Golden rule

- The golden rule allocation is the stationary, feasible allocation that maximizes the utility of the future generations.
- Let the golden rule allocation be denoted by (c_1^{gr}, c_2^{gr}).
- To achieve this allocation, the central planner must be able to:
  - reallocate endowments costlessly between generations;
  - know the exact utility functions of the agents.
- Since these are strong assumptions, we will investigate if there is a way to attain this allocation in a **decentralized** manner.
- Meaning if individuals by themselves, through mutually advantageous trades, can attain this allocation.

Competitive equilibrium

- A competitive equilibrium has the following properties:
  - Individuals make mutually advantageous trades so as to attain the highest possible utility level;
  - Individuals take prices (rates of exchange) as given;
  - Markets clear (supply equals demand).
- Suppose there is no money in the economy. No mutually advantageous trades are possible (there is an absence of **double coincidence of wants**). The resulting equilibrium is **autarkic.**
- Suppose now the government can produce **fiat money** – a nearly costlessly, impossible to counterfeit, and storable commodity with no consumption or production value.

Monetary equilibrium

- Fiat money, by itself, generates no utility, it can only be valuable if it enables individuals to trade for something they wish to consume.
- A **monetary equilibrium** is a competitive equilibrium in which there is a valued supply of fiat money.
- Suppose there is a supply of M units of fiat money, owned by the initial old (each holds $\frac{M}{N}$ units).
- The presence of money opens up a trading possibility. How?

Budget constraints

- This trading possibility only exists if money is valued. If young people **believe** that when old they will be able to obtain goods for their money.
- Let v_t denote the value of one unit of money (a dollar) in terms of units goods.
- It is the inverse of the dollar price of a good $p_t = \frac{1}{v_t}$.
- To find how much money young people decide to acquire we first need to specify their **budget constraint**.
- When young an individual has her endowment, which she can spend in the consumption good or in money:

$$c_{1,t} + v_t m_t \leq y.$$

Budget constraints

- An old individual has no endowment but she can spend her money to acquire goods:

$$c_{2,t+1} \leq v_{t+1} m_t.$$

- We can write these two equations in a consolidated form, a lifetime budget constraint:

$$c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq y.$$

- The (absolute value of the) slope of the budget constraint, $\frac{v_{t+1}}{v_t}$ is the (real) gross **rate of return of fiat money.**
- Using the indifference curves implied by the individual's preferences, and given a rate of return of fiat money, we can find the allocation that maximizes utility, $(c^*_{1,t}, c^*_{2,t+1})$, the tangent point.

Stationary equilibria

- How do we determine the rate of return on fiat money?
- The value of money today, v_t, depends on what people believe the value of money tomorrow, v_{t+1}, will be, which in turn depends on v_{t+2}, etc...
- A reasonable assumption is that these beliefs are the same for every generation.
- This means every generation acts in the same way, choosing $c_{1,t} = c_1$ and $c_{2,t+1} = c_2$, we call the resulting equilibria **stationary equilibria**.
- We also assume individuals have **rational expectations**, which in this non-random economy means future variables are exactly what individuals expect them to be: **perfect foresight**.

Money's rate of return

- The value of money is a price, and like all the prices in competitive markets it is the one at which supply equals demand: $v_t M_t = N_t(y - c_{1,t})$.
- This implies $v_t = \frac{N_t(y-c_{1,t})}{M_t}$.
- The rate of return of fiat money is:

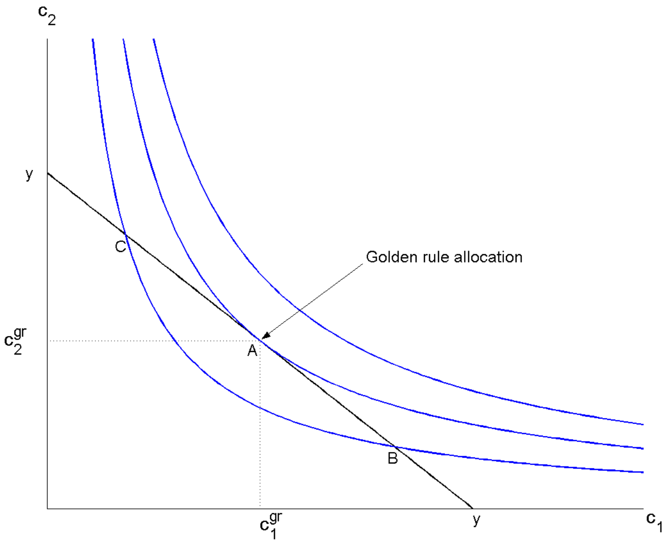
$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_{1,t+1})}{M_{t+1}}}{\frac{N_t(y-c_{1,t})}{M_t}}.$$

- In the stationary case:

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{\frac{N_{t+1}}{M_{t+1}}}{\frac{N_t}{M_t}}.$$

- If the stock of money and population are constant we get $\frac{v_{t+1}}{v_t} = 1$, or $v_{t+1} = v_t$.

The golden rule allocation



The lifetime budget constraint