi) y f_L \, dy = w, \tag{18a}$$

$$W(m, \pi) \equiv \int_m^{m/\pi} (y - m)(f_L - f_H) \, dy + (1 - \pi) \int_{m/\pi}^{\infty} y (f_L - f_H) \, dy. \tag{18b}$$

Let m^* be such that

$$\int_{m^*}^{\infty} (y - m^*) f_L \, dy = w.$$

Since $U_L(m, \pi)$ is strictly decreasing in m and π, one can define a function $\pi : [0, m^*] \to [0, 1]$ by $U_L(m, \pi(m)) = w$. Formally, the function $\pi(m)$ satisfies

$$\int_m^{m/\pi(m)} (y - m) f_L \, dy + \int_{m/\pi(m)}^{\infty} (1 - \pi(m)) y f_L \, dy = w. \tag{19}$$

Implicit differentiation of (19) yields

$$\pi'(m) = -\frac{\int_m^{m/\pi(m)} f_L \, dy}{\int_{m/\pi(m)}^{\infty} y f_L \, dy} < 0,$$

which implies that for a given reservation payoff w for EN, increasing FI’s debt claim requires decreasing the equity claim, so that the participation constraint continues to hold as an equality. To show that a solution with $m > 0$ is superior to pure equity, it suffices to show that as $m \to 0$, we have

$$\frac{dV_L(m, \pi(m))}{dm} > 0 \quad \text{and} \quad \frac{dW(m, \pi(m))}{dm} \geq 0.$$

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Note that as $m \to 0$, we have

$$\frac{\mathrm{d}V_{\mathrm{L}}}{\mathrm{d}m} = \int_m^{m/\pi} f_{\mathrm{L}}\,\mathrm{d}y\left(1 - \frac{\int_{m/\pi}^{\infty}(v'(\pi y)/v'(m))yf_{\mathrm{L}}\,\mathrm{d}y}{\int_{m/\pi}^{\infty} yf_{\mathrm{L}}\,\mathrm{d}y}\right) > \int_m^{m/\pi} f_{\mathrm{L}}\,\mathrm{d}y\left(1 - \frac{\int_{m/\pi}^{\infty} yf_{\mathrm{L}}\,\mathrm{d}y}{\int_{m/\pi}^{\infty} yf_{\mathrm{L}}\,\mathrm{d}y}\right) = 0,$$

where the inequality follows from the fact that $\lim_{y\downarrow 0} v'(y) = \infty$, and hence the ratio $v'(\pi y)/v'(m)$ approaches to zero as $m \to 0$. Similarly, straightforward algebra yields

$$\frac{\mathrm{d}W(.)}{\mathrm{d}m} = \int_m^{m/\pi} f_{\mathrm{H}}\,\mathrm{d}y - \int_m^{m/\pi} f_{\mathrm{L}}\,\mathrm{d}y \underbrace{\left[\frac{\int_{m/\pi}^{\infty} yf_{\mathrm{H}}\,\mathrm{d}y}{\int_{m/\pi}^{\infty} yf_{\mathrm{L}}\,\mathrm{d}y}\right]}_{<1 \text{ by Assumption 1}} > \int_m^{m/\pi} (f_{\mathrm{H}} - f_{\mathrm{L}})\,\mathrm{d}y > 0$$

as $m \to 0$. This follows, since as $m \to 0$ Assumption 1 implies that $f_{\mathrm{H}}(y) > f_{\mathrm{L}}(y)$ for all $y \in (m, m/\pi)$. □

A convertible debt contract has two desirable properties compared to pure debt and pure equity. Unlike a pure equity contract, the debt component of convertible debt assigns the whole payoff to the risk averse FI at the low end of payoff realizations and provides better insurance at the downside. Furthermore, unlike a pure debt contract (or mixed debt–equity), the conversion into equity option of convertible debt creates a convex payoff schedule for FI at the upper end of payoff realizations and corrects EN's high risk incentives arising from FI's debt claim. Accordingly, the convertible debt contract provides better insurance by its debt component, while eliminating EN's preference for high risk with its conversion into equity component.

5. Conclusion

This paper considers a financial contracting problem between a risk neutral entrepreneur who seeks funds for his venture and a risk averse financier who can provide financing. The riskiness of the venture's payoff depends on the action that the entrepreneur takes after the financing is agreement. When the action choice is enforceable, a pure debt contract achieves optimal risk sharing between parties under limited liability and the financier prefers to enforce the low risk action. When the action choice is *not* enforceable, due to the well known risk shifting problem a debt contract induces the entrepreneur a preference for the high risk action due to its convex residual payoff. Accordingly, implementing the low risk action requires a deviation from the optimal risk sharing arrangement provided by the debt contract. I focus on situations where despite this deviation from optimal risk sharing and associated agency cost, implementing the low risk action makes the risk averse financier better off compared to opting for the high risk action. In this setting, I show that a convertible debt contract outperforms pure debt, pure equity and any mixture of debt and equity. This result follows because a convertible debt contract has two desirable properties: (i) at the lower end of payoff realizations it assigns the whole payoff to the risk averse FI and improves risk sharing, (ii) at the upper end of payoff realizations, it creates convexity in FI's payoff schedule and corrects EN's high risk incentives. This role can not be achieved by simple mixtures of debt and equity, since mixed debt–equity contracts also yield a concave payoff for the financier (and hence a convex one for the entrepreneur) and implement the high risk action.

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Appendix A

Appendix A presents an example to illustrate that FI can be better off from implementing a_{L} even with an equity contract $s_{\mathrm{e}} = \pi_{\mathrm{e}} y$ (which satisfies (13b) but not necessarily an optimal solution to (P2)) rather than opting for a_{H} by offering $s_{\mathrm{H}} = \mathrm{Min}(y, m_{\mathrm{H}})$. Let us specify FI's utility function by $v(x) = \ln x$ and set EN's reservation wage as $w = 30$. Suppose that for action a_i with $i \in \{H, L\}$, the probability density function f_i is log-normal and it is given by

$$f_i(y; \mu_i, \sigma_i) = \frac{1}{y\sigma_i\sqrt{2\pi}} \exp\left[-\frac{(\ln y - \mu_i)^2}{2\sigma_i^2}\right].$$

To ensure that F_{H} is a mean preserving spread of F_{L} when both F_{H} and F_{L} are log-normal, one needs the following two conditions to be satisfied

$$\mu_{\mathrm{L}} + \frac{\sigma_{\mathrm{L}}^2}{2} = \mu_{\mathrm{H}} + \frac{\sigma_{\mathrm{H}}^2}{2} \quad \text{and} \quad \sigma_{\mathrm{L}} < \sigma_{\mathrm{H}},$$

which we will ensure with the following parameter specification:

$$\sigma_L = 1, \quad \sigma_H = 2, \quad \mu_L = 4.5, \quad \mu_H = 3.$$

With the above parameter specification, we have

$$\int_0^\infty y f_i \, dy = \exp\left[\mu_i + \frac{\sigma_i^2}{2}\right] = 148.41 \text{ for } i \in \{H, L\}.$$

One can compute the face value m_H of the debt contract under action a_H by solving (6b) which takes the form

$$m_H = \int_0^\infty y f_H \, dy - 30 + \int_0^{m_H} F_H \, dy,$$

and yields $m_H^* = 2584.4$. The debt contract $s_H \equiv \mathrm{Min}(m_H^*, y)$ yields FI an expected utility

$$V_H(s_H) = \int_0^{m_H^*} \ln(y) f_H \, dy + \int_{m_H^*}^\infty \ln(m_H^*) f_H \, dy = 2.995.$$

Let us now compute the equity contract $s_e \equiv \pi_e y$ by solving

$$\pi_e^* = 1 - \frac{w}{\int_0^\infty y f_L \, dy} = 1 - \frac{30}{148.41} = 0.79.$$

This equity contract is not necessarily an optimal solution to (P2), but it implements the low risk action a_L and yields FI an expected utility

$$V_L(s_e) = \int_0^\infty \ln(\pi_e^* y) f_L \, dy = 4.274 > V_H(s_H).$$

References

Bergemann, D., Hege, U., 1998. Venture capital financing, moral hazard and learning. Journal of Banking and Finance 22, 703–735.
Berglof, E., 1994. A control theory of venture capital finance. Journal of Law Economics and Organization 62, 247–267.
Chemmanur, T., Fulghieri, P., 1999. A theory of going public decision. Review of Financial Studies 12, 249–279.
Cornelli, F., Yosha, O., 2003. Stage financing and role of convertible securities. Review of Economic Studies 70, 1–32.
Diamond, D., 1984. Financial intermediation and delegated monitoring. Review of Financial Studies 51, 393–414.
Gompers, P., 1997. Ownership and control in entrepreneurial firms: an examination of convertible securities in venture capital investments. Harvard Business School Working Paper.
Green, R., 1984. Investment incentives, debt and warrants. Journal of Financial Economics 13, 115–136.
Innes, R., 1990. Limited liability and incentive contracting with ex ante action choices. Journal of Economic Theory 52, 45–67.
Jensen, M., Meckling, W., 1976. Theory of the firm: managerial behavior, agency costs and ownership structure. Journal of Financial Economics 3, 305–360.
Kaplan, S., Stromberg, P., 2003. Financial contracting theory meets the real world: an empirical analysis of the venture capital contracts. Review of Economic Studies 70, 281–315.
Marx, L., 1998. Efficient venture capital financing combining debt and equity. Review of Economic Design 3, 371–387.
Rothschild, M., Stiglitz, J., 1970. Increasing risk I: a definition. Journal of Economic Theory 2, 315–329.
Sahlman, W., 1990. The structure and governance of venture capital organizations. Journal of Financial Economics 27, 473–521.
Schmidt, K., 2003. Convertible securities and venture capital finance. Journal of Finance 58, 1139–1166.
Trester, J., 1998. Venture capital contracting under asymmetric information. Journal of Banking and Finance 22, 675–699.

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