Credit Attribution and Collaborative Work*

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Abstract

We examine incentives in research teams where the market, such as the scientific community, attributes credit for success based on its inference of individual efforts. A social planner who could commit to credit ex ante would induce more effort from higher ability agents in exchange for less credit per unit effort. Lacking such commitment, the Bayesian market assigns credit proportional to perceived effort. This inability to distort credit per unit effort leads to an incentive reversal across projects. For "easy" projects with a concave marginal cost of effort, in the unique interior equilibrium, higher ability agents work less and receive lower credit/utility, while the opposite holds for "difficult" projects with a sufficiently convex marginal cost of effort. Moreover, equilibrium may involve overinvestment by some team members who expect to receive most of the credit. The incentives to team up and the implications of effort observability on credit attribution are also investigated.

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1 Introduction

Credit for scientific discovery is vital for the reward system, and hence, the progress of science. According to Flier (2019), "absent credit, it is impossible [for scientists] to secure appointments, promotions, research funding, access to students, and other necessities of research." Essential to credit attribution is the peers' inference of individual contributions to knowledge production. This inference, however, has been complicated by the steady rise in collaborative work across disciplines.[1] While various conventions such as authorship order[2] have developed to overcome this complication, peer recognition of each collaborator's contribution remains the basic currency in assigning credit.

The importance of credit attribution is not specific to scientific teams. In many contexts, it is impractical for outsiders, such as peers or supervisors, to directly observe which team member was instrumental in achieving a certain goal. In his classic study on the way American government agencies work, Wilson (1991) shows how these agencies, despite sharing a common objective, compete for recognition of their work to secure scarce political support and resources. In politics, voters reward or punish coalition parties differently for policy outcomes (Marsh and Tilley, 2010). In business, managers pay discretionary bonuses to their subordinates based on subjective evaluations (Rajan and Reichelstein, 2006). In education, teachers assign individualized grades on group projects (Zhang and Ohland, 2009).

Casual observation suggests that scientific peers view assigning due credit for success as part of their profession. Tenure and promotion guidelines and scientific award committees typically require evidence of contributions independent of those attributable to any mentor, co-author, or supervisor. Peer attribution of scientific credit, therefore, plays a key role in maintaining and improving the ethical and professional standards of a department, discipline, or scientific society. In turn, ex post assignment of credit is instrumental for the ex ante team incentives.

Credit attribution raises some obvious positive and normative questions: Do higher ability (or lower-cost) team members always deserve more credit for the team's success? Do team members always under-invest in the joint project due to free-riding? Does the implicit incentives that peer recognition provides encourage or discourage collaboration? And, do team

[1] Based on all the articles registered from 1980 to 2013 in the Social Science Citation Index (SSCI), Henriksen (2016) documents a significant increase in co-authorship with the average number of authors per article in economics rising from 1.3 to 2.3 over the period. Wuchty, Jones, and Uzzi (2007) report that in the natural sciences, team size has grown each year and nearly doubled from 1.9 to 3.5 authors per article between 1955 and 2000.

[2] As Flier (2019) convincingly argues, it is often difficult to assign credit given the current authorship practices since "broadly accepted conventions that specify the meaning of authorship as regards to type and extent of contributions by each author are lacking." Authorship conventions range from alphabetical (mathematics, economics), to descending importance (biology, high energy physics), to listing the lead author first and the principal investigator last (chemistry, psychology); see Shen and Barabási (2014). Recently the American Economic Association (AEA) has started permitting random order of coauthors (Ray ⓡ Robson, 2018).

members favor close monitoring of their activities for proper credit?

To address these and related questions, we present a simple model of research teams building on Lee and Wilde's (1980) seminal paper on R&D races.[3] We envision that, instead of working independently, agents work as a team toward a single breakthrough, e.g., a scientific discovery. Agents have heterogenous abilities, where higher ability means lower marginal cost of effort. To isolate from career concerns, we assume abilities of team members to be common knowledge, perhaps, because of their past achievements.

Our main departure from the teamwork literature (discussed below) is that the sharing rule for the team's output is not specified exogenously or by an ex ante compensation contract. Rather, each agent's share is determined *in equilibrium* by the outside parties' *ex post* beliefs as to which team member is responsible for the breakthrough. This is because the peers or the public often become aware of a research team upon observing its success. For concreteness, we refer to the outside parties as the "market," which can also be thought of as a planner who cares about allocating due credit ex post but cannot write a contract ex ante. Specifically, unable to observe individual efforts or rates of discovery, the market rationally assigns a probability to each agent representing the likelihood that this agent's contribution led to the breakthrough. We call this probability the agent's "credit" for success. In equilibrium, we require the team's effort profile and the credit allocation to be consistent.

Examination of two benchmarks helps us develop insights for this environment. A social planner who can choose both the effort profile and the credit distribution to maximize team payoff equates agents' marginal flow costs. This first-best solution requires higher ability members to work harder and receive more credit from team's success. When the social planner can choose only the credit distribution, she deviates from the first-best to account for team moral hazard. In this second-best solution, higher ability agents are induced higher effort, but now they also incur a higher marginal flow cost because it is marginally cheaper to induce effort. A key observation in the second-best solution is that the higher ability agents receive less *credit per unit effort*.[4] Relative to the credit he receives, the higher ability agent is made to work disproportionately more.

Our main results are concerned with the properties of the team equilibrium. We focus on the interior equilibrium in which all team members are active and show that one generically exists. As we shortly argue in more detail, an inactive agent can effectively be removed from the team in our model by introducing a small participation cost for being a team member. When individual credits are determined by the market's equilibrium beliefs as to which team member has made the breakthrough, an agent's equilibrium behavior is driven by his "perception" of the project's difficulty, as formally defined in the analysis. This perception

[3] See Reinganum (1989) for a review of this literature.

[4] Able to choose the effort profile, the planner is indifferent to the team's credit distribution in the first-best solution.

generally depends on the discount rate, the anticipation of teammates' efforts, and the shape of the effort cost intrinsic to the project, i.e., common to all agents. For a wide range of projects, however, the intrinsic cost technology dominates the agents' perceptions. We refer to a project as "intrinsically easy" if the *marginal* cost of effort is concave, and "intrinsically difficult" if that marginal cost is sufficiently convex. For the moderately convex marginal cost, we refer to a project as "non-intrinsic" since the agents' perceptions can vary, depending on what each expects of teammates and on the discount rate.

Central to our investigation are the intrinsic projects since they admit a unique interior equilibrium, and more importantly, agents' behaviors and credits display a striking reversal. In particular:

(a) For an intrinsically easy project, the higher ability agent works less and, as a result, receives less credit in equilibrium. Recall that the planner of the second-best solution saves on her scarce credit resource by offering higher ability agents a lower credit per unit effort. Unable to commit to a credit allocation ex ante, the market, in contrast, equates the agents' credit per unit effort. For an intrinsically easy project, the inability of the market to distort the credit per unit allocation in favor of the low ability agent has the following implication: The market can both save credit on the high ability agent and also motivate the low ability one only if the former is "believed" to work less and, in turn, receive less credit.

(b) For an intrinsically difficult project, the higher ability agent works more and, as a result, receives more credit in equilibrium. This is because the market's *constant* credit per unit effort strategy is more in line with the social planner of the second-best solution. This time, the higher ability agent's desire to work harder is not curbed by the reduced credit per unit rule of the social planner, but the convexity of the effort cost function.

(c) Agents also fare very differently across intrinsic projects. In equilibrium, the higher ability agent is worse off (resp. better off) than his lower ability teammates under an intrinsically easy (resp. difficult) project. By working harder in equilibrium, an agent improves his credit but also incurs a greater effort cost. Our result illustrates that the credit effect dominates. Therefore, it is the more diligent, not necessarily the more able, agent who fares better in teamwork.

For non-intrinsic projects, the agents may have different perceptions of the project. As such, there may be multiple interior equilibria, and the equilibrium credit allocation and payoffs can be non-monotonic in ability. To illustrate these points, we provide a numerical example of a non-intrinsic project with a three-member team. When agents have a sufficiently low discount rate, all agents perceive the project as easy; and the unique interior equilibrium has effort and credit inversely related to ability. In the other extreme with a sufficiently high discount rate, all agents perceive the project as difficult, and higher ability agents expend more effort and receive more credit. For intermediate values of the discount rate, however, team members may have different perceptions of the project. Furthermore, multiple interior equilibria in which effort is non-monotonic in ability may arise: it may be the middle ability agent who works the most in one equilibrium and the least in the other one.

Our next set of results focus on inefficiencies that arise from equilibrium credit attribution, regardless of agents' perceptions of the project. Free-riding, a well-known feature of teamwork, often leads to under-investment. Our model also delivers under-investment when team members are identical and thus expect equal credit for success. For a significant ability gap, however, *over-investment* by some member is possible, which, to our knowledge, is a novel feature in the teamwork literature. This inefficiency is obvious for an intrinsically easy project: contrary to desired socially, the low ability agent expends more effort in this case. Consider, thus, two highly heterogeneous agents in an intrinsically difficult project. Anticipating most of the credit, the high ability agent would perform virtually solo in equilibrium. Such a work allocation, however, would be too unequal from the social viewpoint: by shifting some work to the low ability agent, the team's payoff could improve.

While a full analysis is beyond the scope of our paper, in a short extension, we demonstrate endogenous team formation between two agents, e.g., two authors. In particular, we show that incentives to form a team can arise under equilibrium credit attribution by the market: sufficiently patient agents who are also similar in ability prefer to work together to avoid costly competition. Interestingly, this finding implies that a high ability agent can team up with a low ability one on an intrinsically easy project, despite anticipating a lower credit and a lower payoff from its completion than his teammate. In other extensions, we also show (i) how the observability of efforts creates competition for credits and how even higher ability agents may prefer non-observability to avoid such competition, and (ii) how our results are robust to heterogenous cost elasticities.

Related Literature. Aside from those mentioned above, our paper is related to three strands of the literature on team incentives. The first strand examines contracts that elicit efficient actions in teams. These contracts often feature sophisticated sharing rules with large penalties for deviators, e.g., Holmstrom (1982), Rasmusen (1987), Legros and Matthews (1993), and Winter (2004). While such contracts are likely to be available in principal-agent settings, they have very limited applicability for rewarding research teams whose main currency is peer recognition. The second strand fixes the sharing rule and views teamwork as voluntary contributions to a public good, e.g., Olson (1965), Bergstrom, Blume and Varian (1986), and Andreoni (1998). These papers employ static models and identify under-investment among team members due to free-riding. Building on this insight, the third – and more recent – strand of the literature views teamwork as dynamic public good provision. It characterizes non-stationary team dynamics when agents aim to reach a pre-specified project scale (e.g., Admati and Perry, 1991; Marx and Matthews, 2000; Compte and Jehiel, 2004; Yildirim, 2006; Georgiadis, 2015; Bowen, Georgiadis and Lambert, 2019) or they learn about the project's potential (e.g., Bonatti and Horner, 2011; Guo and Roesler, 2018; Cetemen, Hwang and Kaya, 2020). As in Lee and Wilde (1980), our model exhibits stationary strategies. Most importantly, we endogenize the sharing rule as an equilibrium credit allocation driven by peer beliefs.[5] As such, unlike in these studies, equilibrium effort can be non-monotonic in ability, and over-investment by both high and low ability agents can occur in our model.

The scant theoretical work on credit attribution in economics has mostly focused on authorship order as a mechanism to signal relative contributions. Engers, Gans, Grant, and King (1999) show that the alphabetical order may emerge as the unique equilibrium, as any deviation from it hurts the author with the early name in the alphabet more than it benefits the other. Also, in a model with two co-authors, Ray ⓡ Robson (2018) show that a certified random order is fairer, distributes credit evenly on the alphabet, and will invade an environment in which alphabetical order is dominant. While these insights are valuable and empirically relevant (Einav and Yariv, 2006), authorship order can only provide a noisy signal of individual contributions. With more than two authors, this signal can be even noisier, as evident from the lack of consensus on a name-order convention across disciplines and the reliance on peer reviews in tenure/promotion decisions.[6] Therefore, in our model, the public infers relative contributions of an *arbitrary* number of collaborators using no other information than the commonly known and heterogeneous abilities.

Finally, the within-team competition for credit in our model can be viewed as a contest; see Dechenaux, Kovenock, and Sheremeta (2015) for a recent survey. One major difference is that agents also play a game against the public who awards the prize in our setup. Moreover, there are multiple prizes in equilibrium, as in Barut and Kovenock (1998) and Moldovanu and Sela (2001), among others. But in our framework, the designer does not commit to the prize allocation. It is worth noting that although our models and focuses are very different, Moldovanu and Sela also identify the shape of the effort cost as the key driving force in the allocation of prizes.

The paper is organized as follows. Section 2 provides a static example to build some key insights. Section 3 describes the model. Sections 4.1 and 4.2 presents the first and second-best benchmarks, respectively. Sections 5.1 and 5.2 analyze the team equilibrium and contains

[5]With few exceptions, such as Bowen et al. (2019) and Cetemen et al. (2020), dynamic teamwork models assume symmetric agents and focus on symmetric equilibrium, so credit allocation would also be trivial.

[6]The proposals for credit allocation have come a long way from assigning full or equal fractional credit to each co-author to unequal schemes such as arithmetic, geometric and harmonic counting; see Kim and Kim (2015) for a review. Yet, there is always a gap between peer perception and the credit allocated by the specific scheme (Wren et al. 2007). Kim and Kim (2015) quantify this gap by using credit allocations from surveys in chemistry, biomedicine, economics, marketing, and psychology.

our main results. Section 5.3 presents a numerical example and illustrates some observations on non-intrinsic projects. Section 5.4 shows that some team members may overinvest if the team is sufficiently heterogenous. Section 6 includes some extensions and further results: Section 6.1 analyzes endogenous team formation between two agents under the market credit equilibrium. Section 6.2 considers the case when effort is observable. Section 6.3 introduces heterogeneous cost elasticities. Section 7 concludes. The Appendix contains the proofs and the technical details omitted from the main text.

2 Static Example

Consider the following one-shot game.[7] Two risk-neutral agents ($i = 1, 2$) simultaneously exert effort $x_i \geq 0$ to achieve a breakthrough. Agent i's effort cost takes an iso-elastic form:

$$c_i(x) = \frac{x^k}{ka_i},$$

where $k > 1$ and $a_i > 0$.

The parameters k and a_i are commonly known, and a_i is interpreted as agent i's "ability." The breakthrough occurs with probability $X = x_1 + x_2$ and generates a *unit* surplus to the team. To ensure $X \in [0, 1)$, we restrict ability levels here so that $a_1^{1/(k-1)} + a_2^{1/(k-1)} < 1$.

We refer to the outside parties simply as the market.[8] The market does not observe agents' efforts, but observes if there is a breakthrough. Conjecturing individual efforts, the market assigns agent i's share of surplus according to its belief that he is the party responsible for success. In this static example, we assume that the market credits the success to agent i with probability[9]

$$q_i = \frac{x_i}{X}. \tag{1}$$

Hence, agent i's expected payoff is $u_i = Xq_i - c_i(x_i)$.

In a first-best benchmark, a utilitarian planner would choose efforts to maximize team's payoff:

$$\max_{x_1, x_2} u_1 + u_2 = X - c_1(x_1) - c_2(x_2). \tag{2}$$

It is evident from (2) that being able to contract on effort, the planner is indifferent to credit distribution, provided that $q_1 + q_2 = 1$. For consistency, however, we assume that she follows the market's rule in (1) as well.

[7] We thank a referee for suggesting this example.

[8] Depending on the application, the outside party or parties may be the professional peers, a teacher, a manager or a voter.

[9] Though intuitive, the proportional credit rule in (1) is an assumption in this static example. The reason is that the reduced-form probability $x_1 + x_2$ assumed for the collective success does not imply the probability x_i for agent i's individual success. If it did, the team's probability of success would be $x_1 + x_2 - x_1 x_2$, where the last term accounts for the event of a tie when both agents make the breakthrough. As shown below, the credit rule (1) will become exact in the continuous-time model, where ties cannot occur.

The first-order condition reveals that the planner equates agents' marginal costs: $c_i'(x_i^{FB}) = 1$, implying agent i's first-best effort $x_i^{FB} = a_i^{1/(k-1)}$ and, in turn, his first-best utility:

$$u_i^{FB} = (1 - \frac{1}{k})x_i^{FB}.$$

Without loss of generality, let $a_1 > a_2$ so that agent 1 is the high-ability or low-cost team member. It is immediate that

$$x_1^{FB} > x_2^{FB}, \quad q_1^{FB} > q_2^{FB}, \quad \text{and} \quad u_1^{FB} > u_2^{FB}.$$

In the first-best, the high ability agent is asked to work harder in return for more credit from a breakthrough. He also achieves higher utility.

We now examine the equilibrium behavior. In equilibrium, agent i best responds to agent j's effort x_j^* *and* the market's belief q_i^*. Formally,

$$x_i^* = \arg\max_{x_i} u_i = (x_i + x_j^*)q_i^* - c_i(x_i),$$

and thus, agent i's optimal effort satisfies

$$c_i'(x_i^*) = q_i^*. \tag{3}$$

(3) indicates that trivial equilibria in which some agent exerts no effort always exist. In particular, if agent i expects no credit from the breakthrough, i.e., $q_i^* = 0$, then $x_i^* = 0$. Such non-interior equilibria are uninteresting for our purposes, because inactive agents can be removed from the team by requiring an arbitrarily small participation cost.[10] In what follows, we thus focus on the interior equilibrium in which both agents are active.

Inserting the belief-consistency condition $q_i^* = x_i^*/X^*$ into (3) and solving for efforts, the unique interior equilibrium is easily found to be

$$x_i^* = \left(\frac{a_i}{X^*}\right)^{\frac{1}{k-2}} \text{ and } X^* = \left(a_1^{\frac{1}{k-2}} + a_2^{\frac{1}{k-2}}\right)^{\frac{k-2}{k-1}} \text{ for } k \neq 2. \tag{4}$$

Examination of (4) yields the following equilibrium properties:

(i) Higher ability does not always imply higher effort and credit: $a_1 > a_2$ implies $x_1^* \gtrless x_2^*$ if and only if $k \gtrless 2$. In particular, if $k < 2$ so that the cost function is not too convex, it is the low ability team member who works harder and receives more credit from a breakthrough.

[10]The small participation cost $\varepsilon > 0$ can be incurred by agents or a designer recruiting them to the team. In either case, the outside market must expect each team member to exert positive effort and receive some credit in equilibrium. Otherwise, if, in equilibrium, $x_i^* = 0$ for some agent i, then $q_i^* = 0$ and $u_i^* = -\varepsilon < 0$, implying nonparticipation of agent i or his non-inclusion in the team by the designer.

(ii) The same reversal also holds for equilibrium payoffs: it is the more diligent, not necessarily the more able, agent who fares better. To see this, we note that for the iso-elastic cost $c_i(x_i^*) = c_i'(x_i^*)\frac{x_i^*}{k} = q_i^*\frac{x_i^*}{k}$. Thus,

$$\begin{aligned} u_i^* &= X^* q_i^* - c_i(x_i^*) \\ &= x_i^* - \frac{(x_i^*)^2}{kX^*}, \end{aligned}$$

which implies

$$u_1^* - u_2^* = \left(1 - \frac{1}{k}\right)(x_1^* - x_2^*)\,.$$

(iii) Agents underinvest in the joint project, i.e., $x_i^* < x_i^{FB}$. This observation follows from $c_i'(x_i^*) = q_i^* < 1 = c_i'(x_i^{FB})$. Intuitively, when choosing his effort, agent i does not take into account his teammate's share from success.

To gain some intuition for the reversal at $k = 2$, observe that in an interior equilibrium, the market equalizes *credit per unit effort* across team members:

$$\frac{q_1^*}{x_1^*} = \frac{q_2^*}{x_2^*} = \frac{1}{X^*},$$

From (3), this implies that unlike in the first-best outcome, the market equalizes agents' marginal costs per unit effort or simply their *average* marginal costs:

$$\frac{c_1'(x_1^*)}{x_1^*} = \frac{c_2'(x_2^*)}{x_2^*}. \tag{5}$$

Whether the average marginal cost is increasing or decreasing proves key for the agents' equilibrium behavior. In general, for cost functions that start from the origin, i.e., $c_i(0) = c_i'(0) = 0$, we observe[11]

$$sgn\left(\frac{c_i(x)}{x}\right)' = sgn\left(c_i''(x)\right) \quad \text{and thus,} \quad sgn\left(\frac{c_i'(x)}{x}\right)' = sgn\left(c_i'''(x)\right).$$

This last observation allows a simple classification of projects and facilitates our exposition. Note that the average marginal cost is increasing if the marginal cost is convex. In this case, we will say that agent i views the project as "difficult." On the other hand, the average marginal cost is decreasing if the marginal cost is concave, and agent i views the project as "easy." For the iso-elastic cost specification, it can be verified that

$$sgn\left(\frac{c_i'(x)}{x}\right)' = sgn\,(k-2)\,.$$

[11]Simply note that $sgn\left(\frac{c_i(x)}{x}\right)' = sgn\left(c_i'(x) - \frac{c_i(x)}{x}\right) = sgn\left(c_i''(x)\right)$. Substituting for $c_i(x) := c_i'(x)$, we then find $sgn\left(\frac{c_i'(x)}{x}\right)' = sgn\left(c_i'''(x)\right)$. These sign relationships will help us interpret our results and generalize some beyond the iso-elastic cost in the Appendix.

Moreover, since $c_i(x)$ is decreasing in i's ability, (5) implies

$$sgn\,(x_1^* - x_2^*) = sgn\,(k-2) = sgn\,(q_1^* - q_2^*)\,.$$

Accordingly, for $k > 2$, both agents view that project as difficult, the high ability agent exerts greater effort and receives more credit for team's success in equilibrium. However, if the project is easy ($k < 2$) , the reverse holds: it is now the low ability agent who is more diligent and, in turn, deserves more credit from a breakthrough.

As we shall see, the static example of this section captures the key equilibrium reversal well. However, it is based on the ad-hoc credit rule in (1), as explained in Footnote 9. In the next section, we develop a continuous-time model, in which (1) is micro-founded as a conditional probability. Also, note that, in a one-shot game, agents are assumed to be infinitely impatient. As our continuous-time model illustrates, less impatient agents may behave qualitatively differently. In fact, their discount rate may prove crucial on whether they perceive the project as difficult or easy. Furthermore, while the equilibrium of the static example has all agents underinvest, we will show that some agents may overinvest in the dynamic case. We describe this model next.

3 Model

Our dynamic model is adapted from Lee and Wilde's (1980) on R&D races. Rather than competing toward a breakthrough, e.g., a scientific discovery, a team of $n > 1$ risk-neutral agents indexed by $i = 1, ..., n$ undertakes a joint project toward it. They continuously choose their efforts over an infinite time horizon, $t \in [0, \infty)$. Let $x_i(t) \in [0, \infty)$ be agent i's instantaneous effort at time t, which is unobservable to others. The flow cost of effort is given by

$$c_i(x_i(t)) = \frac{c(x_i(t))}{a_i} \tag{6}$$

where $a_i > 0$, $c' > 0$, $c'' > 0$, and $c(0) = c'(0) = 0$.

We refer to the parameter a_i as agent i's "ability" since a higher a_i means a lower marginal flow cost for the same effort. We assume that a_i is publicly known, perhaps due to the agent's track record, ruling out any reputational concern. And we refer to the function $c(.)$ common to all agents as the project's "intrinsic" cost. As usual, the cost of effort is assumed to be strictly increasing and strictly convex. For expositional convenience, we adopt the following iso-elastic form for $c(.)$ in the text:[12]

$$c(x) = \frac{x^k}{k}, \quad k > 1. \tag{7}$$

[12] In the proofs of the formal results, we offer more general cost conditions all of which are satisfied by the iso-elastic specification. Besides its expositional ease, we have adopted the iso-elastic specification in the text for its wide use in the continuous-time teamwork studies; e.g., Bonatti and Horner (2011) and Georgiadis (2015).

As in Lee and Wilde (1980), we assume no knowledge accumulation and that agent i's instantaneous probability of a breakthrough at time t is also his effort, $x_i(t)$.[13] Without loss of generality, we can, therefore, drop the time index and focus on stationary strategies, x_i, throughout. Such stationarity implies that agent i's random time for the breakthrough, denoted by $T_i \in [0, \infty)$, is exponential with rate x_i.[14] The project is completed, and the game ends when the breakthrough is first made. Consequently, the team's random time for completing the project, which is $\min_i T_i$, is also exponential with aggregate rate $X = \sum_i x_i$ (Ross, 2014). Let $\omega_t \in \{0, 1\}$ be the state of the project at time t, with $\omega_t = 1$ representing its completion. The value of a completed project is normalized to *one* while an incomplete project is worth nothing. Agents discount the future benefit and costs at a common rate $r > 0$.

We depart from the existing literature on team incentives (discussed above) and consider an endogenous allocation of the reward based on the market's belief as to who might be responsible for the discovery.[15] We assume that the market such as the scientific community plays an impartial role in allocating due credit for collective success without having any direct stake in it. Specifically, unable to observe the individual efforts leading up to the breakthrough but conjecturing their profile $\mathbf{x} = (x_1, ..., x_n) \neq \mathbf{0}$, the market attributes the breakthrough to agent i with the following probability:

$$\begin{aligned} q_i &\equiv \Pr(T_i = \min_j T_j | \omega_t = 1, \mathbf{x}) \\ &= \frac{x_i}{X}. \end{aligned} \tag{8}$$

We call the probability q_i agent i's "credit" for collaborative success. Since efforts are unobservable to the market, agents take the credit profile $\mathbf{q} = (q_1, ..., q_n)$ as given. Thus, (8) is a belief-consistency condition that needs to hold in equilibrium, which we describe in Section 5. It is worth noting that though absent in (8), q_i will depend on agents' abilities through their effort choices in equilibrium.

When the breakthrough occurs, agent i expects the following gross payoff from the project:

$$q_i(1) + (1 - q_i)(0) = q_i$$

[13]The fact that the rate of discovery is linear and independent of one's ability is without loss of generality. We could assume it to be some strictly increasing and concave function $R_i(x_i)$; e.g., $R_i(x_i) = a_i x_i$. Then, by a change of variables: $x_i := R_i^{-1}(x_i)$, any nonlinearity and ability-dependence in the rate would be absorbed by the cost of effort, $c_i(.)$. The assumption of no knowledge accumulation is obviously unrealistic, but it greatly simplifies the analysis and appears reasonable for highly innovative projects.

[14]The exponential arrival time is assumed by Lee and Wilde (1980), but it need not be. Without knowledge accumulation, agent i's discovery follows a Poisson process, with the rate $x_i(t)$. Moreover, given the stationarity, the process is homogenous, with exponential interarrival times. The stationarity also implies that it is immaterial whether or not agents commit to their effort strategies in our model; see Reinganum (1982) for a similar observation.

[15]We assume that communication between team members and the market is either not feasible or not credible, as each member would claim responsibility for the discovery.

regardless of being responsible for its completion. To derive his expected discounted payoff, note that given the exponential arrival time, the probability of no breakthrough until time t is e^{-Xt}. In the next instant dt, agent i incurs his flow cost $c_i(x_i)dt$ and receives his reward q_i if the team succeeds with probability Xdt. If the team fails, the game is reset to $t = 0$. As a result, agent i's expected discounted payoff at any time without a breakthrough is

$$\begin{aligned} u_i &= \int_0^\infty e^{-rt} e^{-Xt} \left(X q_i - c_i(x_i) \right) dt \\ &= \frac{X}{r+X} q_i - \frac{c_i(x_i)}{r+X}. \end{aligned} \tag{9}$$

4 Two Benchmarks

We begin our analysis by establishing the first- and second-best benchmarks, and then turn our attention to team equilibrium.

4.1 First-best solution

Suppose that a utilitarian planner can choose both the effort profile $\mathbf{x}$ and credit distribution $\mathbf{q}$ to maximize team's payoff: $W = \sum_i u_i$. Using (9) and the credit constraint $\sum_i q_i = 1$, the planner's program becomes

$$\max_{\mathbf{x}} W \equiv \frac{X}{r+X} - \frac{\sum_i c_i(x_i)}{r+X}. \tag{FB}$$

As is evident from (FB), being able to contract on effort, the planner has flexibility in distributing credit so long as the sum is one. For consistency, we break her indifference in favor of (8). Let the effort profile $\mathbf{x}^{FB}$ be a solution to (FB), resulting in the credit $q_i^{FB} = x_i^{FB} / X^{FB}$ and the expected payoff u_i^{FB} for agent i. Lemma 1 establishes the intuitive properties of the first-best.

Lemma 1 *There is a unique solution to (FB). At the solution, $x_i^{FB} > 0$ and $u_i^{FB} > 0$ for all i. Moreover, $a_i > a_j$ implies that $x_i^{FB} > x_j^{FB}$, $q_i^{FB} > q_j^{FB}$, and $u_i^{FB} > u_j^{FB}$.*

At the first-best, the planner allocates positive effort to each agent since $c_i(0) = c_i'(0) = 0$. And she promises the agent enough credit to compensate for his cost of effort. To understand the rest of the lemma, note that solving (FB) requires the planner to minimize the total cost, $\sum_i c_i(x_i)$, of a given team effort, X. Thus, with convex flow costs, a necessary condition for (FB) is that agents' marginal flow costs be equal: $c_i'(x_i) = c_j'(x_j)$. Since the marginal flow cost decreases with ability, it follows that a higher ability team member is asked to work harder in return for more credit from the breakthrough and a greater expected payoff.

4.2 Second-best solution

Suppose now that the planner can only contract on the credit distribution $\mathbf{q}$ for a breakthrough. Agents publicly observe $\mathbf{q}$ and simultaneously exert effort. Thus, given his credit q_i, agent i best responds to teammates' effort $X_{-i} = X - x_i$ by solving

$$\max_{x_i} u_i = \frac{X}{r+X} q_i - \frac{c_i(x_i)}{r+X}. \tag{10}$$

The first-order condition implies[16]

$$c_i'(x_i)(r+X) - c_i(x_i) = rq_i. \tag{11}$$

The left-hand side of (11) can be interpreted as a dynamic marginal cost. Expanding it as $c_i'(x_i)r + [c_i'(x_i)x_i - c_i(x_i)] + c_i'(x_i)X_{-i}$, note that the first term, $c_i'(x_i)r$, accounts for the time value of incurring the marginal flow cost now rather than in the next instant; the second term, $[c_i'(x_i)x_i - c_i(x_i)]$, is the net increase in the flow cost of exerting effort x_i, and the last term, $c_i'(x_i)X_{-i}$, is the opportunity cost of increasing effort in case teammates make the breakthrough now. Similarly, the right-hand side of (11) can be interpreted as a dynamic marginal benefit, where rq_i is the time value of receiving the credit now rather than in the next instant. Clearly, at one extreme, the marginal benefit vanishes as agents grow very patient, $r \to 0$, implying $x_i \to 0$. At the other extreme, as agents grow very impatient, $r \to \infty$, (11) reduces to: $c_i'(x_i) = q_i$, which coincides with the first-order condition in the static example.

Taking (11) as agent i's incentive constraint, the planner's program can be written:

$$\max_{\mathbf{x},\,\mathbf{q}} W \equiv \frac{X}{r+X} - \frac{\sum_i c_i(x_i)}{r+X} \tag{SB}$$
$$\text{subject to } \sum_i q_i = 1, \text{ and (11) for all } i.$$

Let $\mathbf{q}^{SB}$ be a solution to (SB), which induces the effort x_i^{SB} and utility u_i^{SB} for agent i. Lemma 2 characterizes the second-best solution.

Lemma 2 *There is a unique interior solution to (SB) if $k > \underline{k}$ for some $\underline{k} \in \left(\frac{3}{2}, 2\right)$. At the solution, $a_i > a_j$ implies*

- **(a)** $x_i^{SB} > x_j^{SB}$ *and* $c_i'(x_i^{SB}) > c_j'(x_j^{SB})$,
- **(b)** $q_i^{SB} > q_j^{SB}$ *and* $\frac{q_i^{SB}}{x_i^{SB}} < \frac{q_j^{SB}}{x_j^{SB}}$,
- **(c)** $u_i^{SB} > u_j^{SB} > 0$.

[16] The second-order condition is easily verified.

As with the first-best, a higher ability agent is promised more credit from the team's success in return for greater effort. He also fares better. The second-best solution, however, differs from the first-best in two important ways. First, as indicated in Lemma 2(a), a higher ability agent's marginal flow cost exceeds that of a lower ability. Second, as indicated in Lemma 2(b), a higher ability agent earns less credit per unit effort. These differences are best understood by considering the planner's cost-minimization (CM) problem. In particular, as alluded to in the first-best, solving (SB) requires that the planner choose the least costly effort profile to implement a given team total X. Without loss of generality, CM can be relaxed by pooling the agents' incentive constraints since the left-hand side of (11) is nonnegative and uniquely maps into agent i's credit. Hence, CM can be expressed as:

$$\min_{\mathbf{x}} \sum_i c_i(x_i) \tag{CM}$$

$$\text{subject to } \sum_i \left[c_i'(x_i)(r+X) - c_i(x_i)\right] = r$$

$$\sum_i x_i = X.$$

Let

$$\mathcal{L} = \sum_i \left\{c_i(x_i) + \lambda \left[c_i'(x_i)(r+X) - c_i(x_i)\right]\right\} - \lambda r + \mu \left(X - \sum_i x_i\right)$$

be the Lagrangian of (CM), and λ and μ be its respective multipliers for the two constraints. The first-order condition at an interior solution, $\frac{\partial \mathcal{L}}{\partial x_i} = 0$, implies

$$c_i'(x_i) + \lambda \left[c_i''(x_i)(r+X) - c_i'(x_i)\right] = \mu, \tag{12}$$

where the proof of Lemma 2 verifies that $\lambda \in (0,1)$ and $\mu > 0$.

Since $\frac{\partial^2 \mathcal{L}}{\partial x_i \partial x_j} = 0$ for all $i \neq j$, the second-order condition for (CM) is satisfied if $\frac{\partial^2 \mathcal{L}}{\partial x_i^2} > 0$, or

$$c_i''(x_i) + \lambda \left[c_i'''(x_i)(r+X) - c_i''(x_i)\right] > 0. \tag{13}$$

Note that the left-hand side of (12) is the *virtual* marginal cost of eliciting effort x_i, which entails agent i's marginal flow cost $c_i'(x_i)$ and his marginal incentive cost, $\lambda\left[c_i''(x_i)(r+X) - c_i'(x_i)\right]$. As such, (13) is merely a monotonicity condition on the virtual marginal cost. Since $c_i'' > 0$ and $\lambda \in (0,1)$, (13) holds if $c_i''' \geq 0$, or $k \geq 2$ for the iso-elastic cost. But it also holds if c_i''' is not too negative, which explains the lower bound $\underline{k} \in \left(\frac{3}{2}, 2\right)$ in Lemma 2.

With this interpretation, it is intuitive from (12) that the planner minimizes the team's total cost by equating agents' virtual marginal costs – the counterpart of equating marginal costs in the first-best. Then, using (13), it readily follows that $x_i > x_j$ for $a_i > a_j$.

Next, to show $c_i'(x_i) > c_j'(x_j)$, we re-arrange (12):

$$c_i'(x_i)\left[1 - \lambda + \lambda(r+X)\frac{c_i''(x_i)}{c_i'(x_i)}\right] = \mu.$$

Since $\frac{c_i''(x_i)}{c_i'(x_i)} = \frac{k-1}{x_i}$ for the iso-elastic cost,

$$c_i'(x_i) = \frac{x_i}{x_i + \alpha}\beta \tag{14}$$

where $\alpha = \frac{\lambda}{1-\lambda}(k-1)(r+X) > 0$ and $\beta = \frac{\mu}{1-\lambda} > 0$. Hence, $x_i > x_j$ implies $c_i'(x_i) > c_j'(x_j)$. In other words, when faced with a moral hazard problem in the team, the planner deviates from the first-best and implements an effort profile in which the higher ability agent not only exerts a greater effort but also ends up with a higher marginal flow cost. The reason is that the marginal incentive cost for such an agent is lower.

Last, but not least, given the cost-minimizing effort profile, the credit allocation is uniquely pinned down by (11). Re-arranging its terms, we obtain

$$c_i'(x_i)\left[r + X - \frac{c_i(x_i)}{c_i'(x_i)}\right] = rq_i$$

which, by (14) and the fact that $\frac{c_i(x_i)}{c_i'(x_i)} = \frac{x_i}{k}$ for the iso-elastic cost, yields

$$\frac{q_i}{x_i} = \frac{r + X - \frac{x_i}{k}}{x_i + \alpha}\beta. \tag{15}$$

Clearly, given X, the right-hand side of (15) is strictly decreasing in x_i, which implies $\frac{q_i}{x_i} < \frac{q_j}{x_j}$ for $x_i > x_j$. That is, while promised more credit for greater effort, a higher ability agent earns less credit per unit effort in the second-best. Put differently, a higher ability agent is asked to work disproportionately harder than the credit allocation since $\frac{q_i}{q_j} < \frac{x_i}{x_j}$. This insight will be critical for understanding the equilibrium behavior, which we turn to next.

5 Team Equilibrium

Unlike in the two benchmarks, let agents be rewarded by the market, e.g., the scientific community, based on their expected contributions to the breakthrough. The market may be envisioned as a planner who cannot contract on efforts or credit distribution ex ante but cares about allocating due credit ex post. Thus, when choosing their efforts, agents effectively play a simultaneous-move game with each other *and* the market. In this section, we first prove the equilibrium existence and uniqueness of this game. Then, we characterize the equilibrium reversal, followed by a numerical example that delves into the region of multiple equilibria. Last, we show the possibility of overinvestment in teamwork with endogenous credit.

5.1 Existence and uniqueness

In equilibrium, agent i best responds to his teammates' effort X_{-i}^* and the market's belief q_i^* that he is the one responsible for success. As such, his best-response x_i^* must solve (9) and satisfy its first-order condition (11):

$$c_i'(x_i^*)(r+X^*) - c_i(x_i^*) = rq_i^*. \tag{16}$$

Together with the market's belief-consistency condition: $q_i^* = x_i^*/X^*$, (16) characterizes the equilibrium effort profile $\mathbf{x}^*$. As explained in the static example above, we focus on the (interior) equilibrium in which $x_i^* > 0$ for all i.

Notice that in equilibrium, the market equates the agents' credit per unit effort:

$$\frac{q_i^*}{x_i^*} = \frac{1}{X^*}. \tag{17}$$

This contrasts with the second-best, because unlike the planner, the market ignores the ex ante incentive effect of credit allocation. Given (17), it is also useful to define the average (dynamic) marginal cost:

$$AMC^i(x_i, X, r) \equiv \frac{c_i'(x_i)(r+X) - c_i(x_i)}{x_i} \tag{18}$$

and write (16) as:

$$AMC^i(x_i^*, X^*, r) = \frac{r}{X^*}. \tag{19}$$

(19) reveals that in equilibrium, having the same marginal benefit per unit effort, $\frac{r}{X^*}$, agents must also have the same average marginal costs. To compare their equilibrium efforts, it is, therefore, important to understand the properties of $AMC^i(x_i, X, r)$; in particular, whether it is increasing or decreasing in x_i *given* the team aggregate, X, which is common to all agents.

This "aggregative game" approach is more convenient in our setting and can be viewed as a two-step procedure for solving (19). First, we find each agent's "best-response," $f_i(X)$, to the team aggregate, i.e., we fix X and solve for x_i in (19). Second, summing them over, we find X^* and, in turn, x_i^*, from the fixed point equation: $X = \sum_i f_i(X)$.[17]

For ease of reference and discussion, we label the properties of the average marginal cost in the following definition, though such labeling is not essential for our results.

Definition 1 *(**easy versus difficult projects**) Agent i is said to perceive the project as intrinsically easy if* $\frac{\partial AMC^i}{\partial x_i} < 0$*, and intrinsically difficult if* $\frac{\partial AMC^i}{\partial x_i} > 0$ *for all* x_i*,* X*, and* r*. Moreover, agent i is said to perceive the project as easy if* $\left.\frac{\partial AMC^i}{\partial x_i}\right|_{x_i^*,\, X^*} < 0$*, and difficult if* $\left.\frac{\partial AMC^i}{\partial x_i}\right|_{x_i^*, X^*} > 0$.

[17] An aggregative game is defined as one in which each player's payoff is a function of his/her own action and some aggregate of all; see Acemoglu and Jensen (2013) and the references therein. It is evident from (9) that our game falls into this category. Acemoglu and Jensen report that Selten (1970) offered the first systematic analysis of aggregative games by writing one's best reply as a function of the aggregate, which he called the *backward best reply*. We are essentially following his footsteps here.

As is evident from (19), an agent's perception of the project as "easy" or "difficult" is an equilibrium object because it is likely to be influenced by his anticipation of the team's total effort, X^* – a feature that was absent in the static example. His perception is also influenced by the project's intrinsic cost, $c(.)$, and discount rate, r.[18] It is possible that the project's intrinsic cost dominates the agent's perception, in which case we call the project "intrinsically easy" or "intrinsically difficult." The following result sharpens Definition 1 for the iso-elastic cost specification.

Lemma 3 *For $1 < k \leq 2$, the project is intrinsically easy whereas, for $k \geq \frac{3+\sqrt{5}}{2} \approx 2.62$, it is intrinsically difficult. For $2 < k < \frac{3+\sqrt{5}}{2}$, team members' perceptions of the project may differ, depending on the equilibrium efforts and discount rate.*

Lemma 3 reveals that the project is intrinsically easy if the marginal flow cost, c_i', is concave or $k \leq 2$ for the iso-elastic specification, and intrinsically difficult if it is sufficiently convex or $k \geq 2.62$. Otherwise, if the marginal flow cost is moderately convex, the equilibrium efforts and the discount rate also affect how an agent views the project.[19] To examine agents' behaviors across project types, we first establish the existence of an equilibrium.

Proposition 1 *There is an (interior) equilibrium if and only if $k \neq 2$, or $k = 2$ and agents are not too heterogenous in terms of ability. Furthermore, if the project is intrinsically easy or intrinsically difficult, then the equilibrium is unique.*

For $k \neq 2$, an interior equilibrium obtains independent of the team's ability profile since, in this case, the marginal cost of effort satisfies Inada-like conditions: $c_i''(0) = 0$ or ∞. For $k = 2$ (the quadratic cost), $c_i''(0) = 1/a_i$, so the existence of an interior equilibrium requires that agents not be too heterogenous in ability; otherwise, the free-riding incentive would be too strong to elicit positive effort from all team members. More importantly, the interior equilibrium is unique when the project is intrinsically easy or intrinsically difficult. When the project is non-intrinsic, there can be multiple interior equilibria because each agent's perception of the project depends on what he expects of his teammates and on the discount rate.

Next, we characterize an equilibrium reversal across intrinsic projects and then numerically discuss non-intrinsic projects in Section 5.3.

[18] Given the separable cost of effort $c_i(x_i) = c(x_i)/a_i$, the agent's perception of the project is not directly affected by his ability, a_i, but it is scaled.

[19] To understand why the project difficulty closely tracks the shape of the flow effort cost, note that agent i needs to incur 2^k times the cost to double his rate of discovery x_i. Lemma 3 implies that while the effort cost increases at most four times for an intrinsically easy project, it grows at least six times for an intrinsically difficult project. For instance, the reader will probably agree that given the state of knowledge, proving the Pythagorean Theorem is intrinsically easy, say $k = 1.01$, whereas proving the Riemann Hypothesis, one of the Clay Mathematics Institute's millennium problems, is intrinsically difficult, say $k = 100$.

5.2 Equilibrium reversal

Intrinsic projects are of special interest to us because they yield unique equilibrium and generate behaviors that sharply differ, as Proposition 2 reports.

Proposition 2 *Suppose $a_1 \geq a_2 \geq ... \geq a_n$. Then, in the unique (interior) equilibrium*

- **(a)** *for an intrinsically easy project, the higher ability agent works less and thus receives less credit: $x_1^* \leq x_2^* \leq ... \leq x_n^*$ and $q_1^* \leq q_2^* \leq ... \leq q_n^*$,*
- **(b)** *for an intrinsically difficult project, the higher ability agent works more and thus receives more credit: $x_1^* \geq x_2^* \geq ... \geq x_n^*$ and $q_1^* \geq q_2^* \geq ... \geq q_n^*$,*

with strict inequalities whenever $a_i \neq a_j$.

The equilibrium reversal in Proposition 2 is better grasped in conjunction with the second-best benchmark. Consider a two-member team with abilities $a_1 > a_2$. Recall from Lemma 2 that in the second-best, the planner elicits greater effort from agent 1 by promising him more credit: $q_1 > q_2$. But, to also elicit effort from agent 2, the planner saves her scarce credit resource on agent 1 by promising him a lower rate of credit: $q_1/x_1 < q_2/x_2$.

For an intrinsically easy project, as would be the case for a quadratic flow cost, $c_i(x_i) = x_i^2/(2a_i)$, the market cannot replicate the planner's strategy since, unable to commit to the credit allocation ex ante, it is believed to keep the rate of credit constant (at $1/X$). Thus, for an intrinsically easy project, the only way the market can save its credit resource on the high ability agent and also motivate the low ability is if the former is believed to work less and, in turn, obtain lower credit.[20]

For an intrinsically difficult project, as would be the case for a cubic flow cost, $c_i(x_i) = x_i^3/(3a_i)$, the market's strategy is more aligned with the planner's: in equilibrium, agent 1 works harder and receives more credit. In this case, despite the constant rate of credit, the cost of effort is sufficiently convex to curb the high ability agent's incentive, leaving enough credit for the low ability to exert effort.

The equilibrium credit allocation in Proposition 2 raises a natural question: can a higher ability agent be worse off than his lower ability teammate in equilibrium? The following result shows that it is not the more able but the more diligent agent who fares better in equilibrium.

Proposition 3 *Suppose $a_1 \geq a_2 \geq ... \geq a_n$. Then, in the unique (interior) equilibrium*

[20] Our focus on interior equilibrium is important for this intuition since it implies the constant rate of credit from (17). As we explained in the motivating example, non-interior equilibria can be eliminated by an arbitrarily small team participation cost: with it, each team member would expect some credit from the breakthrough, which would require positive effort in equilibrium.

(a) *for an intrinsically easy project, the higher ability agent fares worse:* $u_1^* \leq u_2^* \leq ... \leq u_n^*$,

(b) *for an intrinsically difficult project, the higher ability agent fares better:* $u_1^* \geq u_2^* \geq ... \geq u_n^*$,

with strict inequality whenever $a_i \neq a_j$.

To see an agent's trade-off between receiving more credit for success and working harder, we employ (16) to simplify his equilibrium payoff:

$$\begin{aligned} u_i^* &= \frac{X^*}{r+X^*}q_i^* - \frac{c_i(x_i^*)}{r+X^*} \\ &= q_i^* - c_i'(x_i^*). \end{aligned} \tag{20}$$

That is, agent i's expected payoff is the difference between his expected credit and the marginal flow cost. Recall from Lemma 3 that for an intrinsically easy project, the marginal flow cost is concave, i.e., $c_i''' \leq 0$. In contrast, the market's credit increases linearly in one's effort. Since the lower ability agent is expected to work harder for an intrinsically easy project, he fares better. Part (b) follows because, as we explain in the next result, agents view their efforts as "strategic substitutes" when taking on an intrinsically difficult project. Hence, the diligence of the higher ability agent discourages his teammates, allowing him to obtain disproportionately more credit for success than the increase in his cost.

We complete this section by reporting some intuitive comparative statics with respect to agents' abilities and the discount rate.

Proposition 4 *Suppose* $a_1 \geq a_2 \geq ... \geq a_n$. *Then, for intrinsic projects, the equilibrium total effort is increasing in each agent's ability and the discount rate:* $\partial X^*/\partial a_i > 0$ *for all* i, *and* $\partial X^*/\partial r > 0$. *Moreover, as his teammates grow more able, each agent works harder in equilibrium for an intrinsically easy project, i.e.,* $\partial x_i^*/\partial a_j > 0$ *for* $i \neq j$, *but less hard for an intrinsically difficult project, i.e.,* $\partial x_i^*/\partial a_j < 0$ *for* $i \neq j$.

The fact that the inclusion of a more able agent improves the team's success rate, X^*, is expected. It is also expected that the team's success rate increases with discounting since impatient agents frontload their efforts. The second part of Proposition 4 follows because each agent views his effort as a "strategic complement" to the team effort (including his own), i.e., $\partial x_i/\partial X > 0$, for the intrinsically easy project while he views it to be a "strategic substitute" for the intrinsically difficult project, i.e., $\partial x_i/\partial X < 0$.[21] In particular, the agent is more concerned about cost savings when working for the latter type of project.

[21] Due to the aggregative nature of the game, we find it more convenient to define the concepts of strategic complement and strategic substitute based on an agent's response to the total effort in our team setting. The reason is that, as mentioned above, an agent's optimal strategy can be expressed in terms of the team effort: $x_i = f_i(X)$.

5.3 A numerical example

This section numerically illustrates our team equilibrium result with a three-member team whose ability profile is $(a_1, a_2, a_3) = (3.1, 3.05, 3)$. Panel (a) in Figure 1 considers an intrinsically easy project in which $k = 2$ and plots the individual efforts in the unique interior equilibrium as a function of the discount rate r.[22] In this case, in light of Lemma 3, regardless of r all agents perceive the project as easy: the equilibrium efforts are inversely related to ability with the lowest (highest) ability agent working the most (least). Panel (b) in Figure 1 looks at an intrinsically difficult project with $k = 3$. Here, regardless of r, all agents perceive the project as difficult: in the unique interior equilibrium, higher ability agents work harder.

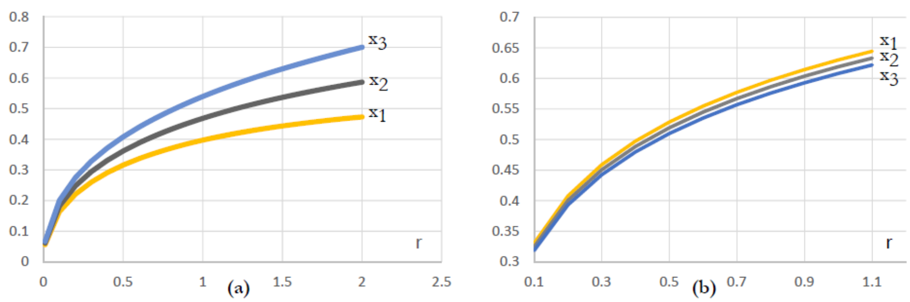

Figure 1: For ability profile (a_1,a_2,a_3)= (3.1, 3.05, 3), panel (a) illustrates equilibrium efforts for an intrinsically easy project with k=2 and panel (b) illustrates equilibrium efforts for an intrinsically difficult project with k=3.

Our numerical example yields some observations for non-intrinsic projects as well. In this case, there can be multiple equilibria because each agent's perception of the project depends on what he expects of his teammates and on the discount rate. Specifically, Figure 2 considers the same ability profile $(a_1, a_2, a_3) = (3.1, 3.05, 3)$ but considers a non-intrinsic project with

[22]Simulations were done with Mathematica, using six different initial points to check convergence.

$k = 2.1$. Below we discuss how agents' perceptions differ across different values of r.

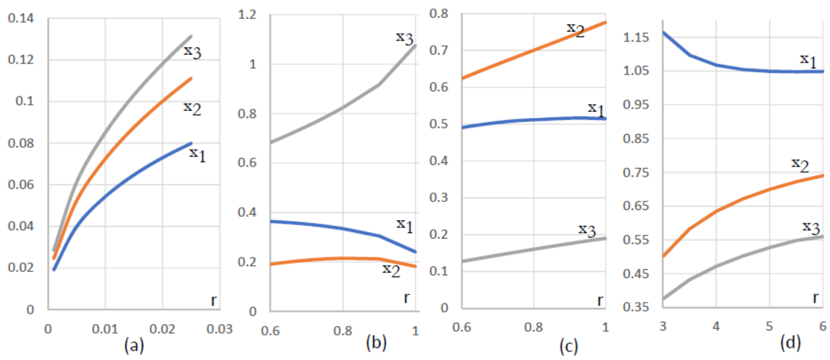

Figure 2: A non-intrinsic project k = 2.1; (a_1,a_2,a_3)= (3.1, 3.05, 3)

Panel (a) illustrates the equilibrium for sufficiently low discount rate r. The unique interior equilibrium has effort inversely related to ability. While the project is non-intrinsic with $k = 2.1$, all agents perceive it as easy in this range according to Definition 1 above. The intuition is as follows. Sufficiently patient agents exert very little effort. Evaluated at such low effort, average marginal cost is decreasing for all agents, as the intrinsic cost technology is not too convex. For example, when $r = 0.01$ in equilibrium, we have

$\left.\frac{\partial AMC^1}{\partial x_1}\right\rvert_{x_1^*,\, X^*} = -0.08$	$\left.\frac{\partial AMC^2}{\partial x_2}\right\rvert_{x_2^*,\, X^*} = -0.16$	$\left.\frac{\partial AMC^3}{\partial x_3}\right\rvert_{x_3^*,\, X^*} = -0.20.$

Panel (d) shows the other extreme for sufficiently high r. In the unique interior equilibrium, agents with higher ability expend more effort. All agents perceive the project as difficult in this range. This is because sufficiently impatient agents frontload their efforts: the convexity of the intrinsic cost technology now proves high enough to increase average marginal cost for all agents. For example, when $r = 5$, this equilibrium has

$\left.\frac{\partial AMC^1}{\partial x_1}\right\rvert_{x_1^*,\, X^*} = 0.17$	$\left.\frac{\partial AMC^2}{\partial x_2}\right\rvert_{x_2^*,\, X^*} = 0.49$	$\left.\frac{\partial AMC^3}{\partial x_3}\right\rvert_{x_3^*,\, X^*} = 0.80.$

Panels (b) and (c) consider the same non-intrinsic project for intermediate values of the discount rate r. The key observation here is that, given our definition, team members may have different perceptions of the project. Furthermore, interior equilibria in which effort is non-monotonic in ability may arise.

Panel (b) shows such an equilibrium: the agent with middle ability expends the least effort, and the one with the lowest ability works hardest. In this equilibrium, both high and middle ability agents perceive the project as difficult because they both view their efforts as strategic substitutes for the total team effort. The lowest ability agent, however, perceives the project as easy. For example, when $r = 0.75$, we have

$\frac{\partial AMC^1}{\partial x_1}\Big\|_{x_1^*, X^*} = 0.07$	$\frac{\partial AMC^2}{\partial x_2}\Big\|_{x_2^*, X^*} = 0.39$	$\frac{\partial AMC^3}{\partial x_3}\Big\|_{x_3^*, X^*} = -0.25.$

Panel (c) illustrates that for the same range of r, equilibria with the middle ability working the most and the lowest ability working the least also exist. In this latter type of equilibria, both highest and middle ability agents view their efforts as strategic complements for the total team effort, and thus both perceive the project as easy. The lowest ability agent now perceives it as difficult. For example, when $r = 0.75$, this equilibrium has

$\frac{\partial AMC^1}{\partial x_1}\Big\|_{x_1^*, X^*} = -0.10$	$\frac{\partial AMC^2}{\partial x_2}\Big\|_{x_2^*, X^*} = -0.20$	$\frac{\partial AMC^3}{\partial x_3}\Big\|_{x_3^*, X^*} = 0.70.$

While not reported here for brevity, equilibrium payoffs follow the same order as efforts for non-intrinsic projects, too.[23]

5.4 Overinvestment

Given that each team member maximizes his own utility, it seems intuitive that they will all underinvest in the joint project, compared to the first-best optimum. The static example in Section 2, which corresponds to $r \to \infty$, has confirmed this intuition. For $r < \infty$, we show that the underinvestment result continues to hold in a homogenous team, but overinvestment may occur in a sufficiently heterogenous team.

Lemma 4 *Suppose the team is homogenous, i.e., $a_i = a$ for all i. Then, every agent underinvests, i.e., $x_i^* < x_i^{FB}$ for all i.*

In a homogenous team, each agent expects to receive equal credit for the team's success in any equilibrium, which amounts to teamwork with an exogenous sharing rule of $1/n$. Hence, the agent's behavior is plagued with the standard free-riding incentive, resulting in underinvestment.

This straightforward logic, however, does not extend to a highly heterogenous team. To develop insight, consider a two-member team with abilities $(a_1, a_2) = (50, 2)$ and discount rate $r = .1$. Table 1 reports the unique interior equilibrium and first-best effort levels for project technologies $k = 1.5$ and $k = 3$.

[23] Those details can be found in our working paper, Ozerturk and Yildirim (2019).

k	x_1^*	x_1^{FB}	x_2^*	x_2^{FB}
1.5	0.004	5.877	0.516	0.009
3	1.866	1.801	0.052	0.360

Table 1. Over- vs. Under-investment in Teams

Note that for the intrinsically difficult project ($k = 3$), agent 1, being substantially more able, works virtually solo in equilibrium and expects credit $q_1^* = 0.973$ from success. Such equilibrium effort allocation is too unequal compared to that of the first-best: the high ability agent overinvests ($x_1^* > x_1^{FB}$) while the low ability agent underinvests ($x_2^* < x_2^{FB}$) in the project. The reason is that in the first-best, the planner would choose a more balanced workload in the team, equating agents' marginal costs.[24] A similar argument also explains the direction of the inefficiency for the intrinsically easy project ($k = 1.5$): the high ability agent underinvests ($x_1^* < x_1^{FB}$) while the low ability agent overinvests ($x_2^* > x_2^{FB}$). In this case, we know from Proposition 2 that it is the low ability agent who is expected to undertake most of the work, which is clearly in contrast to the first-best effort allocation. We formalize these observations in Proposition 5.

Proposition 5 *Consider a two-member team with $a_1 > a_2$. For a sufficiently large a_1,*

(a) *the high ability agent underinvests whereas the low ability agent overinvests in an intrinsically easy project: $x_1^* < x_1^{FB}$ and $x_2^* > x_2^{FB}$,*

(b) *the high ability agent overinvests whereas the low ability agent underinvests in an intrinsically difficult project: $x_1^* > x_1^{FB}$ and $x_2^* < x_2^{FB}$.*

6 Extensions and Further Results

In this section, we consider three extensions: one that demonstrates endogenous team formation between two agents, the other that examines the impact of effort monitoring on team incentives, and the last that confirms the robustness of our conclusions when agents possess heterogeneous cost elasticities.

6.1 Endogenous team formation

Up to now, we have assumed that agents are committed to collaborating on the project. Such commitment may be unavoidable in many contexts. The project may be part of a broader

[24]Put differently, the virtually solo work in equilibrium would essentially require the planner to ignore the rest of the team, which is not efficient given $c_i(0) = c_i'(0) = 0$.

agenda for researchers or assigned to a group of employees by their employer. For many projects, though, agents are flexible whether or not to collaborate. While a full analysis of team formation for an arbitrary number of agents is beyond the scope of this paper, we show that with two agents, e.g., two authors, teamwork is the unique "outcome" if they are of similar ability and sufficiently patient. In particular, despite expecting to receive less credit and fare worse than his low ability teammate, a high ability agent may collaborate on an intrinsically easy project.

Following Bloch (1996) and Yi (1997), suppose that a randomly selected agent proposes to the other whether to team up or work solo on the project. If the responder accepts a team offer, the game proceeds as in our base model. Otherwise, agents work solo on the project, which is a standard R&D race or contest whose winner receives the full credit of one. Agent i's expected payoff from solo work is, therefore,

$$u_i = \frac{x_i}{r+X} - \frac{c_i(x_i)}{r+X}. \tag{21}$$

Lemma 5 *Consider two agents with $a_1 > a_2$. Under solo work, there is an equilibrium. And, in every equilibrium, the high ability agent competes harder and receives a higher expected payoff: $x_1^S > x_2^S$ and $u_1^S > u_2^S$.*

Under solo work, every equilibrium is interior because, unlike in teamwork, each agent expects an exogenous reward from success. In addition, exploiting his cost advantage toward the same unit prize, the high ability agent exerts greater effort and is better off than his low ability rival. We also know from Lee and Wilde (1980) that an agent is motivated by the rival's effort in an R&D race. As such, the competition is expected to grow most intense among similar agents. Moreover, agents place more weight on the future competition as they become more patient. We, therefore, predict that such agents will team up even if that means sharing the credit for success. Proposition 6 confirms our prediction.

Proposition 6 *If two agents have sufficiently similar abilities and are sufficiently patient, they strictly prefer to team up. Formally, for a given ability level $a > 0$, there exists $\varepsilon > 0$ such that $u_i^S < u_i^*$ for all i whenever $|a_i - a| < \varepsilon$ for all i, and $r < \varepsilon$.*

To understand, consider identical and very patient ($r \to 0$) agents. With little time discounting, the first-best or efficient outcome would require such agents to exert vanishingly small efforts. When they work as a team, patient agents achieve efficiency due to free-riding. However, when agents compete for the breakthrough, they continue to exert significant effort since, despite being very patient, competing agents discount future returns also by the probability of losing the race. Moreover, in a homogenous team, each agent expects to receive a reward of $1/n$ in equilibrium due to equal credit sharing under teamwork and an equal probability of winning under solo work. Hence, patient agents strictly prefer to team up and avoid costly competition when they are identical. In other words, teamwork flattens incentives and, not surprisingly, leads to better outcomes. The same conclusion extends to agents who are not too heterogeneous and not too impatient by the continuity of payoffs.

It is worth noting that Proposition 6 holds regardless of whether agents perceive the project to be easy or difficult. As a numerical example for an intrinsically easy project, consider a quadratic cost of effort, $k = 2$, an ability profile $(a_1, a_2) = (3, 2.8)$, and the discount rate $r = 0.1$. Consistent with Proposition 2(a), the high ability agent fares worse in teamwork, $u_1^* = 0.355 < 0.452 = u_2^*$. However, he would not operate solo since he would then have to compete with the low ability agent for the breakthrough. Although the high ability agent would be better off than his rival in this competition, $u_1^S = 0.331 > 0.313 = u_2^S$, he would be worse off than working as a team, i.e., $u_1^S < u_1^*$. Hence, the high ability agent has a strict incentive to team up. Since the low ability agent shares the same strict preference, the two would collaborate.[25]

6.2 Observable effort and competition for credit

A key obstacle for credit attribution in teamwork is that individual efforts are unobservable to the market. While this monitoring problem is likely to be severe in cases where the market is in an arms-length relationship with the team, it may be less so in others. For instance, in academia, it is not uncommon that researchers discuss their ongoing projects with their peers and regularly present them at conferences.

Since credit attribution affects team incentives, it is natural to ask how the observability of effort by the market may change these incentives. To this end, suppose that, unlike in Section 5, the market can perfectly observe, though cannot dictate, team members' flow of efforts. The same level of monitoring may or may not be available within the team, but this is irrelevant in our setting because efforts are chosen simultaneously and renewed at each instant. Hence, given his teammates' flow effort X_{-i}, agent i expects to receive the credit $q_i = \frac{x_i}{x_i + X_{-i}}$, which is increasing in his own effort, x_i. This means that the observability of effort by the market creates competition for credit within the team.

To see the amount of competition, we substitute for q_i in (9) and find the following expected payoff for agent i under the observability:

$$
\begin{aligned}
u_i &= \frac{X}{r+X}\left(\frac{x_i}{X}\right) - \frac{c_i(x_i)}{r+X} \\
&= \frac{x_i}{r+X} - \frac{c_i(x_i)}{r+X}.
\end{aligned}
\tag{22}
$$

[25] The details of this numerical example can be found in our working paper, Ozerturk and Yildirim (2019). In particular, our simulations indicate that agents would team up for $r \leq .2$, but not for $r > .2$ since the high ability would then refuse teamwork.

Clearly, (22) coincides with (21): teamwork with observable efforts is strategically equivalent to solo work for the agents. In other words, the market's monitoring of effort destroys the "team spirit" since team members act as though they were in a contest. From this equivalence and our results in Section 5.1, two main insights emerge.

First, Lemma 5 implies that when the market can monitor efforts, a higher ability agent always works harder and receives more credit from success. Second, Proposition 6 reveals that team members dislike such public monitoring if they are of similar ability and sufficiently patient. The reason is that the lack of monitoring allows them to commit not to compete for credit. Interestingly, as discussed in the previous subsection, a higher ability agent would also endorse no monitoring by the public even though he would fare worse than his lower ability teammate when collaborating on an intrinsically easy project.

The competition for credit under observable efforts further suggests that team members may have private incentives to disclose their efforts to the market. For instance, each coauthor may be eager to present the joint work to the peers in the hopes of making his or her contribution more visible. Similarly, each student on a group assignment may independently approach the professor to prove how hard he or she has worked on the assignment.[26] We demonstrate how such unilateral actions to make effort known may hurt the team spirit within a simple numerical example.

Consider a team of two symmetric agents with the abilities $a_1 = a_2 = 1$, a cubic cost of effort $c(x) = x^3/3$, and the discount rate $r = 0.1$. Before they work on the joint project, agents independently decide once-and-for-all whether to disclose (d) their efforts to the market, i.e., work in the limelight or not disclose (nd), i.e., work in secrecy. The fixed cost of disclosing effort is $\lambda \geq 0$ for both agents, while nondisclosure is assumed costless.[27] Once disclosure decisions are made, they become public, and agents continuously choose their efforts until a breakthrough, as in the base model.

Note that there are three effort subgames depending on disclosure decisions. If no agent discloses, then they engage in teamwork with endogenous credit. In contrast, if both agents disclose, then, as argued in (22), they effectively work solo. Finally, if one agent discloses but the other does not, then the effort subgame has the solo payoff (22) for the disclosing agent and the team payoff (10) for the non-disclosing agent. Each effort subgame has a unique equilibrium: $(0.79, .079)$ in (d, d), $(0.62, 0.15)$ in (d, nd), and $(0.29, 0.29)$ in (nd, nd). The following

[26]We thank a referee for this observation.

[27]The disclosure cost may involve traveling to professional conferences for researchers and making appointments with the professor for students.

table summarizes the agents' payoffs for the disclosure game.

	d	nd
d	$0.37 - \lambda, 0.37 - \lambda$	$0.62 - \lambda, 0.18$
nd	$0.18, 0.62 - \lambda$	$0.41, 0.41$

Table 2: Disclosure Game

Inspecting Table 2, pure strategy Nash equilibria are easily found to be

$$\begin{cases} (d, d) & \text{if} \quad \lambda < 0.19 \\ (d, nd) \text{ and } (nd, d) & \text{if} \quad 0.19 < \lambda < 0.21 \\ (nd, nd) & \text{if} \quad \lambda > 0.21. \end{cases}$$

Unsurprisingly, the unique equilibrium has both agents' disclosing efforts when the cost of doing so is small, and no one disclosing when the cost is large. Somewhat interestingly, when the cost is intermediate, only one agent discloses in equilibrium. Knowing that he can influence the market's credit allocation through his effort, the disclosing agent works harder than his nondisclosing teammate, 0.62 vs. 0.15, and, indeed, receives more credit in equilibrium, 0.80 vs. 0.20. However, it is evident that agents would collectively prefer a high disclosure cost, $\lambda > 0.21$, to keep the team spirit. While teamwork introduces free-riding as a source of inefficiency, Table 2 indicates that this inefficiency is less severe than two symmetric agents' excessive effort when competing for the breakthrough.

6.3 Heterogenous cost elasticities

In our model, we call the parameter a_i agent i's ability because higher a_i implies a lower marginal flow cost $c_i'(x)$ for all $x > 0$. However, an agent's ability could also be defined based on his cost elasticity, k. Employing the static model in Section 2, we demonstrate that the equilibrium reversals in Propositions 2 and 3 continue to hold under this specification. To this end, suppose $a_i = a$ for all i and that agent i's flow cost is

$$c_i(x) = \frac{x^{k_i}}{k_i a}, \; k_i > 1. \tag{23}$$

So, agents are heterogenous in their cost elasticities. As in Section 2, they simultaneously choose efforts and only once. The breakthrough occurs with probability $X = \sum_i x_i$. And the credit is allocated proportional to effort: $q_i = x_i / X$. To guarantee $X \in (0, 1)$, we assume $\sum_i a^{\frac{1}{k_i - 1}} < 1$. Note that $k_i > k_j$ implies $c_i'(x) < c_j'(x)$ for $x \in (0, 1)$. Thus, a higher cost elasticity refers to a higher ability agent in this specification. The following proposition characterizes the first-best and the interior equilibrium.

Proposition 7 *Consider the static model just described. In the unique first-best solution,* $x_i^{FB} > x_j^{FB}$ *and* $u_i^{FB} > u_j^{FB}$ *for* $k_i > k_j$*. Furthermore,*

(a) *there is a unique interior equilibrium if and only if either* $k_i > 2$ *or* $k_i < 2$ *for all i.*

(b) *In the interior equilibrium,*

$$k_i > k_j > 2 \Longrightarrow x_i^* > x_j^* \text{ and } u_i^* > u_j^*$$

and

$$2 > k_i > k_j \Longrightarrow x_i^* < x_j^* \text{ and } u_i^* < u_j^*, \text{ whenever } a < e^{-\frac{1}{2}} \approx .61.$$

The intuition behind the equilibrium reversals in part (b) parallels that of the static example in Section 2. Specifically, the market continues to equate the agents' average marginal costs in equilibrium, yielding the critical cost elasticity of 2 for the reversal. Since higher k refers to the higher ability, the equilibrium efforts and payoffs are ordered the same as the abilities if agents view the project as difficult. And the reverse order holds if they view it as easy.

7 Conclusion

Proper credit for scientific discovery plays a key role in the progress of science, as it affects appointments, promotions, and funding for researchers. Assigning credit, however, has grown complicated by the increasing dominance of collaborative work across many disciplines. A similar problem also exists in non-academic settings such as business and politics, where teamwork is prevalent. In this paper, we have examined in some detail the endogenous relationship between credit attribution and team incentives in a tractable model.

Our first main result is an equilibrium reversal. We show that it is the most diligent – not necessarily the most able – team member who deserves the most credit for collective success. That member's identity, however, depends on the team's composition and the project's difficulty. In particular, it is the least able member who works the hardest for an "intrinsically easy" project and thus receives the most credit for the success, while the opposite holds for an "intrinsically difficult" project. The most diligent agent also fares the best. Our second main result is that some agents may overinvest in teamwork with endogenous credit. Specifically, we show that in a two-member team with sufficient ability differential, expecting to receive most of the credit, the low (resp. high) ability agent works virtually solo in an intrinsically easy (resp. difficult) project. This workload allocation is inefficient since it would effectively require the social planner to ignore the remaining team members. In two variations of our base model, we further offer conditions under which even the high ability agent would strictly prefer to team up to avoid excessive competition. Last, we argue that although teamwork obscures credit attribution, team members may nonetheless dislike the market's monitoring of their efforts for proper credit because such monitoring also creates competition within the team.

While contributing to the ongoing debate on credit attribution in teamwork, our investigation has only scratched the surface. For one, unlike in our model, many team projects require complementary efforts, such as when projects involve a clear division of labor among team members. We conjecture that the observability and thus, the credit attribution problem will be less severe for these projects. Furthermore, team projects may come with a deadline. The deadline is likely to render effort choices non-stationary and introduce credit attribution dynamics. For instance, if the project is completed earlier than the deadline, will the low- or high-ability agent get more credit? Another extension would recognize that agents often accumulate knowledge, i.e., learn from their past failures. Such learning is likely to affect their perceptions of the project's difficulty. Thus, as with deadlines, knowledge accumulation is expected to generate a nontrivial effort and credit attribution dynamics.

Last but not least, it would be valuable to test our equilibrium reversal result. For instance, one might define a co-authored article's difficulty based on the journal's ranking in the scientific field, each co-author's ability as his or her experience on the topic, such as related publications, and the individual credit based on each co-author's career trajectory after the article's publication. In two recent studies, Jin et al. (2019) and Sarsons et al. (2021) estimate credit attribution in scientific teams but with different focuses.

Appendix A: Proofs for Section 4

Proof of Lemma 1. Differentiating the objective function, W, in (FB), we find

$$\frac{\partial}{\partial x_i} W = \frac{r + \sum_i c_i(x_i)}{(r+X)^2} - \frac{c_i'(x_i)}{r+X}. \tag{A-1}$$

Since $c_i'(0) = 0$ and $r > 0$, (A-1) implies that if it exists, the solution to (FB) must be interior: $x_i^{FB} > 0$ for all i. And, the first-order conditions must bind at the optimum:

$$\frac{\partial}{\partial x_i} W = 0 \text{ for all } i. \tag{A-2}$$

Now we establish that there is a unique effort profile $\mathbf{x}$ that satisfies (A-2). Note from (A-2) that

$$c_i'(x_i) = z \text{ for all } i. \tag{A-3}$$

Since $c_i(x_i) = \frac{c(x_i)}{a_i}$ and $c' > 0$, (A-3) can be inverted as

$$x_i = \phi(a_i z), \tag{A-4}$$

where $\phi \equiv c'^{-1}$, $\phi(0) = 0$, and $\phi' > 0$.

Using (A-4), (A-2) can be written:

$$\Gamma(z) \equiv (r + \sum_i \phi(a_i z))z - \sum_i c_i(\phi(a_i z)) - r = 0. \tag{A-5}$$

Clearly, $\Gamma(0) = -r < 0$. Moreover,

$$\begin{aligned}
\Gamma(1) &= \sum_i \phi(a_i) - \sum_i c_i(\phi(a_i)) \\
&= \sum_i (x_i - c_i(x_i)) \\
&> \sum_i x_i \left(1 - c_i'(x_i)\right) \\
&= 0,
\end{aligned}$$

where $\frac{c_i}{x_i} < c_i'$ since $c_i'' > 0$ and $c_i'(0) = 0$, and $c_i'(x_i) = z = 1$ by assumption.

Hence, $\Gamma(1) > 0$ and, by continuity, there is an interior solution, $z^{FB} \in (0,1)$, to (A-5). Next, we observe that

$$\Gamma'(z) = r + \sum_i \phi(a_i z) > 0,$$

which establishes that z^{FB} is unique. As a result, there is a unique solution to (FB) such that $x_i^{FB} = \phi(a_i z^{FB}) > 0$.[28]

[28]The unique solution to the first-order conditions cannot be minimizing welfare since the unique minimizer is trivially $\mathbf{x} = \mathbf{0}$, yielding $W = 0$.

Next, for $a_i > a_j$,

$$x_i^{FB} = \phi(a_i z^{FB}) > \phi(a_j z^{FB}) = x_j^{FB},$$

and, in turn, $q_i^{FB} = x_i^{FB}/X^{FB} > x_j^{FB}/X^{FB} = q_j^{FB}$.

Finally, by definition,

$$u_i^{FB} = \frac{X^{FB}}{r + X^{FB}} q_i^{FB} - \frac{c_i(x_i^{FB})}{r + X^{FB}} = \frac{x_i^{FB} - c_i(x_i^{FB})}{r + X^{FB}}.$$

Note that $x - c_i(x)$ is strictly decreasing in x for $c_i'(x) < 1$, which is true at the optimum (since $z^{FB} < 1$). Moreover, $x - c_i(x)$ is strictly increasing in a_i since $c_i(x) = c(x)/a_i$. Hence, $x_i^{FB} > x_j^{FB} > 0$ implies $u_i^{FB} > u_j^{FB} > 0$. ■

Proof of Lemma 2. As discussed in the text for the cost-minimization program (CM), without loss of generality, we can re-write (SB) by pooling the incentive constraints (11):

$$\max_{\mathbf{x}} W \equiv \frac{X}{r + X} - \frac{\sum_i c_i(x_i)}{r + X}$$

$$\text{subject to } \sum_i \left[c_i'(x_i)(r + X) - c_i(x_i)\right] = r. \tag{IC}$$

Let

$$\mathcal{L} = \frac{X}{r + X} - \frac{\sum_i c_i(x_i)}{r + X} + \eta \left(r - \sum_i \left[c_i'(x_i)(r + X) - c_i(x_i)\right] \right)$$

be the Lagrangian where η is the multiplier for (IC). To save on notation, we drop arguments of functions in the remaining of this proof.

In an interior solution,

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{r + \sum_i c_i - (r + X)c_i'}{(r + X)^2} + \eta \left(-\sum_i c_i' - (r + X)c_i'' + c_i' \right) = 0. \tag{A-6}$$

Substituting for $r + \sum_i c_i = (r + X)\sum_i c_i'$ from (IC), (A-6) reduces to

$$\eta(r + X) = \frac{\sum_i c_i' - c_i'}{\sum_i c_i' - c_i' + (r + X)c_i''} \in (0, 1). \tag{A-7}$$

Note that $\eta(r + X) = \lambda$ at the second-best optimum where λ is the Lagrange multiplier for (IC) in (CM). Then, re-arranging (A-7), we have

$$c_i' + \lambda[(r + X)c_i'' - c_i'] = (1 - \lambda)\sum_i c_i'. \tag{A-8}$$

Comparing (A-8) with (12), we conclude that $\mu = (1 - \lambda)\sum_i c_i' > 0$ and, in turn, $\beta = \frac{\mu}{1-\lambda} = \sum_i c_i'$, where μ and β are defined in Section 4.2.

Differentiating (A-6) further and re-arranging terms, the second-order condition holds if:[29]

$$\left.\frac{\partial^2 \mathcal{L}}{\partial x_i^2}\right|_{\frac{\partial \mathcal{L}}{\partial x_i}=0} = -4\lambda c_i'' - 2\frac{\lambda}{r+X}\underbrace{\left(\sum_i c_i' - c_i'\right)}_{>0} - \underbrace{\left\{c_i'' + \lambda\left[(r+X)c_i''' - c_i''\right]\right\}}_{>0 \text{ by the assumption (13)}} < 0. \tag{A-9}$$

Below we show that (13) is satisfied if $k > \underline{k}$ for some $\underline{k} \in \left(\frac{3}{2}, 2\right)$.

Let $a_1 \geq a_2 \geq ... \geq a_n$. In Section 4.2, we have derived the following for the second-best:

$$\begin{cases} x_1 \geq x_2 \geq ... \geq x_n \\ c_1' \geq c_2' \geq ... \geq c_n' \\ \frac{q_1}{x_1} \leq \frac{q_2}{x_2} \leq ... \leq \frac{q_n}{x_n}, \end{cases} \tag{A-10}$$

with strict inequalities whenever $a_i \neq a_j$. Next we show $u_1 \geq u_2 \geq ... \geq u_n$.

Inserting (11) into (10), agent i's second-best utility reduces to:

$$u_i = q_i - c_i' = \frac{c_i' X - c_i}{r}. \tag{A-11}$$

Moreover, inserting $\beta = \sum_i c_i'$ into (14), we find

$$c_i' = \frac{x_i}{x_i + \alpha}\sum_i c_i'. \tag{A-12}$$

where $\alpha = \frac{\lambda}{1-\lambda}(k-1)(r+X)$. Since $\frac{c_i}{c_i'} = \frac{x_i}{k}$ for the iso-elastic cost, (A-11) and (A-12) reveal

$$u_i = \frac{x_i}{x_i + \alpha}\left(X - \frac{x_i}{k}\right)\frac{\sum_i c_i'}{r}.$$

From here,

$$\begin{aligned} u_i - u_j &\propto \frac{x_i}{x_i+\alpha}\left(X - \frac{x_i}{k}\right) - \frac{x_j}{x_j+\alpha}\left(X - \frac{x_j}{k}\right) \\ &= \frac{x_i\left(x_j+\alpha\right)\left(X - \frac{x_i}{k}\right) - x_j\left(x_i+\alpha\right)\left(X - \frac{x_j}{k}\right)}{\left(x_i+\alpha\right)\left(x_j+\alpha\right)}. \end{aligned}$$

[29]To be sure, for the second-order condition of (SB), it is also necessary to check the cross-partials of the Lagrangian, which is tedious. There is, however, a more direct way. Consider the program (CM) in the text, and let $C(X)$ be its value function, i.e., the minimum total cost of implementing X. By the envelope theorem, $C'(X) = (r+X)\lambda\sum c_i' + \mu > 0$. Since $x_i' = \partial x_i/\partial X > 0$, we find $C''(X) = \lambda\left[\sum c_i' + (r+X)\sum c_i'' x_i'\right] > 0$. Next, note that (SB) can be stated as:

$$\max_X W(X) = \frac{X}{r+X} - \frac{C(X)}{r+X}. \tag{SB'}$$

Clearly,

$$W''(X)\big|_{W'(X)=0} = -rC''(X) < 0.$$

Hence, $W(X)$ is strictly quasi-concave. Other than verifying the second-order condition, we find working with (SB') as tedious as (SB) in the proof.

$\Longrightarrow$

$$
\begin{aligned}
u_i - u_j &\propto x_i\left(x_j+\alpha\right)\left(X-\frac{x_i}{k}\right) - x_j\left(x_i+\alpha\right)\left(X-\frac{x_j}{k}\right) \\
&= \left(\alpha X - \frac{x_i x_j}{k}\right)x_i - \frac{\alpha}{k}x_i^2 - \left(\left(\alpha X - \frac{x_i x_j}{k}\right)x_j - \frac{\alpha}{k}x_j^2\right) \qquad \text{(A-13)} \\
&= \left(x_i - x_j\right)\left(\alpha X - \frac{x_i x_j}{k} - \frac{\alpha}{k}(x_i+x_j)\right) \\
&= \left(x_i - x_j\right)\{\alpha\underbrace{(X - x_i - x_j)}_{\geq 0} + \frac{1}{k}\underbrace{\left[\alpha(k-1)(x_i+x_j) - x_i x_j\right]}_{\equiv \Omega}\}.
\end{aligned}
$$

To prove $sgn\left(u_i - u_j\right) = sgn\left(x_i - x_j\right)$, it suffices to prove $\Omega > 0$.

Solving (A-12) for x_i yields

$$
x_i = \alpha\frac{c_i'}{A + c_j'}
$$

where $A \equiv \sum_i c_i' - (c_i' + c_j') \geq 0$. Hence,

$$
\begin{aligned}
\Omega &= \alpha(k-1)\left(\alpha\frac{c_i'}{A+c_j'} + \alpha\frac{c_j'}{A+c_i'}\right) - \alpha^2\frac{c_i'c_j'}{\left(A+c_j'\right)\left(A+c_i'\right)} \\
&\propto (k-1)\left[c_i'\left(A+c_i'\right) + c_j'\left(A+c_j'\right)\right] - c_i'c_j' \\
&= (k-1)\left[\left(c_i' - c_j'\right)^2 + 2c_i'c_j' + A\left(c_i'+c_j'\right)\right] - c_i'c_j' \\
&= (k-1)\left[\left(c_i' - c_j'\right)^2 + A\left(c_i'+c_j'\right)\right] + \left(2(k-1)-1\right)c_i'c_j'.
\end{aligned}
$$

So, $\Omega > 0$ if $2(k-1) - 1 \geq 0$ or $k \geq \frac{3}{2}$. We argue that $k \geq \frac{3}{2}$ must hold for an interior second-best.

Note that since $\frac{c_i'''}{c_i''} = \frac{k-2}{x_i}$, (13) becomes

$$
c_i'' + \lambda\left[(r+X)c_i''' - c_i''\right] > 0
$$

$\Longleftrightarrow$

$$
1 + \underbrace{\frac{\lambda(r+X)}{1-\lambda}}_{=\alpha/(k-1)}\frac{k-2}{x_i} > 0
$$

$\Longleftrightarrow$

$$
x_i > \alpha\frac{2-k}{k-1}. \qquad \text{(A-14)}
$$

Next, from (A-10) and (A-12),

$$
c_i' = \frac{x_i}{x_i + \alpha}\sum_i c_i'
$$

$\Longrightarrow$

$$c_i' \geq \frac{x_n}{x_n + \alpha} \sum_i c_i'$$

$\Longrightarrow$

$$\sum_i c_i' \geq n \frac{x_n}{x_n + \alpha} \sum_i c_i'$$

$\Longrightarrow$

$$x_n \leq \frac{\alpha}{n-1}.$$

Together with (A-14), we have

$$\alpha \frac{2-k}{k-1} < x_n \leq \frac{\alpha}{n-1}.$$

Since $\frac{\alpha}{n-1} \leq \alpha$ for $n > 1$, this implies

$$\frac{2-k}{k-1} < 1 \Longrightarrow k > \frac{3}{2},$$

and $\Omega > 0$, as desired. Hence, (A-13) reveals that $sgn\left(u_i - u_j\right) = sgn\left(x_i - x_j\right)$, or

$$u_1 \geq u_2 \geq ... \geq u_n. \qquad \text{(A-15)}$$

Finally, since $u_i = q_i - c_i'$ by (A-11), and $c_1' \geq c_2' \geq ... \geq c_n'$ by (A-10), (A-15) implies

$$q_1 \geq q_2 \geq ... \geq q_n.$$

■

Appendix B: Proofs for Section 5

Proof of Lemma 3. First, since $c_i(x) = c(x)/a_i$, define the following function from (18):

$$\Phi(x_i, X, r) \equiv a_i AMC^i(x_i, X, r) = \frac{c'(x_i)(r + X) - c(x_i)}{x_i}. \qquad \text{(B-1)}$$

Then,

$$sgn\left(AMC^i_{x_i}\right) = sgn\left(\Phi_{x_i}\right), \qquad \text{(B-2)}$$

where the subscripts refer to partial derivatives throughout. Clearly,

$$\Phi_{x_i} = \left(\frac{c'(x_i)}{x_i}\right)'(r + X) - \left(\frac{c(x_i)}{x_i}\right)'. \qquad \text{(B-3)}$$

Since

$$sgn\left(\frac{c(x_i)}{x_i}\right)' = sgn\left(c''(x_i)\right) > 0 \quad \text{and} \quad sgn\left(\frac{c'(x_i)}{x_i}\right)' = sgn\left(c'''(x_i)\right),$$

it follows that if $c'''(x_i) \leq 0$ for all x_i, then

$$\Phi_{x_i} < 0 \text{ for all } x_i, X, r. \tag{B-4}$$

Conversely, suppose (B-4) holds, but $c'''(x_i) > 0$ for some x_i. Then, (B-3) would imply that $\Phi_{x_i} > 0$ for $r + X_{-i} \to \infty$, a contradiction. Hence, by (B-2), the project is *intrinsically easy* if and only if $c'''(x_i) \leq 0$ for all x_i. For the iso-elastic specification, $c(x_i) = x_i^k/k$, it is readily verified that

$$c'''(x_i) \leq 0 \Longleftrightarrow 1 < k \leq 2.$$

Next, by (B-2), the project is *intrinsically difficult* if and only if $\Phi_{x_i} > 0$ for all x_i, X, r, which, by (B-3), requires that $c'''(x_i) > 0$ for all x_i. Hence, the project is intrinsically difficult if and only if $\Phi_{x_i} \geq 0$ for $r = X_{-i} = 0$ and all x_i, or equivalently,

$$\left(\frac{c'(x_i)}{x_i}\right)' x_i - \left(\frac{c(x_i)}{x_i}\right)' \geq 0 \text{ for all } x_i. \tag{B-5}$$

For $c(x_i) = x_i^k/k$, (B-5) is satisfied if and only if $(k-2) - \frac{k-1}{k} \geq 0$, or $k \geq \frac{3+\sqrt{5}}{2} \approx 2.62$, as claimed. ■

Proof of Proposition 1. We offer a more general proof here by imposing the following three cost conditions. It is straightforward to verify that the iso-elastic specification, $c(x) = x^k/k$ with $k > 1$, satisfies them.

$$\lim_{x\to\infty} \frac{c(x)}{xc'(x)} < 1 \tag{COND 1}$$

$$\lim_{x\to 0} \frac{xc''(x)}{c'(x)} < 1 \text{ if } c''(0) = \infty. \tag{COND 2}$$

$$c''(0) = 0 \text{ if } c''' > 0. \tag{COND 3}$$

As a preliminary, we use $\Phi(x_i, X, r)$ from (B-1) and define

$$m(x_i, X, r) \equiv X\Phi(x_i, X, r). \tag{B-6}$$

Then, given that $c_i(x_i) = \frac{c(x_i)}{a_i}$, the first-order condition in (19) can be written as

$$m(x_i, X, r) = ra_i. \tag{B-7}$$

Clearly,

$$sgn\,(m_{x_i}) = sgn\,(\Phi_{x_i})\,. \tag{B-8}$$

Depending on the $sgn\,(\Phi_{x_i})$, we consider two cases in turn.

Case 1. *$c'''(x_i) \leq 0$ for all x_i ($k \in (1,2]$ for $c(x) = x^k/k$).*

From Lemma 3, this case refers to the intrinsically easy project. In particular, $\Phi_{x_i} < 0$ and thus, $m_{x_i} < 0$. Given $X > 0$, there is a unique solution $x_i \in [0, X]$ to (B-7) if and only if

$$A(X) \leq ra_i \leq B(X)$$

where

$$A(X) \equiv m(X,X,r) = c'(X)(r+X) - c(X), \tag{B-9}$$

$$B(X) \equiv m(0,X,r) = c''(0)(r+X)X.$$

Clearly, $A(0) = 0$ and, by (COND 1), $A(\infty) = \infty$. Furthermore, $A'(X) = c''(X)(r+X) > 0$. Hence, there is a unique cutoff $0 < X_i < \infty$ such that $A(X) \leq ra_i$ for $X \leq X_i$. For $B(X) \geq ra_i$, we consider two subcases.

Case 1.1. *$c''(0) = \infty$ ($k < 2$ for $c(x) = x^k/k$).*

Then, $B(X) > ra_i$ for all $X > 0$. This implies that there is a unique solution

$$x_i = f(X, r, a_i), \tag{B-10}$$

to (B-7) if and only if $X \in [0, X_i]$, with $f(0, r, a_i) = 0$ and $f(X_i, r, a_i) = X_i$.

Substituting (B-10) into (B-7), we re-write (B-7) as

$$\Phi(f(X,r,a_i), X, r) = \frac{ra_i}{X}, \tag{B-11}$$

which implies

$$f_X(., a_i) = -\frac{\frac{ra_i}{X^2} + \Phi_X}{\Phi_{x_i}} \tag{B-12}$$

and

$$f_{a_i}(., a_i) = \frac{r}{X\Phi_{x_i}}. \tag{B-13}$$

From (B-1), we obtain

$$\Phi_X = \frac{c'(x_i)}{x_i} > 0 \text{ and} \tag{B-14}$$

$$\Phi_{x_i} = \frac{1}{x_i}\left[c''(x_i)(r+X) - c'(x_i) - \Phi\right]. \tag{B-15}$$

Substituting (B-14) and (B-15) into (B-12), and using $\Phi_{x_i} < 0$, it follows that

$$\begin{aligned} f_X(.,a_i) &= \frac{\frac{x_i}{X}\Phi + c'(x_i)}{\Phi + c'(x_i) - c''(x_i)(r+X)} \\ &> \frac{\frac{x_i}{X}\Phi + c'(x_i)}{\Phi + c'(x_i)} \\ &\geq \frac{x_i}{X}. \end{aligned} \tag{B-16}$$

Hence, using (B-10),

$$f_X(.,a_i) > \frac{f(X,r,a_i)}{X}. \tag{B-17}$$

Next, note that summing (B-10) across agents, the equilibrium total effort X must solve

$$h(X,r,\mathbf{a}) \equiv \sum_i f(X,r,a_i) - X = 0. \tag{B-18}$$

Moreover, note from (B-17) that

$$h_X(.) = \sum_i f_X(.,a_i) - 1 > 0 \;\text{ whenever } h(.) = 0. \tag{B-19}$$

Hence, if a solution to (B-18) exists, it must be unique.

We now establish the existence. First, observe that for any j,

$$\begin{aligned} h(X_j,r,\mathbf{a}) &= \sum_{i\neq j} f(X_j,r,a_i) + \underbrace{f(X_j,r,a_j) - X_j}_{=0} \\ &> 0. \end{aligned} \tag{B-20}$$

Second, since $h(0,.) = 0$, to complete the proof, we need to establish that $h(\widehat{X},.) < 0$ for some $\widehat{X} > 0$. To do so, it suffices to show that $\lim_{X\to 0} f_X(.,a_i) = 0$, which, by (B-19), will reveal that $\lim_{X\to 0} h_X(.) = -1 < 0$.

Dividing both the numerator and denominator on the right-hand side of (B-16) by Φ, we have that

$$f_X(.,a_i) = \frac{\frac{x_i}{X} + \frac{c'(x_i)}{\Phi}}{1 + \frac{c'(x_i)}{\Phi} - \frac{c''(x_i)(r+X)}{\Phi}}. \tag{B-21}$$

Using the definition of $\Phi(.)$ in (B-1), we can re-write the last term in the denominator in (B-21) as

$$\frac{c''(x_i)(r+X)}{\Phi} = \frac{x_i c''(x_i)}{c'(x_i)(r+X) - c(x_i)}(r+X). \tag{B-22}$$

Since $x_i = f(X,r,a_i)$ and $f(0,r,a_i) = 0$, it follows, from (B-22) and (COND 2), that

$$\lim_{X\to 0} \frac{c''(x_i)(r+X)}{\Phi} = \lim_{x_i\to 0} \frac{x_i c''(x_i)}{c'(x_i)} < 1. \tag{B-23}$$

For the other terms in (B-21), we observe

$$\lim_{X\to 0}\Phi(x_i,X,r)=\infty,\quad \lim_{X\to 0} f(X,r,a_i)=0 \text{ and } \quad \lim_{X\to 0}\frac{f(X,r,a_i)}{X}=0. \tag{B-24}$$

Applying (B-23) and (B-24) in (B-21), we, therefore, obtain

$$\begin{aligned}\lim_{X\to 0} f_X(.,a_i) &= \lim_{X\to 0}\frac{\frac{x_i}{X}+\frac{c'(x_i)}{\Phi}}{1+\frac{c'(x_i)}{\Phi}-\frac{c''(x_i)(r+X)}{\Phi}}\\ &= 0.\end{aligned}$$

Hence, there exists some $\widehat{X}>0$ such that $h(\widehat{X},.)<0$, and in turn, a unique solution $X^*>0$ to (B-18). Given $f_X(.,a_i)>0$ by (B-17), we find $x_i^*=f(X^*,r,a_i)>0$, proving the existence and uniqueness of an interior equilibrium when $c''(0)=\infty$.

Case 1.2. *$c''(0)<\infty$ ($k=2$ for $c(x)=x^k/k$).*

Then, from (B-9), we have that $B(X)>ra_i$ if and only if $X>Z_i$, where

$$Z_i=\frac{1}{2}\left(-r+\sqrt{r^2+\frac{4ra_i}{c''(0)}}\right)>0.$$

Recall that $A(X)\le ra_i$ for $X\le X_i$. We next show $Z_i<X_i$. To do so, it suffices to show that $A(X)<B(X)$ for $X>0$. Let $\Delta(X)\equiv B(X)-A(X)$. Then, from (B-9),

$$\Delta(X)=c''(0)(r+X)X-c'(X)(r+X)+c(X).$$

Clearly, $\Delta(0)=0$, and

$$\begin{aligned}\Delta'(X) &= c''(0)(r+2X)-c''(X)(r+X) \\ &\ge c''(X)(r+2X)-c''(X)(r+X) \text{ (since } c'''(X)\le 0) \\ &= Xc''(X). \\ &> 0 \text{ for } X>0.\end{aligned} \tag{B-25}$$

Hence, $\Delta'(X)>0$ and, in turn, $\Delta(X)>0$ for $X>0$, establishing that $A(X)<B(X)$ for $X>0$. As a result, $f(X,a_i,r)\in[0,X]$ if and only if $X\in[Z_i,X_i]$, with $f(Z_i,r,a_i)=0$.

In sum, together with (B-20), when $c''(0)<\infty$, an interior equilibrium exists if and only if

$$h(Z_i,r,\mathbf{a})<0 \text{ for all } i.$$

Case 2. *$c'''(x_i)>0$ for all x_i ($k>2$ for $c(x)=x^k/k$).*

First, by (COND 3), $c''(0)=0$. Hence, $B(X)=0$ in (B-9). Moreover, $ra_i\le A(X)$ whenever $X\ge X_i$. Hence, there is a solution

$$x_i=f(X,r,a_i)\in[0,X]$$

to (B-7) if and only if $X \geq X_i$, with $f(X_i, r, a_i) = X_i$. It follows from (B-18) that $h(X_i, r, \mathbf{a}) > 0$ for any i. Furthermore, $\lim_{X\to\infty} h(X, r, \mathbf{a}) = -\infty$. Therefore, there exists an $X^* > 0$ that solves (B-18). In particular, $X^* > \max_i X_i$, implying that $x_i^* > 0$ for all i.

Suppose now the project is intrinsically difficult, i.e., $AMC^i_{x_i} > 0$ (and hence $\Phi_{x_i} > 0$) for all i. It follows from (B-12) that $f_X(., a_i) < 0$. This further implies, from (B-19), that $h_X < 0$ whenever $h(.) = 0$, proving the uniqueness of the interior equilibrium in this case. ■

Proof of Proposition 2. From (18), $AMC^i(x_i, X, r)$ is strictly decreasing in a_i since $c_i(x_i) = c(x_i)/a_i$. Moreover, by definition, $AMC^i(x_i, X, r)$ is strictly decreasing in x_i for an intrinsically easy project. Hence, it is immediate from the equilibrium condition (19) that $x_i^* \leq x_j^*$ and in turn, $q_i^* \leq q_j^*$ by (8), with strict inequalities for $a_i \neq a_j$, proving part (a). Part (b) similarly follows because, by definition, $AMC^i(x_i, X, r)$ is strictly increasing in x_i for an intrinsically difficult project. ■

Proof of Proposition 3. To prove part (a), suppose the project is intrinsically easy: $c'''(x_i) \leq 0$ and thus $\Phi_{x_i} < 0$, where $\Phi = \Phi(x_i, X, r)$ is as defined in (B-1). In this part, we also impose the following cost condition:

$$\left(\frac{c(x_i)}{c'(x_i)}\right)'' \leq 0 \text{ for all } x_i, \tag{COND 4}$$

which is, again, satisfied by the iso-elastic cost: $c(x_i) = x_i^k/k$ with $k > 1$.

Consider agent i's equilibrium utility described in (20). Given $q_i^* = x_i^*/X^*$, it can be rewritten as

$$u_i^* = \frac{x_i^*}{X^*} - \frac{c'(x_i^*)}{a_i}. \tag{B-26}$$

Next recall from the proof of Proposition 1 that $x_i^* = f(X^*, r, a_i)$. Substituting for x_i^* and dropping the star sign for simplicity here, (B-26) becomes

$$u_i = \frac{f(X, r, a_i)}{X} - \frac{c'(f(X, r, a_i))}{a_i} \equiv U(X, r, a_i). \tag{B-27}$$

It is evident from (B-27) that to obtain the reverse utility ordering in part (a), it suffices to show that

$$U_{a_i}(X, r, a_i) < 0.$$

To this end, we partially differentiate (B-27) with respect to a_i and substitute back for $x_i = f(X, r, a_i)$ to find

$$U_{a_i} = \left[\frac{1}{X} - \frac{c''(x_i)}{a_i}\right] f_{a_i} + \frac{c'(x_i)}{a_i^2}. \tag{B-28}$$

Since $f_{a_i} = \frac{r}{X\Phi_{x_i}}$ by (B-13), we observe

$$\begin{aligned} U_{a_i} \; < \; 0 \quad &\Longleftrightarrow \quad \frac{c'(x_i)}{a_i^2} < -\left[\frac{1}{X} - \frac{c''(x_i)}{a_i}\right]\frac{r}{X\Phi_{x_i}} \\ &\Longleftrightarrow \quad c'(x_i) < -\left[\frac{a_i}{X} - c''(x_i)\right]\frac{ra_i}{X\Phi_{x_i}} \\ &\Longleftrightarrow \quad -c'(x_i)\Phi_{x_i} < \left[\frac{a_i}{X} - c''(x_i)\right]\frac{ra_i}{X}, \end{aligned} \tag{B-29}$$

where the last line follows because $\Phi_{x_i} < 0$.

Collecting terms, we further observe

$$\begin{aligned} U_{a_i} \; < \; 0 \quad &\Longleftrightarrow \quad -c'(x_i)\Phi_{x_i} + c''(x_i)\frac{ra_i}{X} < \frac{ra_i^2}{X^2} \\ &\Longleftrightarrow \quad r\left[-c'(x_i)\Phi_{x_i} + c''(x_i)\frac{ra_i}{X}\right] < \left(\frac{ra_i}{X}\right)^2 \\ &\Longleftrightarrow \quad r\left[-c'(x_i)\Phi_{x_i} + c''(x_i)\Phi\right] < \Phi^2 \\ &\Longleftrightarrow \quad r\frac{-c'(x_i)\Phi_{x_i} + c''(x_i)\Phi}{\Phi^2} < 1 \\ &\Longleftrightarrow \quad r\frac{\partial}{\partial x_i}\left[\frac{c'(x_i)}{\Phi}\right] < 1, \end{aligned} \tag{B-30}$$

where the third line follows because $\Phi = \frac{ra_i}{X}$ by (B-6) and (B-7).

Next, by the definition of Φ from (B-1), note that

$$\begin{aligned} \frac{c'(x_i)}{\Phi} &= \frac{c'(x_i)}{\frac{c'(x_i)(r+X)-c(x_i)}{x_i}} \\ &= \frac{x_i}{r + X - \underbrace{\frac{c(x_i)}{c'(x_i)}}_{\equiv g(x_i)}}. \end{aligned}$$

Hence,

$$\frac{\partial}{\partial x_i}\left[\frac{c'(x_i)}{\Phi}\right] = \frac{r + X - g(x_i) + x_i g'(x_i)}{(r + X - g(x_i))^2}.$$

Since $g''(x_i) \leq 0$ by the cost condition (COND 4) above, we have that $-g(x_i) + x_i g'(x_i) \leq 0$. Moreover, $g(x_i) \leq x_i$ since $c''(x_i) > 0$. Therefore, from (B-30),

$$U_{a_i} < 0 \text{ if } \frac{r(r+X)}{(r + X - x_i)^2} < 1.$$

Now recall from Proposition 2 that for an intrinsically easy project, the ability profile $a_1 \geq ... \geq a_n$ implies $x_1 \leq ... \leq x_n$ in equilibrium. Hence, $x_i \leq \frac{X}{2}$ for all $i \neq n$ since for a positive set of real numbers, there can be *at most* one that is strictly larger than the half of their sum. For $i \neq n$, we thus conclude

$$\frac{r(r+X)}{(r+X-x_i)^2} \leq \frac{r(r+X)}{(r+X-\frac{X}{2})^2} = \frac{r(r+X)}{r(r+X)+\frac{X^2}{4}} < 1,$$

which proves that $U_{a_i}(X, r, a_i) < 0$ for $i \neq n$ and in turn,

$$U(X, r, a_1) \leq U(X, r, a_2) \leq ... \leq U(X, r, a_n),$$

with strict inequality whenever $a_i \neq a_j$, as desired.

To prove part (b), suppose the project is intrinsically difficult: $\Phi_{x_i} > 0$ from (B-1). From (9), it follows that in equilibrium,

$$\begin{aligned} u_i^* &= \frac{X^*}{r+X^*} q_i^* - \frac{c_i(x_i^*)}{r+X^*} \\ &= \frac{x_i^*}{r+X^*} - \frac{c_i(x_i^*)}{r+X^*} \\ &\propto x_i^* - \frac{c(x_i^*)}{a_i} \\ &= f(X^*, r, a_i) - \frac{c(f(X^*, r, a_i))}{a_i}. \end{aligned} \quad \text{(B-31)}$$

Recall from (B-13) that $f_{a_i} > 0$ when $\Phi_{x_i} > 0$. Hence, given X^*, we find

$$\frac{\partial u_i^*}{\partial a_i} \propto \underbrace{[1 - c_i'(f(.))]}_{(+)} \underbrace{f_{a_i}}_{(+)} + \frac{c(.)}{a_i^2} > 0, \quad \text{(B-32)}$$

where $1 - c_i'(f(.)) > 0$ by (20). From (B-32), the utility ordering in part (b) is immediate. ■

Proof of Proposition 4. We prove the results for each intrinsic project.

(a) Consider an intrinsically easy project: $c''' \leq 0$ and thus $\Phi_{x_i} < 0$, where $\Phi = \Phi(x_i, X, r)$ is as defined in (B-1). Below we show that **(i)** $\frac{\partial X^*}{\partial a_i} > 0$ for all i, **(ii)** $\frac{\partial x_i^*}{\partial a_j} > 0$ for $i \neq j$, and **(iii)** $\frac{\partial X^*}{\partial r} > 0$.

(a-i) Differentiating $h(X^*, r, \mathbf{a}) = 0$ in (B-18) with respect to a_i reveals

$$\frac{\partial X^*}{\partial a_i} = -\frac{h_{a_i}}{h_X} = -\frac{f_{a_i}(., a_i)}{h_X} > 0,$$

since $h_X > 0$ by (B-19) and $f_{a_i}(., a_i) < 0$ by (B-13) when $\Phi_{x_i} < 0$.

(a-ii) Given $x_i^* = f(X^*, r, a_i)$ and $f_X(., a_i) > 0$ by (B-17), we find

$$\frac{\partial x_i^*}{\partial a_j} = f_X(., a_i) \frac{\partial X^*}{\partial a_j} > 0 \text{ for } i \neq j. \quad \text{(B-33)}$$

(a-iii) Using (B-6), the first-order condition in (B-7) for agent i can be re-written as

$$X\Phi(x_i, X, r) = ra_i. \tag{B-34}$$

Differentiating both sides of (B-34) with respect to r and re-arranging terms yield

$$x_i' + \underbrace{\left(\frac{x_i\Phi + c'X}{x_iX}\right)}_{\equiv T_i}\frac{X'}{\Phi_{x_i}} = \frac{a_i}{x_i\Phi_{x_i}}\underbrace{\left(\frac{x_i}{X} - \frac{c'}{a_i}\right)}_{=q_i - c_i'}. \tag{B-35}$$

where we let $x_i' \equiv \partial x_i/\partial r$ and $X' \equiv \sum x_i'$ here for convenience. Hence, summing both sides of (B-35) across agents, we obtain

$$X'\left(1 + \sum_i \frac{T_i}{\Phi_{x_i}}\right) = \sum_i \frac{a_i\left(q_i - c_i'\right)}{x_i\Phi_{x_i}}. \tag{B-36}$$

Since $\Phi_{x_i} < 0$, and $q_i - c_i' = u_i > 0$ by (20),

$$X' > 0 \Longleftrightarrow \left(1 + \sum_i \frac{T_i}{\Phi_{x_i}}\right) < 0. \tag{B-37}$$

Substituting for Φ_{x_i} from (B-15) and T_i from (B-35) into (B-37) reveals

$$X' > 0 \Longleftrightarrow \sum_i \frac{x_i\Phi + c'X}{X\left(\Phi + c' - (r+X)c''\right)} > 1. \tag{B-38}$$

First, observe that since $c'' > 0$,

$$Z_i \equiv x_i\Phi = (r+X)c' - c > 0. \tag{B-39}$$

Next, observe that multiplying both the numerator and denominator by x_i, we can rewrite (B-38) as

$$X' > 0 \Longleftrightarrow \Omega \equiv \sum_i \underbrace{\left(\frac{Z_i + c'X}{Z_i + x_ic'(x_i) - x_i(r+X)c''}\right)}_{\equiv Y_i}\frac{x_i}{X} > 1. \tag{B-40}$$

Note that $Y_i > 1$ because $Z_i + c'X > 0$ and

$$\begin{aligned} Z_i + x_ic' - x_i(r+X)c'' &= \left[(r+X)c' - c\right] + x_ic' - x_i(r+X)c'' \\ &> (r+X)c' - x_i(r+X)c'' \\ &= (r+X)\left(c' - x_ic''\right) \\ &\geq 0. \end{aligned}$$

where the second line follows from $\frac{c}{x_i} < c'$ since $c'' > 0$, and the last line follows from $\frac{c'}{x_i} \geq c''$ since $c''' \leq 0$. Therefore, as desired,

$$\Omega = \sum_i Y_i \frac{x_i}{X} > \sum_i \frac{x_i}{X} = 1.$$

(b) Consider an intrinsically difficult project: (B-5) or equivalently, $\Phi_{x_i} > 0$ holds. Below we show that **(i)** $\frac{\partial X^*}{\partial a_i} > 0$ for all i, **(ii)** $\frac{\partial x_i^*}{\partial a_j} < 0$ for $i \neq j$ and $\frac{\partial x_i^*}{\partial a_i} > 0$, and **(iii)** $\frac{\partial X^*}{\partial r} > 0$.

(b-i) Recall from (B-12) that $f_X(.,a_i) < 0$ for all i when $\Phi_{x_i} > 0$. Thus, from (B-19), $h_X = \sum f_X(.,a_i) - 1 < 0$. Recall also that, when $\Phi_{x_i} > 0$, we have $f_{a_i}(.,a_i) > 0$ from (B-13). Differentiating $h(X^*, r, \mathbf{a}) = 0$ in (B-18) with respect to a_i, therefore, yields

$$\frac{\partial X^*}{\partial a_i} = -\frac{h_{a_i}}{h_X} = -\frac{f_{a_i}(.,a_i)}{h_X} > 0.$$

(b-ii) From $x_i^* = f(X^*, r, a_i)$, we obtain

$$\frac{\partial x_i^*}{\partial a_j} = f_X(.,a_i)\frac{\partial X^*}{\partial a_j} < 0. \tag{B-41}$$

Furthermore,

$$\begin{aligned} \frac{\partial x_i^*}{\partial a_i} &= f_X(.,a_i)\frac{\partial X^*}{\partial a_i} + f_{a_i}(.,a_i) \\ &= f_X(.,a_i)\left(-\frac{f_{a_i}(.,a_i)}{h_X}\right) + f_{a_i}(.,a_i) \\ &= f_{a_i}(.,a_i)\left(1 - \frac{f_X(.,a_i)}{h_X}\right). \end{aligned} \tag{B-42}$$

Since $h_X < 0$ and $f_{a_i}(.,a_i) > 0$, we observe from the last line in (B-42) that

$$sgn\left(\frac{\partial x_i^*}{\partial a_i}\right) = sgn\left(1 - \frac{f_X(.,a_i)}{h_X}\right) = sgn\left(f_X(.,a_i) - h_X\right).$$

Given $h_X = \sum_j f_X(.,a_j) - 1 < 0$ and $f_X(.,a_j) < 0$, it follows that

$$sgn\left(\frac{\partial x_i^*}{\partial a_i}\right) = sgn\left(1 - \sum_{j \neq i} f_X(.,a_j)\right) > 0. \tag{B-43}$$

(b-iii) Since $T_i > 0$ by (B-35) and $q_i - c_i' = u_i > 0$ by (20), we observe from (B-36) that $\frac{\partial X^*}{\partial r} > 0$ since $\Phi_{x_i} > 0$.

$\blacksquare$

Proof of Lemma 4. Suppose $a_i = a$ for all i. Then, by Proposition 1, there is an (interior) equilibrium, which must be symmetric: $x_i^* = x^* > 0$ and $q_i^* = \frac{1}{n}$ for all i. In addition, (11) implies that x^* uniquely solves

$$c'(x^*)(r + nx^*) - c(x^*) = \frac{ra}{n}. \tag{B-44}$$

The uniqueness in Lemma 1 implies that $x_i^{FB} = x^{FB} > 0$, which solves (A-2):

$$c'(x^{FB})(r + nx^{FB}) - nc(x^{FB}) = ra. \tag{B-45}$$

Suppose, to the contrary, that $x^* \geq x^{FB}$. Since $(c'(x)(r+nx) - c(x))' > 0$, (B-44) and (B-45) reveal

$$\begin{aligned} \frac{ra}{n} &= c'(x^*)(r + nx^*) - c(x^*) \\ &\geq c'(x^{FB})(r + nx^{FB}) - c(x^{FB}) \\ &= ra + (n-1)c(x^{FB}), \end{aligned}$$

and, in turn, $\frac{ra}{n} > ra$, a contradiction. Hence, $x^* < x^{FB}$. $\blacksquare$

Proof of Proposition 5. Consider a two-member team with abilities $a_1 > a_2$, and fix r and a_2. To prove part (a), suppose that the project is intrinsically easy. Then, by Proposition 2, $x_1^* < x_2^*$. Moreover, $x_2^* < \infty$ by (20). Next, we show that $x_1^* \to 0$ as $a_1 \to \infty$. From the first-order condition in (11), note that

$$c'(x_1^*)(r + X^*) - c(x_1^*) = rq_1^* a_1. \tag{B-46}$$

Clearly, the left-hand side of (B-46) is finite, but its right-hand side would grow unbounded as $a_1 \to \infty$ if $x_1^* \nrightarrow 0$ and thus $q_1^* \nrightarrow 0$. Hence, $x_1^* \to 0$, which implies $q_2^* \to 1$ and in turn, $x_2^* \nrightarrow 0$. To compare with the first-best, recall from Lemma 1 that $0 < x_2^{FB} < x_1^{FB}$, and by (A-3),

$$\frac{c'(x_1^{FB})}{a_1} = \frac{c'(x_2^{FB})}{a_2} = z^{FB} \in (0,1). \tag{B-47}$$

Suppose that $z^{FB} \nrightarrow 0$ as $a_1 \to \infty$, which would imply $x_2^{FB} \nrightarrow 0$. But inspecting (FB), it is clear that the planner could do strictly better by shifting the effort x_2^{FB} to agent 1 with $a_1 \to \infty$. Hence, $z^{FB} \to 0$ and thus $x_2^{FB} \to 0$. Moreover, $x_1^{FB} \nrightarrow 0$. Together, we conclude that $x_1^* < x_1^{FB}$ and $x_2^* > x_2^{FB}$ for a sufficiently large a_1.

To prove part (b), suppose that the project is intrinsically difficult. Then, $x_2^* < x_1^*$. Moreover, since $c_i(x_i) = \frac{x_i}{k} c_i'(x_i)$ for the iso-elastic cost, (11) reveals

$$\begin{aligned} c_i'(x_i^*) &= \frac{rq_i^*}{r + X^* - \frac{x_i^*}{k}} \\ &= \frac{rx_i^*}{\left(r + X^* - \frac{x_i^*}{k}\right) X^*} \end{aligned} \tag{B-48}$$

and, in turn, $c_2'(x_2^*) < c_1'(x_1^*)$. For part (b), it, therefore, suffices to show that $c_2'(x_2^*) < z^{FB} < c_1'(x_1^*)$.

To the contrary, suppose that $z^{FB} \leq c_2'(x_2^*)$. Then, $x_2^{FB} \leq x_2^*$ and $x_1^{FB} < x_1^*$ by (B-47) since $c_i'' > 0$. From the first-order conditions (A-2) and (11), this implies

$$rq_1^* = c_1'(x_1^*)(r + x_1^* + x_2^*) - c_1(x_1^*) > c_1'(x_1^{FB})(r + x_1^{FB} + x_2^{FB}) - c_1(x_1^{FB}) = r + c_2(x_2^{FB}), \quad \text{(B-49)}$$

because $c_i'(x_i)(r + x_i + x_j) - c_i(x_i)$ is strictly increasing in x_i. Hence, $rq_1^* > r + c_2(x_2^{FB})$ and, in turn, $q_1^* > 1$, a contradiction.

Next, suppose, to the contrary, that $c_1'(x_1^*) \leq z^{FB}$ for a sufficiently large a_1. Then, $x_2^* < x_2^{FB}$ and $x_1^* \leq x_1^{FB}$, implying $X^* < X^{FB}$. By the iso-elastic cost, $c_i(x_i^{FB}) = \frac{x_i^{FB}}{k} c_i'(x_i^{FB}) = \frac{x_i^{FB}}{k} z^{FB}$. Inserting this fact into (A-2), we obtain

$$z^{FB}(r + X^{FB}) - \sum_i \frac{x_i^{FB}}{k} z^{FB} = r,$$

which yields

$$z^{FB} = \frac{r}{r + X^{FB} - \frac{X^{FB}}{k}}. \quad \text{(B-50)}$$

Using (B-48) and (B-50), we then observe

$$\begin{aligned} c_1'(x_1^*) \ \leq \ z^{FB} \quad &\Longleftrightarrow \quad \frac{rx_1^*}{\left(r + X^* - \frac{x_1^*}{k}\right) X^*} \leq \frac{r}{r + X^{FB} - \frac{X^{FB}}{k}} \\ &\Longleftrightarrow \quad x_1^* \leq \frac{X^*(r + X^*)}{r + X^{FB} - \frac{X^{FB}}{k} + \frac{X^*}{k}}. \end{aligned} \quad \text{(B-51)}$$

We now claim that $x_1^* \approx X^*$ for a sufficiently large a_1. To prove, suppose that $x_1^* \nrightarrow \infty$ as $a_1 \to \infty$. Then, since $x_2^* < x_1^* < \infty$, the left-hand side of (B-46) is finite, but its right-hand side would grow unbounded since $q_1^* \nrightarrow 0$. Hence, $x_1^* \to \infty$, which implies that $q_1^* \to 1$ (since $x_2^* < \infty$) and $x_2^* \to 0$, proving the claim. Plugging $x_1^* \approx X^*$ into (B-51) and simplifying terms, we find that $X^{FB} \leq X^*$, a contradiction.

As a result, $c_2'(x_2^*) < z^{FB} < c_1'(x_1^*)$ for a sufficiently large a_1, and in turn,

$$x_2^* < x_2^{FB} \text{ and } x_1^* > x_1^{FB}$$

by (B-47), as desired. ■

Appendix C: Proofs for Section 6

Proof of Lemma 5. In (Nash) equilibrium, agent i maximizes (21) with respect to x_i given X_{-i}. The first-order condition of this maximization requires that

$$c_i'(x_i)(r + X) - c_i(x_i) = r + X_{-i}. \quad \text{(C-1)}$$

Substituting (C-1) into (21), and simplifying terms, we observe that in equilibrium,

$$u_i = 1 - c_i'(x_i) > 0. \tag{C-2}$$

(In equilibrium, $u_i > 0$ because agent i would otherwise choose $x_i = 0$). Hence, in equilibrium,

$$x_i < c'^{-1}(a_i).$$

Next, given $X_{-i} = X - x_i$, we define

$$\Lambda(x_i, X, r, a_i) = c_i'(x_i)(r + X) - c_i(x_i) + x_i - X,$$

so that (C-1) becomes

$$\Lambda(x_i, X, r, a_i) = r. \tag{C-3}$$

Proceeding as in the proof of Proposition 1, fix X and note that

$$\Lambda(0, X, r, a_i) = -X < 0 \text{ and } \Lambda(X, X, r, a_i) = c_i'(X)(r + X) - c_i(X) > 0.$$

Moreover, $\Lambda_X = -[1 - c_i'(x_i)] < 0$. Hence, (C-3) admits a unique solution:

$$x_i = \widehat{f}(X, r, a_i) \tag{C-4}$$

if and only if

$$\Lambda(X, X, r, a_i) \geq r. \tag{C-5}$$

Since $\Lambda(0, 0, r, a_i) = 0$ and $d\Lambda(X, X, r, a_i)/dX = c_i''(X)(r + X) > 0$, (C-5) is satisfied if and only if

$$X > \underline{X}_i$$

where $\underline{X}_i > 0$ is the unique cutoff such that $\widehat{f}(\underline{X}_i, r, a_i) = \underline{X}_i$.

Now note that summing up (C-4) across agents, the equilibrium total effort X must solve

$$\widehat{h}(X, r, \mathbf{a}) \equiv \sum_i \widehat{f}(X, r, a_i) - X = 0.$$

Clearly, since $\widehat{f}(\underline{X}_i, a_i, r) - \underline{X}_i = 0$,

$$\widehat{h}(\min_i \underline{X}_i, r, \mathbf{a}) > 0 \text{ and } \widehat{h}(\max_i c'^{-1}(a_i), r, \mathbf{a}) < 0.$$

Hence, by continuity, there is a solution to

$$\widehat{h}(X, \mathbf{a}, r) = 0,$$

which constitutes an equilibrium by (C-4).

To prove the rest of the proposition, we implicitly differentiate $\Lambda(\widehat{f}(X,r,a_i),X,r,a_i) = r$, and find that

$$\widehat{f}_{a_i}(.,a_i) = -\frac{\Lambda_{a_i}}{\Lambda_{x_i}} > 0, \tag{C-6}$$

since $\Lambda_{a_i} = -\frac{c_i'(x_i)(r+X)-c_i(x_i)}{a_i} < 0$ and $\Lambda_{x_i} = c_i''(x_i)(r+X)+1-c_i'(x_i) > 0$.

Next let $a_1 > a_2$ for some $i = 1,2$. Then, (C-6) implies

$$x_1^S = \widehat{f}(X^S,r,a_1) > \widehat{f}(X^S,r,a_2) = x_2^S.$$

Finally, to show $u_1^S > u_2^S$, note that

$$\begin{aligned} u_1^S &= \max_{x_1} \frac{x_1 - c_1(x_1)}{r + x_1 + X_{-1}^S} \\ &> \frac{x_2^S - c_1(x_2^S)}{r + x_2^S + X_{-1}^S} \\ &> \frac{x_2^S - c_2(x_2^S)}{r + x_2^S + X_{-2}^S} \\ &= u_2^S, \end{aligned}$$

where the second line follows by the optimality of x_1^S, and the third line follows because $X_{-1}^S < X_{-2}^S$ given $x_1^S > x_2^S$, and $c_1(x) < c_2(x)$ given $a_1 > a_2$. ■

Proof of Proposition 6. We first show the result in the limit case where $r \to 0$ and $a_i = a$ for all i. Under both team- and solo work, the (interior) equilibrium must be symmetric by Proposition 1 and (C-4). That is, in teamwork, $x_i^* = x^* > 0$ and $q_i^* = \frac{1}{n}$, reducing the first-order condition in (11) to

$$c'(x^*)(r + nx^*) - c(x^*) = \frac{ra}{n}.$$

Let $\lim_{r\to 0} x^* = x_\ell^*$, which must satisfy

$$c'(x_\ell^*)nx_\ell^* - c(x_\ell^*) = 0. \tag{C-7}$$

If $x_\ell^* > 0$, then the left-hand side of (C-7) would be strictly positive since $c'' > 0$, yielding a contradiction. Hence, $x_\ell^* = 0$ and in turn, by (20),

$$\lim_{r\to 0} u^* \equiv u_\ell^* = \frac{1}{n}.$$

Next, consider the solo work. By symmetry, $x_i^S = x^S > 0$, and the first-order condition in (C-1) becomes

$$c'(x^S)(r + nx^S) - c(x^S) = ra + (n-1)ax^S.$$

As $r \to 0$, it further becomes

$$c'(x_\ell^S) n x_\ell^S - c(x_\ell^S) = (n-1) a x_\ell^S. \tag{C-8}$$

Clearly, $x_\ell^S = 0$ is a solution to (C-8) since $c'(0) = c(0) = 0$, but it would not constitute an equilibrium because $\frac{\partial u_i}{\partial x_i}\Big|_{x^S=0,r=0} = \frac{1}{r+X^S}\Big|_{x^S=0,r=0} = \infty$. Hence, we look for a positive solution, $x_\ell^S > 0$. To this end, divide both sides of (C-8) by x_ℓ^S:

$$c'(x_\ell^S) n - \frac{c(x_\ell^S)}{x_\ell^S} = (n-1) a. \tag{C-9}$$

As $x_\ell^S \to 0$, the left-hand side of (C-9) approaches 0 (since $c'(0) = 0$) whereas the right-hand side remains $(n-1)a > 0$. Moreover, as $x_\ell^S \to \infty$, the left-hand side of (C-9) grows unbounded because of the cost assumption (COND 3) above, i.e., $\lim_{x\to\infty} \frac{c(x)}{xc'(x)} < 1$, and the fact that $c'' > 0$, which, again, are satisfied by the iso-elastic cost. Hence, there is a positive solution to (C-9), i.e., $x_\ell^S > 0$. Finally, note from (C-9) that $c'(x_\ell^S)/a = \frac{n-1}{n} + \frac{c(x_\ell^S)/a}{n x_\ell^S}$, and from (C-2), we observe that

$$\begin{aligned}
\lim_{r\to 0} u^S &\equiv u_\ell^S \\
&= 1 - c'(x_\ell^S)/a \\
&= 1 - \left[\frac{n-1}{n} + \frac{c(x_\ell^S)/a}{n x_\ell^S}\right] \\
&= \frac{1}{n} - \frac{c(x_\ell^S)/a}{n x_\ell^S} \\
&< u_\ell^*.
\end{aligned}$$

Now define $\overline{\delta} = \frac{u_\ell^* - u_\ell^S}{2} > 0$ and take $\delta \in [0, \overline{\delta})$. Then, by the continuity of equilibrium payoffs in discount rate r and the ability profile $\mathbf{a}$, there is some $\varepsilon > 0$ such that $u_i^* - u_i^S > \delta$ for all i whenever $r < \varepsilon$ and $|a_i - a| < \varepsilon$ for all i. ■

Proof of Proposition 7. As with the static example in Section 2, the first-best program is given by

$$\max_{\mathbf{x}} W = X - \sum_i c_i(x_i), \tag{C-10}$$

where

$$c_i(x_i) = \frac{x_i^{k_i}}{k_i a}, k_i > 1.$$

Assume that $\sum_i a^{\frac{1}{k_i-1}} < 1$, which requires $a < 1$. From (C-10), the unique first-best optimum is easily determined to be

$$x_i^{FB} = a^{\frac{1}{k_i-1}},$$

which implies $x_i^{FB} > x_j^{FB}$ for $k_i > k_j$. Moreover, since $u_i = Xq_i - c_i(x_i)$ and $q_i = x_i/X$, we have $u_i^{FB} = (1 - \frac{1}{k_i})x_i^{FB}$. Thus, $u_i^{FB} > u_j^{FB}$ for $k_i > k_j$.

In equilibrium, agent i best responds to his teammates, X_{-i}^*, and the market's belief, q_i^*:

$$x_i^* = \arg\max_{x_i} u_i = (x_i + X_{-i}^*)q_i^* - c_i(x_i).$$

The first-order condition, after substituting for $q_i^* = x_i^*/X^*$, becomes

$$\frac{(x_i^*)^{k_i-2}}{a} = \frac{1}{X^*}, \tag{C-11}$$

or

$$x_i^* = \left(\frac{a}{X^*}\right)^{\frac{1}{k_i-2}} \text{ for } k_i \neq 2. \tag{C-12}$$

We first argue that no interior equilibrium exists if $k_i < 2 < k_j$ for some i and j. Suppose it did. Then, (C-12) would imply $x_i^* = \left(\frac{X^*}{a}\right)^{\frac{1}{2-k_i}}$ and $x_j^* = \left(\frac{a}{X^*}\right)^{\frac{1}{k_j-2}}$, which, given $x_i^* < 1$ and $x_j^* < 1$, would reveal $X^* < a$ and $a < X^*$, a contradiction.

Second, an interior equilibrium does not exist if $k_i = 2 \neq k_j$ for some i and j, either. Otherwise, $k_i = 2$ would imply $X^* = a$ from (C-11), and that $x_j^* = 1 > X^*$ from (C-12), a contradiction. Third, if $k_i = 2$ for all i, then, from (C-11), any effort profile such that $X^* = a$ would be an interior equilibrium, which would not be unique.

In light of these observations, first consider $k_i < 2$ for all i.

Then, $x_i^* = \left(\frac{X^*}{a}\right)^{\frac{1}{2-k_i}}$, and X^* solves

$$\sum_i \left(\frac{X^*}{a}\right)^{\frac{1}{2-k_i}} - X^* = 0,$$

or dividing both sides by X^*,

$$h(X^*) \equiv \sum_i \frac{1}{a^{\frac{1}{2-k_i}}} (X^*)^{\frac{k_i-1}{2-k_i}} - 1 = 0. \tag{C-13}$$

Note that $\frac{k_i-1}{2-k_i} > 0$ for $1 < k_i < 2$; hence, $h' > 0$. Moreover, $h(0) = -1$ and $h(a) = \frac{n}{a} - 1 > 0$ (since $a < 1$). Therefore, there exists a unique $X^* \in (0,1)$. From (C-13), it further follows that

$$\frac{1}{a^{\frac{1}{2-k_i}}} (X^*)^{\frac{k_i-1}{2-k_i}} < 1 \Longrightarrow X^* < a^{\frac{1}{k_i-1}} \text{ for all } i. \tag{C-14}$$

Now consider $k_i > 2$ for all i.

Proceeding as above, X^* solves

$$g(X^*) \equiv \sum_i \left(\frac{a}{X^*}\right)^{\frac{1}{k_i-2}} - X^* = 0. \tag{C-15}$$

Clearly, $g' < 0$. Moreover, $g(a) = n - a > 0$, and

$$g(1) = \sum_i a^{\frac{1}{k_i - 2}} - 1 < \sum_i a^{\frac{1}{k_i - 1}} - 1 < 0.$$

Therefore, there is a unique $X^* \in (0, 1)$. From (C-15), we further observe $X^* > a^{\frac{1}{k_i - 1}}$ for all i, proving part (a).

Next we prove the effort reversal.

Suppose $k_i < 2$ for all i. Then, $x_i^* = \left(\frac{X^*}{a}\right)^{\frac{1}{2-k_i}}$ by (C-12). Since $\frac{X^*}{a} \in (0, 1)$, and $\frac{1}{2-k_i} > 1$ (given also $k_i > 1$) and is strictly increasing in k_i, it follows that x_i^* is strictly decreasing in k_i. Conversely, suppose $k_i > 2$ for all i. Then, $x_i^* = \left(\frac{a}{X^*}\right)^{\frac{1}{k_i - 2}}$, where $\frac{a}{X^*} \in (0, 1)$, and $\frac{1}{k_i - 2} > 0$ and is strictly decreasing in k_i. Hence, x_i^* is strictly increasing in k_i in this case, as desired.

Last, we show the payoff reversal.

For brevity, we ignore the star signs for equilibrium objects in this part. Note that agent i's equilibrium payoff is:

$$U(x_i, k_i, X) \equiv Xq_i - c_i(x_i) = x_i - \frac{x_i^2}{k_i X},$$

where x_i is given by (C-12), and we used the facts that $c_i(x_i) = c_i'(x_i)\frac{x_i}{k}$, and $c_i'(x_i) = q_i$ by the first-order condition.

As in the proof of Proposition 3, it suffices to show the monotonicity of U in k_i by keeping X *fixed* since it is common to all agents. Differentiating with respect to k_i, we have

$$\begin{aligned} dU/dk_i &= \left(1 - \frac{2x_i}{k_i X}\right) x_i' + \frac{x_i^2}{k_i^2 X} \\ &= \left(1 - \frac{2x_i}{k_i X}\right) \frac{x_i \ln x_i}{2 - k_i} + \frac{x_i^2}{k_i^2 X}, \end{aligned} \tag{C-16}$$

where, fixing X, $x_i' = \partial x_i / \partial k_i$ is found from (C-12). Since $x_i \in (0, 1)$, (C-16) implies $dU/dk_i > 0$ for $k_i > 2$. Thus, $u_i > u_j$ for $k_i > k_j > 2$.

Finally, let $k_i < 2$ for all i. We want to show that $dU/dk_i < 0$ or, simplifying terms in (C-16),

$$\frac{k_i X - 2x_i}{2 - k_i} \ln x_i + \frac{x_i}{k_i} \overset{?}{<} 0. \tag{C-17}$$

Note that $x_i \leq \frac{X}{2}$ for all i but possibly $i_{\max}$ where $i_{\max} = \max_i x_i$. Take $i \neq i_{\max}$. Then, $k_i X - 2x_i > 0$, and the left-hand side of (C-17) is strictly increasing in x_i. Thus, (C-17) is satisfied if

$$\frac{k_i X - 2\frac{X}{2}}{2 - k_i} \ln \frac{X}{2} + \frac{\frac{X}{2}}{k_i} < 0 \Longleftrightarrow \frac{k_i - 1}{2 - k_i} \ln \frac{X}{2} + \frac{1}{2k_i} < 0$$

$\iff$

$$\ln\frac{X}{2} < -\frac{2-k_i}{2k_i(k_i-1)} \iff X < 2e^{-\frac{2-k_i}{2k_i(k_i-1)}}. \tag{C-18}$$

Since, in equilibrium, $X < a^{\frac{1}{k_i-1}}$ from (C-14), (C-18) holds if

$$a^{\frac{1}{k_i-1}} < 2e^{-\frac{2-k_i}{2k_i(k_i-1)}} \Leftrightarrow a < 2^{k_i-1}e^{-\frac{2-k_i}{2k_i}},$$

which is true if $a < e^{-\frac{1}{2}}$, as assumed. Hence, $dU/dk_i < 0$ for all $i \neq i_{\max}$. In other words, if, without loss of generality, $1 < k_1 \leq k_2 \leq ... \leq k_n < 2$, then in equilibrium, $u_1 \geq u_2 \geq ... \geq u_n$ whenever $a < e^{-\frac{1}{2}}$, as claimed. ■

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