

# Direct sale of information when precision is unobservable

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*Abstract.* This paper studies the incentives of an information seller to provide precise information when precision is not observable and investors with rational expectations can extract information from the equilibrium asset price. I show that the seller can verify her precision by employing a non-linear contract. I derive the equilibrium fee for information as a function of the seller's incentives, the sales volume, and buyers' trading intensity. I also analyse the implications of allowing the seller to trade on her own account for truthfulness and precision choice. JEL Classification: G11, G14, D42

*La vente directe d'information quand la précision n'est pas observable.* Ce mémoire étudie les incitations d'un vendeur d'information à fournir de l'information précise quand la précision n'est pas observable et que des investisseurs qui ont des anticipations rationnelles peuvent extraire de l'information du prix d'équilibre d'un actif. L'auteur montre que le vendeur peut vérifier sa précision à l'aide d'un contrat non linéaire, et dérive le prix d'équilibre pour l'information comme fonction des incitations du vendeur, du volume de ventes, et de l'intensité de transactions des acheteurs. On analyse aussi les implications qu'entraîne la possibilité pour le vendeur de transiger pour son propre compte sur le choix du degré de précision et de vérité.

I am grateful to two anonymous referees of the Journal for many useful comments. This paper is a much revised chapter of my dissertation at New York University, Department of Economics. I would like to thank Charles A. Wilson, Alberto Bisin, Boyan Jovanovic, Levent Kockesen, Valerie Meunier, Kamal Saggi, and seminar participants at New York University and 2003 International Industrial Organization Conference at Northeastern University, Boston. The usual disclaimer applies.  
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## 1. Introduction

There have been long-standing complaints about the reliability of security analysts' forecasts and the investment advice provided by brokerage firms in the forms of newsletters and buy/sell recommendations. The reliability issue has started a hot debate recently, thanks to a set of startling emails made public, where stocks that were publicly graded were called 'horrible . . . a piece of crap' in internal communications between the same analysts who rated the stocks high (*Businessweek* 2002). Although even the best analyst can be wrong in her prediction, the current debate concerning the workings of the information market suggests that the problem runs deeper than just being wrong: the information providers simply do not seem to have the correct incentives to provide precise investment information.

Regarding the reliability of analyst services, the two main issues are how to motivate the analyst to gather precise information and how to ensure that the analyst truthfully reveals this information to her clients. The prevailing compensation schemes in the industry, on the other hand, hardly seem to be equipped with any such incentives. As noted by Brennan and Chordia (1993), information produced by security analyst research is typically provided free of charge to investors such as pension funds and money managers in the expectation that it will stimulate trade, rewarding the brokerage house with commissions (Brennan and Hughes 1991). Analysts play a key role in investment-banking teams that help public companies sell new blocks of stock. With attractive fees at stake, commission-based schemes give analysts good reason to tout the stocks of client companies.<sup>1</sup> Moreover, commission-based schedules do not give any direct incentives to gather precise information in the first place.

Despite its clear importance, surprisingly little research has addressed the reliability issue in the sale of investment information by financial brokerage firms. In a notable contribution, Bhattacharya and Pfleiderer (1985) (henceforth BP (1985)) identify a simple non-linear quadratic contract that induces a portfolio manager to disclose her information truthfully. In this paper, I contribute to the literature by identifying how an information seller can use this quadratic contract to verify the precision of her information in a setting where (i) the seller privately chooses her precision, and this choice is unobservable by her clients; (ii) the value of the information is determined by the equilibrium in the asset market, and therefore the information and asset markets are not isolated. The

<sup>1</sup> Michaely and Womack (1999) provide empirical evidence on the credibility of underwriter recommendations. They argue that there is a conflict of interest between the corporate finance division, which is primarily responsible for completing transactions such as IPOs and seasoned equity offerings and the brokerage operation and its equity research department, which is motivated to maximize commissions and spreads by providing timely and high-quality information for their clients. They refer to a Morgan Stanley internal memo (from *Wall Street Journal*, 14 July 1992), which indicates a dim view for an analyst's negative report on one of its investment banking clients: 'Our objective . . . is to adopt a policy, fully understood by the entire firm, including the Research Department, that we do not make negative or controversial comments about our clients, as a matter of sound business practice.'

analysis extends the partial equilibrium setting of BP (1985) to an equilibrium setting by linking the information and asset markets and endogenizing the seller's sales volume and her unobservable precision choice. This is done by explicitly characterizing the effect of the information sale contract on the portfolio problem of the information buyers and on the asset market equilibrium. The extent that the information is incorporated in the equilibrium asset price determines the value of information and thus the incentives of the seller to provide precise information. Furthermore, I analyse an extension of the basic framework where the seller is allowed to trade on her own account, and I derive further implications for the seller's truthfulness and precision choice.

I describe a setting where a security analyst (seller) can generate an information signal about the return of a risky asset. A signal with better precision is costly to produce, and the seller's precision choice and the signal realization are *not* observable to potential buyers. In this standard agency setting, the seller cannot charge a fixed fee contingent on her unobservable precision. To convince the market that she will indeed provide precise information and reveal it truthfully, the seller has to offer an incentive contract. I show that the quadratic contract gives incentives to produce precise information by making the seller's compensation depend on the accuracy of her disclosure. In equilibrium, although the precision choice is not observable, buyers perfectly deduce it from the contract. Apart from giving incentives to the seller, the non-linearity in the information sale contract also distorts the portfolio decision of the buyers and causes them to trade more aggressively and take a larger position in the asset. This increases the extent of information that is 'built into' the asset price and makes the equilibrium asset price more informative. In equilibrium, the fee a seller can charge depends on her inferred precision, the distortion in the portfolio choice, and the extent of information that leaks from the equilibrium asset price.

In the main body of the analysis, I assume that the seller does not trade on her own account. Interesting further results are derived when the seller is allowed to trade. When her trading position is unobservable, the seller has an incentive to manipulate the price by a false disclosure, even when she behaves competitively in the asset market. This follows because the seller can undo the forecast error component of the quadratic contract (which induces truthfulness) by hedging her lie with her portfolio choice. I show that when the seller's portfolio choice is unobservable to her clients, then the quadratic contract no longer induces truthful disclosure. However, if the clients observe the seller's portfolio as well, then they can perfectly infer the true signal after any disclosure. Furthermore, provided that the seller is truthful in the disclosure stage, then allowing her to trade as a competitive trader results in a less informative equilibrium asset price. This follows because the seller's optimal portfolio choice offsets the adverse effect of the aggressive trading of her clients on the value of information. With less information being revealed from the asset price, the seller has better incentives to provide precise information, compared with the case when she is not allowed to trade.

The link between the information market and the asset market is important, since the equilibrium value of information and thus the incentives to provide precise information depend on how much information can be extracted from the asset price costlessly. This feature distinguishes the present paper from the work of Allen (1990) and Stoughton (1993), who present partial equilibrium analysis of the reliability problem in settings where there is no interaction between the information and asset markets.<sup>2</sup> Admati and Pfleiderer (1986, 1990) provide the first and only other analysis of an information market that functions together with an asset market.<sup>3</sup> In building this link, however, they assume away reliability issues. In both papers, they consider a setting where (i) the precision and the signal are common knowledge, and therefore there is no incentive problem to address; (ii) they derive fixed fee schedules that are a direct function of this *known* precision. In contrast, I study the reliability issue in an agency framework where the seller's precision choice and the information signal are not observable by the buyers. Furthermore, and by the same reason of unobservability, the seller cannot charge a fixed fee directly contingent on the precision.

Finally, this paper is also close to a recent paper by Biais and Germain (2002), who study optimal incentive compatible contracts when information is sold *indirectly* by setting up a fund. In their setting, the agency problem is not related to the precision of the information, and, since information is sold indirectly, there is no issue of truthful disclosure.<sup>4</sup> Their seller has perfect information and this is common knowledge. However, apart from the fund she manages, the seller also trades on her own account. Her investment strategy in the fund may not coincide with that of the investors who buy shares in her fund. In contrast, I study direct sale of information, where the seller discloses the information and the agency problem stems from reliability of the seller's information.

The plan of the paper is as follows. In the next section I describe the model, introduce the contract, and characterize the seller's precision choice. In section 3 I study how the information sale contract affects the portfolio decision of buyers and determine the extent of information that leaks from the equilibrium asset

2 In Allen (1990), the seller establishes that she is indeed informed by describing a portfolio to invest for every signal realization. The optimal compensation scheme Allen derives has a similar quadratic structure as BP (1985), but details and modelling assumptions are quite different. Stoughton (1993) also investigates the incentive properties of the quadratic contract, but he only analyses a principal agent setting with one investor and one portfolio manager. More important, he does not consider the asset market equilibrium and informational role of the asset price at all, which is central in my analysis.

3 Admati and Pfleiderer (1986) study direct sale of information, whereas Admati and Pfleiderer (1990) consider the case where the seller sets up a fund and sells the information indirectly, through selling shares from the fund.

4 The relevant incentive problem depends on the mode in which information is sold. With direct sale, the practically relevant agency issue is the truthfulness of the seller's disclosure and the precision of her information. In case of indirect sale, the seller does not disclose the information but makes a portfolio decision on behalf of the clients, as in creating a mutual fund. The relevant agency issue in indirect sale, therefore, is whether the seller will *choose* the portfolio that maximizes investor welfare (see Biais and Germain 2002).

price by explicitly solving for the asset market equilibrium. In section 4 I derive the equilibrium value of information and solve for the equilibrium contract, taking into account the subsequent precision choice and trading in the asset market. In section 5 I analyse the implications of allowing the seller to trade on her own account for truthfulness and precision choice. Section 6 concludes. All proofs are presented in the appendix.

## 2. The model

The model builds upon the familiar normal-exponential setting of Grossman and Stiglitz (1980) and Admati and Pfleiderer (1986). There is a continuum of atomistic investors with a total mass one. There are four dates,  $t = 0, 1, 2,$  and  $3$ . Trading in the asset market involves a risky asset and a riskless asset, which serves as the numeraire. I normalize the riskless rate of return to one. The risky asset generates a stochastic return  $\tilde{v}$ , which is normally distributed with mean  $\bar{v}$  and variance  $\sigma_v^2$ . Without loss of generality, I set  $\sigma_v^2 = 1$ . The basic ingredients and assumptions are as follows:

(A1) HOMOGENEOUS INVESTORS. *Each investor is assumed to be endowed with an initial wealth  $w_0$ . Investors have identical CARA preferences represented by the utility function*

$$U(W) = -\exp(-aW) \quad (1)$$

where  $W$  is the final wealth and  $a > 0$  is the coefficient of absolute risk aversion common to all investors.<sup>5</sup> Furthermore, a priori, all investors possess the same information on the return of the risky asset, summarized by the distribution of the return.

(A2) COMPETITIVE RATIONAL EXPECTATIONS EQUILIBRIUM. *Trading takes place at date 2. The market for the risky asset clears in per capita sense and each investor maximizes the expected utility from final wealth conditional on all the information available, including the equilibrium asset price. In choosing their portfolios, investors take the price distribution and price realizations as given.*

(A3) NOISY SUPPLY. *The per capita supply  $\tilde{x}^s$  of the risky asset is also normally distributed with zero mean and variance  $\sigma_x^2$  and is independent of any information that traders may have.*<sup>6</sup>

5 As pointed out by a referee, portfolio choice problems with CARA preferences are not empirically appealing, since optimal investment does not depend on the initial wealth. On the other hand, owing to the analytical tractability they buy, they have been widely popular in the market microstructure literature, which is the reason I adopt them.

6 The supply noise is a standard assumption to prevent the asset price from being fully revealing, and it ensures that information is valuable in equilibrium. As pointed out by a referee, this assumption is conceptually equivalent to introducing liquidity traders who trade a random amount. I do not model the source of the randomness in supply, but it can be justified through the randomness in endowments or trading by liquidity traders. For a model where noise trading is not assumed but stems from the career concerns of a portfolio manager, see Dow and Gorton (1997).

(A4) INFORMATION SELLER. *There is an information seller who can provide investors information on the return of the risky asset before trading takes place. This seller can be thought of as a security analysis or financial services firm that issues newsletters or provides investment advice to investors. In particular, the seller has access to an information signal  $\tilde{s}$  correlated with the return of the risky asset, where*

$$\tilde{v} = \tilde{s} + \tilde{\varepsilon}. \quad (2)$$

*Furthermore,  $\tilde{s}$  and  $\tilde{\varepsilon}$  have a multivariate normal distribution with*

$$E[\tilde{\varepsilon}] = 0, E[\tilde{s}\tilde{\varepsilon}] = 0 \text{ and } \text{Var}[\tilde{\varepsilon}] \equiv h^{-1} > 0. \quad (3)$$

*Since  $\tilde{s}$  and  $\tilde{\varepsilon}$  are uncorrelated, (3) implies that the posterior mean is  $E[\tilde{v} | s] \equiv \mu(s) = s$ , and the posterior variance conditional on signal is  $\text{Var}[\tilde{v} | s] = h^{-1}$ . I refer to the precision of the seller's information simply by  $h$ . For expositional simplicity, I assume that the seller is risk neutral; however, none of the results depend on this.*

(A5) SELLER DOES NOT TRADE. *For the main part of the analysis, I assume that the seller does not trade on her own account in the asset market. In section 5, I analyse the implications of allowing the seller to trade.*

Unlike Admati and Pfleiderer (1986, 1990), I introduce the following two assumptions to address the reliability problem in the provision of precise information.

(A6) COST OF PRECISION. *Providing a signal with better precision is costly. In particular, when the seller chooses the precision of her signal, she incurs a private cost  $c(h)$ , which is convex and increasing in  $h$ . Seller's cost function is common knowledge.*

(A7) UNOBSERVABILITY. *At date 1, the seller chooses the precision  $h$  and observes the signal privately. The signal and the precision are not observable to investors at any point in time. This assumption and the fact that better precision has private cost for the seller introduce a standard agency framework. One immediate implication is then, in the absence of any incentive mechanism, potential buyers would rationally anticipate that the seller would avoid the cost of precision and disclose an uninformative signal. Therefore, the seller has to overcome a reliability problem and offer an information sale contract that assures the buyers about the truthfulness of her disclosure and her choice of precision. This rules out the fixed fee schedules of the type derived in Admati and Pfleiderer (1990), which are directly contingent on the seller's precision. This difference is also in line with the point made by Brennan and Chordia (1993).<sup>7</sup> They argue that it is unlikely, in practice, to make the payment for*

<sup>7</sup> Brennan and Chordia (1993) analyse the efficiency of selling investment return information in return for a brokerage commission. In order to focus on risk-sharing and price discrimination considerations, they assume that the seller's information is reliable and asset price does not reveal any information.

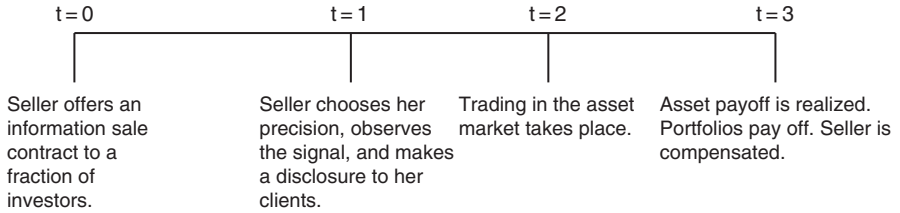


FIGURE 1 The sequence of events

*information depend on the precision of information, since precision is typically of a qualitative nature and cannot be verified by an independent third party.*

(A8) DIRECT SALE. *I restrict attention to direct sale of information where the seller discloses the information to her clients and they make their own portfolio decisions based on the information disclosed to them. An alternative setting is to sell the information indirectly by creating a fund and selling shares of the fund (see Admati and Pfleiderer 1990). In this case, the seller would not be disclosing the information to her clients; instead she would be investing on their behalf. The agency problem I study is concerned with the truthfulness of the seller's disclosure and the precision of her information, which is rather relevant for direct sale. In an indirect sale, traders do not observe the seller's information but pay for the response to the information taken on their behalf. For that reason, the practically relevant agency problem in indirect sale is whether the seller really follows the investment strategy that is in the best interests of her clients, and this incentive problem has been addressed by Biais and Germain (2002).*

For readers' convenience, I summarize the sequence of events in the time Readers' line in figure 1. At date 0, the seller offers an information sale contract to a fraction  $\lambda \in [0, 1]$  of investors. This contract specifies a mechanism that ensures the buyers that they will be receiving a certain precision, and the seller discloses the signal truthfully. At date 1, the seller chooses her precision, observes the signal, and makes a disclosure to her clients. At date 2, the buyers use the information disclosed to them to choose their portfolios. Those investors who do *not* buy the seller's contract use the equilibrium asset price as an informative signal. Finally, at date 3, the risky asset's payoff is realized, and the seller is compensated according to the contract.

### 2.1. The contract

Following Bhattacharya and Pfleiderer (1985), I restrict attention to the following non-linear compensation schedule for the seller:

$$F(f, k; \hat{\mu}, v) \equiv f - k(v - \hat{\mu})^2, \quad (4)$$

where  $f$  is a fixed fee,  $k$  is a positive constant,  $\hat{\mu}$  is the seller's disclosure of conditional mean given the signal, and  $v$  is the *realized* asset return. This

contract has a very simple and intuitive interpretation. The seller's compensation has two components: a fixed fee minus a rebate related to the forecast error of the seller's disclosure, given by  $k(v - \hat{\mu})^2$ . The main result (proposition 2) of BP (1985) is that the above schedule induces truthful disclosure of conditional mean for any *weakly risk averse* agent.<sup>8</sup> Since in my setting the information seller is *risk neutral*, I first extend their truthful disclosure result to the case of risk neutrality.<sup>9</sup>

PROPOSITION 1. (i) *Bhattacharya and Pfleiderer (1985): Suppose the seller is weakly risk averse. Then, given the quadratic compensation scheme in (5), the unique equilibrium disclosure is the truthful disclosure. (ii) Truthful disclosure is also the unique equilibrium if the seller is risk neutral.*

*Proof.* For part (i), see BP (1985). The proof of the extension, part (ii), is given in the appendix.

In BP (1985), the precision of the seller's information is exogenous and the focus is on screening agents with inferior precision. In what follows, I depart from their screening framework and study how the above non-linear contract form, (i) gives incentives to the seller to produce precise information and how buyers deduce this precision from the information sale contract and (ii) affects the portfolio decision of buyers and the equilibrium in the asset market. This is crucial, since in this model the equilibrium value of information and hence the incentives to produce precise information depend on the asset market equilibrium.

## 2.2. Seller's choice of precision

At date 1, given a contract  $(f, k)$  and a sales volume  $\lambda$ , the seller chooses the precision that maximizes her expected profits. In doing so, she also takes into account that, once she observes the signal, she will be making a truthful disclosure. Given the truthful disclosure and the fact that the true conditional mean is  $\mu(s) = s$ , the seller's compensation from a single contract becomes

$$\tilde{\pi}(f, k) \equiv f - k(\bar{v} - \bar{s})^2. \quad (5)$$

The seller's expected profits from a sales volume  $\lambda$  is then given by

$$E[\tilde{\Pi}(f, k; \lambda)] = E[\lambda(f - k(\bar{v} - \bar{s})^2) - c(h)]. \quad (6)$$

<sup>8</sup> This follows, since disclosures other than the true one yields payoff distributions that are second-order stochastically dominated. Therefore, a weakly risk averse seller makes a truthful disclosure.

<sup>9</sup> Assuming risk aversion for the seller does not change any qualitative results; however, it complicates the equilibrium analysis and exposition. When the seller is risk averse, her incentives to produce precise information are even stronger with the quadratic contract.



In order to evaluate the expectation of the quadratic stochastic component  $k(\tilde{v} - \tilde{s})^2$ , recall that  $\tilde{v} = \tilde{s} + \tilde{\varepsilon}$ . Therefore,  $E[k(\tilde{v} - \tilde{s})^2] = kE[\tilde{\varepsilon}^2]$ . Since  $\tilde{\varepsilon} \sim N(0, h^{-1})$ , it follows that  $E[\tilde{\varepsilon}^2] = h^{-1}$ . Therefore, the seller's expected profits become

$$E[\tilde{\Pi}(f, k; \lambda)] = \lambda \left( f - \frac{k}{h} \right) - c(h). \quad (7)$$

Now one can see the incentive effect of the quadratic payment schedule. For a given sales volume  $\lambda$ , the contract parameter  $k$  introduces a forecast error loss given by  $(\lambda k/h)$  to the seller's compensation. She can reduce this loss by setting a higher precision  $h$ . The following proposition characterizes the optimal precision choice under the quadratic payment schedule.

PROPOSITION 2. *The seller's optimal precision  $h$  is unique and solves*

$$\frac{\lambda k}{h^2} = c'(h). \quad (8)$$

*Precision  $h$  is uniquely determined by the contract parameter  $k$  and sales volume  $\lambda$ , and it is an increasing function of  $k$  and  $\lambda$ .*

Proposition 2 tells us that although the precision the seller provides is not observable, the buyers can uniquely deduce it from the contract. For every coefficient  $k$  and sales volume  $\lambda$ , there is a unique precision choice  $\hat{h}(k, \lambda)$ , and this precision is increasing in  $k$  and  $\lambda$ . In what follows, I refer to  $k$  as the incentive parameter. The fixed fee  $f$ , on the other hand, does not play any role in the precision choice. In order to emphasize that buyers do not observe the precision choice, but they infer it perfectly from the contract, I write the inferred precision as  $\hat{h}$ , which is a function of  $k$  and  $\lambda$ .

One motivation of this paper is the case against the prevailing commission-based schedules in resolving the reliability problem to provide precise investment information. If the seller, as in Brennan and Chordia (1993), was compensated with a commission-based schedule (which is a function of the trading volume of her clients), she would always misrepresent both the signal and her precision to boost demand. Furthermore, a commission-based schedule would not give the seller any direct incentives to produce precise information in the first place. In contrast, the analysis up to this point illustrates that a non-linear contract might be desirable in a security analyst context, because (i) it provides direct incentives to produce precise information, (ii) it ensures that even a risk neutral seller makes a truthful disclosure once she obtains the information, and (iii) buyers perfectly infer the precision the seller provides from the contract. Now, I turn to the asset market and derive the equilibrium trading behaviour of investors, given the information sale contract.

### 3. Asset market equilibrium

As in Grossman and Stiglitz (1980), the equilibrium in the asset market is a competitive noisy rational expectations equilibrium, where the asset price partially reveals the information that informed investors have. Although the analysis is similar to that of Grossman and Stiglitz (1980), an important difference is the distortion in the portfolio problem of informed investors, arising from the non-linear component of the information sale contract.

#### 3.1. Portfolio problem of investors

##### 3.1.1. Informed investors

Upon the seller’s truthful disclosure of the conditional mean  $\tilde{\mu}(s) = s$ , an information buyer chooses her position in the risky asset to maximize her final portfolio wealth. One can write the *conditional* distribution of her portfolio wealth as

$$\tilde{W}_I(s, p) = w_0 + D_I(\tilde{v} - p) - f + k(s - \tilde{v})^2, \tag{9}$$

where  $D_I$  is the position in the risky asset. The distribution of portfolio wealth depends on the final asset value  $\tilde{v}$  in two ways:  $D_I(\tilde{v} - p)$  is the payoff from the position in the risky asset. The second term,  $k(s - \tilde{v})^2$ , introduces a distortion in the portfolio problem: given the signal  $s$ , deviations of the asset return  $\tilde{v}$  from the signal adds to the portfolio value regardless of whether this deviation is because  $\tilde{v}$  turns out to be low or it turns out to be high. In that sense, for higher (lower) realizations of  $\tilde{v}$ , the term  $k(s - \tilde{v})^2$  further improves the return from a long (short) position; whereas for lower (higher) realizations, it protects the downside of this position. Therefore, the non-linearity induces over-investment in the risky asset. Formally, the informed investor’s portfolio problem is

$$\text{Max}_{D_I} E[-\exp(-a\tilde{W}_I)|s]. \tag{10}$$

To evaluate the expectation in the objective function, consider again the two stochastic terms in (9). With algebraic manipulations, one can rewrite the *conditional* expected utility of the information buyer as

$$E\left[-\exp\left(-a\left\{w_0 - f + D_I(s - p) - \frac{(D_I)^2}{4k} + k\left(\tilde{v} - s + \frac{D_I}{2k}\right)^2\right\}\right)|s\right]. \tag{11}$$

The following proposition describes the portfolio choice of an informed investor and qualifies the intuition that, for a given precision  $\hat{h}$ , she trades more aggressively and overinvests in the risky asset due to the nonlinear stochastic part of the information sale contract.

PROPOSITION 3. *The information buyer's demand for the risky asset is given by*

$$D_l^*(s,p) = (\hat{h} + 2ak) \left( \frac{s-p}{a} \right). \tag{12}$$

*Proof.* See the appendix

Owing to the distortion described, now the informed investor trades as if she has precision  $\hat{h} + 2ak$ , although she *infers* from the contract that the seller discloses a signal with precision  $\hat{h}$ . This distortion is more pronounced the more risk averse is the investor. Next, I identify how this trading behaviour affects the signal extraction problem of those investors who *do not* buy the information.

### 3.1.2. Uninformed investors

In the former analysis of the rational expectations equilibrium (e.g., Admati and Pfleiderer 1986; Grossman and Stiglitz 1980; Hellwig 1980), there is no non-linear stochastic component in the informed investor's decision problem, and the resulting wealth distribution is normal. This yields an equilibrium price distribution that is linear in the signal realization  $s$  and per capita supply  $x^s$ . I now show that despite the impact of the non-linearity, there is an equilibrium price distribution  $\tilde{p}_\lambda(s, x^s)$  still linear in  $s$  and  $x^s$ , and it clears the market for the risky asset in a per capita sense. Suppose that the uninformed investors conjecture that the equilibrium asset price is of the form,

$$p_\lambda^*(s, x^s) = \beta s - \gamma x^s, \tag{13}$$

where  $\beta$  and  $\gamma$  are equilibrium coefficients. An uninformed investor chooses her demand  $D_u$  for the risky asset to maximize the expected utility from her portfolio,

$$E[U(\tilde{W}_U)|p_\lambda^*] = E[-\exp(-a(D_u(\tilde{v} - p) + w_0))|p_\lambda^*]. \tag{14}$$

In doing so, she takes the price realization as given, but uses the equilibrium asset price as an informative signal through her conjecture. In order to see how an uninformed investor forms posteriors on  $\tilde{v}$  using her conjecture  $p_\lambda^*$ , define a random variable  $\tilde{\theta}$  using the conjecture as

$$\tilde{\theta}(s, x_s) \equiv \frac{p_\lambda^*(s, x_s)}{\beta} = \tilde{s} - \frac{\gamma}{\beta} \tilde{x}_s. \tag{15}$$

Conditioning on  $\tilde{\theta}$  is statistically equivalent to conditioning on the conjecture  $p_\lambda^*(s, x_s)$ . To see how  $\tilde{\theta}$  conveys information about  $\tilde{v}$ , substitute  $\tilde{v} = \tilde{s} + \tilde{\epsilon}$  in the above expression:

$$\tilde{\theta} = \tilde{v} - \left( \tilde{\epsilon} + \frac{\gamma}{\beta} \tilde{x}_s \right) \Rightarrow \tilde{v} = \tilde{\theta} + \left( \tilde{\epsilon} + \frac{\gamma}{\beta} \tilde{x}_s \right). \tag{16}$$

Without any supply uncertainty, conditioning on  $\tilde{\theta}$  is informationally equivalent to conditioning on the signal, and therefore all the information is revealed through the equilibrium asset price. With supply uncertainty, the asset price becomes a noisy indicator of the signal. The optimal uninformed demand  $D_u$  that maximizes (14) is given by

$$D_u^* = \frac{E[\tilde{v}|p_\lambda^*] - p}{a \text{Var} [\tilde{v}|p_\lambda^*]}. \tag{17}$$

The extent of information revealed by price is given by the uninformed posterior variance  $\text{Var}[\tilde{v}|p_\lambda^*]$ . In an equilibrium where the equilibrium asset price has the linear form, as in (13), it follows that the uninformed posterior variance (inverse of precision) is

$$\frac{1}{\hat{h}^u} \equiv \text{Var} [\tilde{v}|p_\lambda^*] = 1 - \frac{(1 - \hat{h}^{-1})^2}{(1 - \hat{h}^{-1}) + (\frac{\gamma}{\beta})^2 \sigma_x^2}. \tag{18}$$

The ratio of equilibrium coefficients  $\beta$  and  $\gamma$  is of particular importance for the extent of information revealed from the equilibrium asset price. If  $\gamma/\beta$  is small in absolute terms, then the supply uncertainty becomes less important relative to the information signal in determining price movements, and uninformed traders are able to extract more information from the equilibrium asset price. The following proposition derives the equilibrium in the asset market and identifies the impact of the non-linear part of the information sale contract on the informativeness of the equilibrium asset price.

PROPOSITION 4

(i) *There is a rational expectations equilibrium where the equilibrium asset price is linear in signal realization and per capita supply;  $p(s, x^s) = \beta s - \gamma x^s$ . The ratio of equilibrium coefficients is given by*

$$\frac{\gamma}{\beta} = \frac{a}{\lambda[\hat{h}(k, \lambda) + 2ak]}. \tag{19}$$

(ii) *The informativeness of the equilibrium asset price is increasing in the incentive parameter  $k$  and the sales volume  $\lambda$ .*

*Proof.* See the appendix.

The equilibrium in the asset market illustrates that, apart from giving the seller incentives to produce precise information and reveal it truthfully, the information sale contract has also a direct effect on the equilibrium asset price. This is due to the distortion in the portfolio decision of information buyers as

described in proposition 3. With the resulting aggressive trading, *more information* is built into the equilibrium asset price. The better incentives the seller has (a higher  $k$ ) to produce precise information, the more information is revealed through the equilibrium asset price through two channels. The first one is through the better precision the seller provides as precision  $\hat{h}$  is increasing in  $k$ . This is *the incentive effect of the contract on the seller*. The second is for a given precision  $\hat{h}$ , *some extra information* is revealed from the equilibrium asset price due to the *distortion in the portfolio decision of buyers*.

#### 4. Equilibrium fee for information

In this section, I derive the fixed fee  $f$  that an investor is willing to pay when the seller sells a contract with  $k > 0$  to a fraction  $\lambda$  of investors. This corresponds to deriving the demand schedule of the investors for the information sale contract. It is important to emphasize that the fixed fee  $f$  cannot be directly contingent on the precision  $h$ , since it is privately chosen by the seller after the contract is sold and her choice is unobservable by the investors. Instead, the fee depends on the seller's incentives and her sales volume,  $k$  and  $\lambda$ .

Suppose the seller fixes a contract parameter  $k$  and offers it to a fraction  $\lambda$  of investors. Investors *infer* the precision  $\hat{h}(k, \lambda)$  they will receive, described by (8). They also take into account their own optimal portfolio choice  $\tilde{D}_I^*$  and how much information leaks from the equilibrium asset price. An investor buys the contract  $(f, k)$  offered to a fraction  $\lambda$  as long as her *unconditional* expected utility with the contract is greater than or equal to being uninformed and simply conditioning on the equilibrium asset price, or, formally,  $E(U(\tilde{W}_I^*)) \geq E(U(\tilde{W}_U^*))$ , where

$$\begin{aligned} \tilde{W}_I^* &= w_0 + \tilde{D}_I^*(\tilde{v} - \tilde{p}) - (f - k(\tilde{s} - \tilde{v})^2) \\ \tilde{W}_U^* &= w_0 + \tilde{D}_U^*(\tilde{v} - \tilde{p}). \end{aligned} \tag{20}$$

In equilibrium, the fee  $f$  will be such that both types of investors are indifferent, and the seller extracts all the surplus. The following proposition derives the payment schedule  $f(k, \lambda)$ .

PROPOSITION 5. *The fixed fee  $f$  the seller can charge for an incentive parameter  $k$  and sales volume  $\lambda$  is given by*

$$f(k, \lambda) = \frac{1}{2a} \log \left[ \left( 1 + \frac{2ak}{\hat{h}(k, \lambda)} \right) \left\{ 1 + (\hat{h}(k, \lambda) + 2ak) \underbrace{(\text{Var}[\tilde{s}|p_\lambda^*])}_{\text{leakage}} \right\} \right], \tag{21}$$

where the extent of information that leaks from the equilibrium asset price is given by

$$\text{Var}[\tilde{s}|p_\lambda^*] = [1 - (\hat{h}(k, \lambda))^{-1}] \left\{ 1 - \frac{1 - (\hat{h}(k, \lambda))^{-1}}{1 - (\hat{h}(k, \lambda))^{-1} + \frac{a^2 \sigma_x^2}{\lambda^2 [\hat{h}(k, \lambda) + 2ak]^2}} \right\}$$

and  $\hat{h}(k, \lambda)$  is the precision the seller provides (and buyers perfectly infer) as defined in proposition 2.

*Proof.* See the appendix.

The equilibrium fixed fee seller can charge combines the various effects present in our framework. The first term  $(1/2a) \log[1 + (2ak/\hat{h})]$  stands for the certainty equivalent of the expected rebate  $k(\tilde{v} - \tilde{s})^2$  due to seller’s forecast error. If the seller could provide perfect information, this term would disappear, since there would be no forecast error. The term  $\hat{h} + 2ak$  captures the distortion in the portfolio choice due to the non-linear component of the contract. Although buyers perfectly infer that the seller discloses a signal with precision  $\hat{h}$ , they trade as if they receive precision  $\hat{h} + 2ak$ . The term  $\text{Var}[\tilde{s}|p_\lambda^*]$  describes the leakage of information from the equilibrium asset price. When price does not convey any information ( $\sigma_x^2 = \infty$ ), this effect on the equilibrium value of information disappears. At the other extreme when  $\sigma_x^2 = 0$ , then  $\text{Var}[\tilde{s}|p_\lambda^*] = 0$ , and all information is revealed from the equilibrium asset price.

Better incentives for the seller (a higher  $k$ ) and a higher sales volume  $\lambda$  increase the equilibrium precision but also increase the extent of information revealed from the equilibrium asset price. An interesting aspect of the equilibrium fee schedule is that the fixed fee seller can charge is *not* monotonically increasing in the incentive parameter  $k$ . To see this directly, note that when  $k$  is a very large number, the equilibrium ratio  $\gamma/\beta$  in (19) approaches zero. This implies that the equilibrium asset price is almost fully revealing and the uninformed precision  $1/\text{Var}[\tilde{v}|p_\lambda^*]$  approaches informed precision  $\hat{h}$ , as one can verify from (18). In this case, value of information should approach zero. At the other extreme, when  $k = 0$ , the seller provides a completely uninformative signal. In this case, as well, the value of information is zero.

It should be clear from the analysis up to this point that the seller’s incentives to provide precise information (by offering a higher incentive contract  $k$ ) and her equilibrium sales volume  $\lambda$  depend on the extent of information revealed from the asset price. An increase in supply noise  $\sigma_x^2$  decreases the extent of information revealed from the equilibrium asset price and thus increases the fee seller can charge for a given  $k$  and  $\lambda$ . Furthermore, with the equilibrium asset price less revealing, the seller can increase her sales volume  $\lambda$ . To formalize this conjecture, I conclude the analysis by characterizing the equilibrium quadratic contract in the information market.

Taking into account her subsequent precision choice  $\hat{h}(k, \lambda)$ , as described in proposition 2 and the fee schedule  $f(k, \lambda)$  in (21), the seller’s expected profits in (7) becomes

$$E[\tilde{\Pi}(f, k; \lambda)] = \lambda[f(k, \lambda)] - V(k, \lambda), \tag{22}$$

where

$$V(k, \lambda) \equiv \frac{\lambda k}{\hat{h}(k, \lambda)} + c(\hat{h}(k, \lambda)) \tag{23}$$

is the reduced *total* cost function of the seller, which incorporates the cost of the subsequent precision choice at date 1 and the total expected forecast error rebate. For a given incentive parameter  $k > 0$ , each sales volume  $\lambda \in [0, 1]$  implies a unique fee  $f$ . This follows, since given  $k > 0$ , there is a unique precision choice  $\hat{h}$  for each sales volume  $\lambda \in [0, 1]$  as described in (8). By the fee schedule in (21), then, each  $\lambda$  implies a unique  $f$  that extracts all the surplus (again for a given  $k > 0$ ). The seller's problem at date 0 is

$$\text{Max}_{k > 0 \text{ and } \lambda \in [0, 1]} \lambda[f(k, \lambda)] - V(k, \lambda). \tag{24}$$

The following proposition characterizes the equilibrium contract and sales volume and shows that the higher the supply noise  $\sigma_x^2$ , the better information the seller produces and sells to a higher fraction of investors.

PROPOSITION 6. *Given the equilibrium fee schedule  $f(k, \lambda)$  in (21), the seller offers an incentive contract  $k^*$  to a fraction  $\bar{\lambda} \equiv \text{Min}(\lambda^*, 1)$  of investors, charges a fee  $f^* = f(k^*, \bar{\lambda})$  and sets her precision  $\hat{h}(k^*, \bar{\lambda})$  according to (8) where the equilibrium  $(\lambda^*, k^*)$  solves*

$$\lambda^* f_k(k^*, \lambda^*) - V_k(k^*, \lambda^*) = 0 \tag{25}$$

$$f(k^*, \lambda^*) + \lambda^* f_\lambda(k^*, \lambda^*) - V_\lambda(k^*, \lambda^*) = 0 \tag{26}$$

*The equilibrium incentive parameter  $k^*$ , the precision  $\hat{h}$  and the equilibrium sales volume  $\bar{\lambda}$  are all increasing in the supply noise  $\sigma_x^2$ .*

*Proof.* See the appendix.

With too much noise arising from liquidity trade (or in general from trade that is not based on superior information), the equilibrium asset price is not a good indicator of the information investors may have. This increases the equilibrium value of information and thus the seller's incentives to provide precise information. Consequently, as the market noise parameter  $\sigma_x^2$  increases, the seller sets a higher  $k$  and offers the contract to a larger fraction of investors without diluting the fee investors are willing to pay. At the extreme case, when  $\sigma_x^2 = \infty$  and equilibrium asset price does not reveal any information, the fee  $f$  the seller can charge in a single contract is actually increasing in her sales volume, owing to the incentive effect of  $\lambda$  on precision choice. Hence, it is optimal for the seller to sell the information to the whole market and set  $\lambda$  to 1.

It is worth noting that the observability of the seller's sales volume  $\lambda$  plays a crucial role in our analysis. First, if the clients do not observe the sales volume  $\lambda$ , then they cannot infer the seller's precision choice perfectly. This can be readily observed from the seller's precision choice as described in proposition 2. Second and more important, in our noisy rational expectations framework *observability of  $\lambda$  is crucial even if the seller's precision is observable*. This equilibrium concept is based on the idea that before trading takes place, all market participants know the allocation of information, that is, the fraction of informed traders and the precision of their information. Without this knowledge, uninformed traders cannot have a correct conjecture on the equilibrium asset price. Suppose the clients know the seller's precision perfectly, but only the seller knows the fraction that buys information. If the supply noise is large enough, then sales volume would have no effect on the value of information. Observability of  $\lambda$  would not matter, since the seller would sell the information to all traders. However, if price reveals some information, then the seller has an incentive to pretend to every client that her sales volume is lower than it actually is, in order to charge more for her information. These issues, while interesting, are beyond the scope of our analysis.

## 5. Allowing the seller to trade

Up to this point in the analysis, I assumed that the seller does not trade in the asset market on her own account, and revenues from information sale are her only source of income. In this section, I consider the possibility that the seller, as well as selling information, also trades and chooses an asset portfolio. My focus is to investigate how this possibility affects (i) the asset market equilibrium and informativeness of the asset price and (ii) the precision choice and truthfulness of the seller with the quadratic contract.

In order to address these issues as simply as possible and without departing from my main framework, I assume that the seller can also trade in the asset market, but like the other market participants, she behaves competitively in choosing her asset portfolio. Furthermore, I assume that the seller also maximizes a CARA-type utility function with a coefficient of risk aversion  $b > 0$ . These assumptions allow me to introduce the seller as a competitive trader into the tractable Grossman and Stiglitz (1980) framework.<sup>10</sup>

I first analyse the seller's portfolio decision. Suppose the seller observes a signal  $s$  and makes a disclosure  $\hat{s}$  to her clients. Following the disclosure, she

10 If the seller is a risk neutral competitive trader, then all private information is revealed from the equilibrium asset price and information has no value in equilibrium. Introducing the seller as a strategic trader would imply possible price manipulation with not one but two instruments: manipulation with false disclosure and manipulation with market power. This would also require a complete departure from the competitive noisy rational expectations framework and call for a Kyle (1989) type large strategic trader model. Instead, I introduce the seller as a competitive trader and analyse her incentives to manipulate the asset price only with a false disclosure to her clients.



chooses her position  $D_s$  in the risky asset to maximize her conditional expected utility from final wealth  $\tilde{W}_s$ , where

$$\tilde{W}_s = \lambda[f - k(\hat{s} - \tilde{v})^2] + D_s(\tilde{v} - p). \tag{27}$$

The seller’s wealth distribution now also incorporates the wealth from the asset portfolio given by  $D_s(\tilde{v} - p)$ . The following proposition describes the seller’s optimal portfolio choice.

PROPOSITION 7. *After observing a signal  $s$  and making a disclosure  $\hat{s}$  to her clients, the seller’s optimal demand for the risky asset is*

$$D_s^*(s, \hat{s}, p) = (h - 2\lambda kb) \left( \frac{s - p}{b} \right) + 2\lambda k(s - \hat{s}). \tag{28}$$

The examination of the above optimal portfolio demand illustrates two important implications of allowing the seller to trade on her own account:

(i) *Hedging the lie.* If the seller makes a false disclosure, she hedges this lie with her portfolio choice. This is captured by the part of her portfolio that depends on the disclosure  $\hat{s}$ , given by  $2\lambda k(s - \hat{s})$ . To illustrate how this ‘*hedging the lie*’ works, suppose the seller has disclosed  $\hat{s} < s$ . By understating the signal, the seller faces an extra risk of forecast error loss, which is increasing in the asset payoff realization  $v$ . She can hedge this lie by taking a position for which the payoff is increasing in  $v$ , and this causes the additional *long* position given by  $2\lambda k(s - \hat{s})$ , above. The argument is similar if the seller *overstates* the signal, that is, if  $\hat{s} > s$ . In this case, the seller hedges by a *short* position  $2\lambda k(s - \hat{s})$ , which is negative.

(ii) *If the seller is truthful, allowing her to trade makes the asset price less informative.* Suppose the seller is truthful. Then her optimal portfolio choice reduces to

$$D_s^* = (h - 2\lambda kb) \left( \frac{s - p}{b} \right). \tag{29}$$

Notice that the quadratic contract distorts the portfolio choice of the seller in exactly the opposite way it distorts the portfolio choice of her clients. Recall from proposition 3 that the non-linearity in the contract causes the clients to trade aggressively. For a sales volume  $\lambda$  and without the seller trading, the aggregate distortion in the total informed demand was  $2\lambda k(s - p)$ . When the seller trades as well, and if she is truthful, then her optimal portfolio strategy eliminates this aggregate distortion completely with a position  $-2\lambda k(s - p)$  in the risky asset. Therefore, we have the following proposition,

PROPOSITION 8. *If the seller is truthful in her disclosure and if she behaves competitively, then there is a linear asset market equilibrium price  $p(s, x^s) = \beta s - \gamma x^s$ , where the ratio of equilibrium coefficients is given by*

$$\frac{\gamma}{\beta} = \frac{a}{\lambda h}. \quad (30)$$

Therefore, provided that the seller is truthful, for the same precision and sales volume, the equilibrium asset price is less informative when the seller trades compared with the case when she does not trade.

*Proof.* See the appendix.

An immediate corollary is, then, if the seller is truthful in the disclosure stage, allowing her to trade in the asset market as a competitive trader increases her ex ante incentives to provide precise information. This follows, since her trading position offsets the negative impact of the aggressive trading of her clients. The resulting equilibrium asset price is less informative for the same contract compared with the case when the seller does not trade. With less information being revealed from the equilibrium asset price, information with a certain precision is now more valuable. So, the seller will offer an incentive contract with a higher  $k$  and provide more precise information.

*Truthfulness.* The analysis above takes the truthfulness as given and solves for the asset market equilibrium. Now I analyse the equilibrium in the disclosure stage given the seller's trading strategy derived in proposition 7. The ability to trade and to hedge a lie changes the seller's incentives to reveal her information truthfully. Without this ability, we know that the seller is always truthful with the quadratic contract, since true disclosure minimizes the forecast error. In contrast, now the seller's optimal disclosure strategy trades off the additional forecast error loss when she lies against the possible portfolio gains from price manipulation with a false disclosure. Within this trade-off, however, note that the seller can hedge against the additional forecast error loss when she lies. By doing so, she can undo the forecast error pressure of the non-linear contract which induces truthfulness. Therefore, if price manipulation with a false disclosure is possible, the seller has an incentive to lie when she trades.

However, a remedy for this problem is to make the trading position of the seller observable to her clients. Then the seller can not manipulate the equilibrium asset price with a false disclosure.<sup>11</sup> This can be easily observed from the trading position of the seller derived in Proposition 7. For any disclosure  $\hat{s}$ , the client perfectly infers the true signal  $s$  if she conditions on the seller's position as well. In this case, the seller has no incentive to lie since the only consequence of lying is the extra hedging position she has to take, but the equilibrium asset price distribution will not depend on her disclosure.

11 As pointed out by an anonymous referee, recent projects of the SEC to force security analysts to disclose their trading positions aim to prevent possible price manipulation by analysts. I am grateful to the referee for this point.

To formalize this intuition, consider the seller's problem of optimal disclosure choice. Substituting the optimal portfolio demand  $D_s^*$  back to the seller's wealth distribution, one gets

$$\tilde{W}_s^*(\hat{s}, s) = \lambda f - \lambda k(\hat{s} - \tilde{v})^2 + 2\lambda k(\tilde{p} - \hat{s})(\tilde{v} - \tilde{p}) + h\left(\frac{s - \tilde{p}}{b}\right)(\tilde{v} - \tilde{p}). \quad (31)$$

Given the actual signal  $s$  she observes, the seller chooses a disclosure  $\hat{s}$  to maximize the expected utility from the above final wealth. In doing so, she takes into account the effect of her disclosure on the equilibrium asset price  $\tilde{p}$  and her own portfolio choice following the disclosure. Simplifying the above expression and excluding the terms that do not depend on disclosure  $\hat{s}$ , the seller's problem of disclosure choice can be expressed as

$$\text{Max}_{\hat{s}} E[-\exp(-b\tilde{Z})|s], \quad (32)$$

where  $\tilde{Z}$ , the part of the seller's wealth distribution that depends on  $\hat{s}$  is given by

$$\tilde{Z} \equiv -\lambda k[\hat{s}^2 - 2s(\hat{s} - \tilde{p})] + h\left(\frac{s - \tilde{p}}{b}\right)(\tilde{v} - \tilde{p}). \quad (33)$$

Now, suppose the seller's portfolio choice is *not* observable. To check whether truthful disclosure is a Perfect Bayesian Equilibrium, fix the clients' beliefs such that they believe that the seller is truthful. In this case, from the seller's point of view, the equilibrium asset price is  $\tilde{p} = \beta\hat{s} - \gamma\tilde{x}^s$  (see the appendix for details). Maximizing (32) in this case shows that truthful disclosure is *not* a Perfect Bayesian Equilibrium. Now, consider the alternative case, where the seller's trading position is *observable* to her clients. This time, for any disclosure  $\hat{s}$ , buyers perfectly infer the true  $s$ . From the point of view of the seller, the equilibrium asset price  $\tilde{p}$  now does not depend on  $\hat{s}$ . Actually, the equilibrium asset price is exactly the same as in proposition 8, where the seller is truthful (see the appendix). Therefore we have the following proposition

**PROPOSITION 9.** *When the seller is allowed to trade and if her portfolio choice is unobservable, then the quadratic contract no longer induces truthful disclosure. However, if the clients observe the seller's trading position, they perfectly infer the true signal for any disclosure.*

*Proof.* See the appendix.

## 6. Conclusion

Motivated by the recent controversy on the reliability of information provided by financial brokerage firms, I have investigated a model where the information

seller employs a non-linear contract to ensure the reliability of her information. The analysis shows that, unlike the prevailing commission-based schedules, the non-linear contract provides direct incentives to produce precise information and reveal it truthfully to investors. The compensation scheme I consider resolves the reliability issue by making the seller's compensation depend on the accuracy of her forecast. In equilibrium, the seller's precision is perfectly inferred from the information sale contract, and truthful disclosure is the unique equilibrium even for a risk neutral seller. The information sale contract affects the portfolio choice of the information buyers and causes them to take a larger position in the risky asset. For a given precision, this increases the extent of information built into the equilibrium asset price. I derive the equilibrium fee the seller can charge for her information, as a function of her incentives and the sales volume, by taking into account her subsequent precision choice, the portfolio choices of clients, and the extent of information revealed from the equilibrium asset price.

I also consider the possibility that the seller can trade on her own account as a competitive trader after disclosing her information to her clients. In this case, following a false disclosure, the seller can hedge her lie with an appropriate position in the asset and undo the incentives the quadratic contract provides to be truthful. If her trading position is not observable to her clients, then truthfulness is no longer the optimal revelation. However, truthfulness can be restored by making the seller's position observable to her clients. In this case, clients infer the true signal after any disclosure, and the seller cannot manipulate the asset price with a false disclosure. Finally, if the seller is truthful, then allowing her to trade as a competitive trader results in a less informative asset price and increases her ex ante incentives to provide precise information.

This paper is a small step towards understanding the incentive problems involved in selling information and how these problems can be dealt through contracts. Many issues deserve further research. I assumed that the seller's only function is to provide investment information. In reality, the brokerage firms also provide investment banking services to firms that issue new stock. It is clearly an important issue to understand how the different roles of brokerage firms determine their incentives to provide precise information. For instance, what are the incentives of the research department to provide an honest and precise forecast on the stock of a firm, which is also an important client of the investment banking branch? Can reliability in this context be assured only by severing ties between investment banking and research departments, as suggested by some critics, or can the financial services firm provide contractual arrangements to remedy such problems? Another important issue for further research is the existence of different modes to sell information. In this paper, I restrict attention to direct sale. Information is sold also indirectly through creating a fund and selling shares of the fund. In indirect sale, there is no disclosure of information, instead the seller makes an investment decision on behalf of the clients. Endogenizing the optimal sales mode in the presence of agency considerations between the seller and clients is an unexplored research avenue.

**Appendix**

*A.1. Proof of part (ii) in proposition 1*

Let  $\mu \equiv E[\tilde{v}|s]$ . The seller makes a disclosure  $\hat{\mu}$  to maximize her expected revenue given by  $E[\lambda(f - k(\tilde{v} - \hat{\mu})^2)|s]$ . Maximizing this is equivalent to minimizing  $E[(\tilde{v} - \hat{\mu})^2|s]$ . Now rewrite

$$E[(\tilde{v} - \hat{\mu})^2|s] = E[(\tilde{v} - \mu)^2 + 2(\tilde{v} - \mu)(\mu - \hat{\mu}) + (\mu - \hat{\mu})^2|s]. \tag{A1}$$

Note that  $E[(\tilde{v} - \mu)^2]$  is independent from  $\hat{\mu}$ , and since  $E[\tilde{v}|s] \equiv \mu$ , the expectation of the second term in the brackets is zero. The only relevant part is the non-stochastic third term  $(\mu - \hat{\mu})^2$ . The disclosure that minimizes this term is unique and it is  $\hat{\mu} = \mu$ .

*A.2. Proof of proposition 2*

Maximizing (7) with respect to  $h$  yields the equilibrium condition (8). Uniqueness is ensured by the convexity of  $c(\cdot)$ . To see that  $\hat{h}$  is increasing in  $k$  and  $\lambda$ , totally differentiate (8) to get

$$\frac{d\hat{h}}{dk} = \frac{\lambda}{c''h^2 + 2hc'} > 0 \text{ and } \frac{d\hat{h}}{d\lambda} = \frac{k}{c''h^2 + 2hc'} > 0.$$

*A.3. Proof of proposition 3*

Consider (11) and define  $(\tilde{z}|s) \equiv \tilde{v} - s + (D_I/2k)$ . Note that  $(\tilde{z}|s) \sim N((D_I/2k), \hat{h}^{-1})$ . It follows that  $\hat{h}(\tilde{z}|s)^2$  has a non-central chi-square distribution with a non-centrality parameter  $(D_I/2k)$ . Therefore, one can evaluate the expectation of the quadratic term in (11) as

$$-(1 + \frac{2ak}{\hat{h}})^{-\frac{1}{2}} \times \exp \left[ -a \left\{ \frac{\hat{h}(D_I)^2}{4k(\hat{h} + 2ak)} + D_I(s - p) - \frac{(D_I)^2}{4k} - f + w_0 \right\} \right].$$

Maximizing the above expression with respect to  $D_I$  yields (12).

*A.4. Proof of proposition 4*

(i) The market clearing condition is  $\lambda D_I^*(p, s) + (1 - \lambda) D_U^*(p, p_\lambda^*) = x^s$ . In order to compute the uninformed posterior mean and variance for  $\tilde{v}$ , define  $\tilde{\theta}$  as in (15) to obtain  $E[\tilde{v}|\theta] = t\theta$  and  $\text{Var}[\tilde{v}|\theta] \equiv h^u = 1/[1 - t(1 - \hat{h}^{-1})]$ , where

$$t \equiv \frac{(\hat{h} - 1)}{(\hat{h} - 1) + \hat{h} \left( \frac{\gamma}{\beta} \right)^2 \sigma_x^2}.$$

Substituting these expressions together with (12) and (16) into the market-clearing condition and solving for  $p$  gives

$$p(s, x^s) = \left( \frac{\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u t}{\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u} \right)^s - \left( \frac{a + (1 - \lambda)h^u t \frac{\gamma}{\beta}}{\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u} \right) x^s.$$

Equating this to the conjecture  $p_\lambda^*(s, x^s) = \beta \tilde{s} - \gamma \tilde{x}^s$ , we get

$$\beta = \frac{\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u t}{\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u} \text{ and } \gamma = \frac{a + (1 - \lambda)h^u t(\gamma/\beta)}{\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u},$$

which implies

$$\frac{\gamma}{\beta} [\lambda(\hat{h} + 2ak) + (1 - \lambda)h^u t] = a + (1 - \lambda)h^u t \frac{\gamma}{\beta}.$$

Simplifying and solving for  $\gamma/\beta$  gives (19).

(ii) The informativeness of the equilibrium asset price is given by  $1/\text{Var} [\tilde{v}|p_\lambda^*]$  and the expression for  $\text{Var} [\tilde{v}|p_\lambda^*]$  is given in the text by (18). It is straightforward to see that the ratio  $\gamma/\beta$  as computed above is decreasing in the sales volume  $\lambda$  and in the incentive parameter  $k$ . Hence, from (18),  $\text{Var} [\tilde{v}|p_\lambda^*]$  is decreasing in  $\lambda$  and  $k$ .

*A.5. Proof of proposition 5*

For saving notation, I set  $w_0 = 0$ , which is inconsequential. To find the conditional expected utility of an uninformed investor, substitute  $D_u^*$  in (17) back into (14) and obtain

$$E[U(\tilde{W}_U)|p_\lambda^*] = -\exp \left[ -\frac{(E[\tilde{v}|p_\lambda^*] - p)^2}{2 \text{Var} [\tilde{v}|p_\lambda^*]} \right]. \tag{A2}$$

Similarly, substituting  $D_f^*$  in (12) back into (11), one gets the informed conditional expected utility as

$$E[U(\tilde{W}_I)|s] = -\left(1 + \frac{2ak}{\hat{h}}\right)^{\frac{-1}{2}} \times \exp \left[ -\frac{(\hat{h} + 2ak)(E[\tilde{v}|s] - p)^2}{2} + af \right]. \tag{A3}$$

Now, define  $\Gamma$  and  $\tilde{Z}$  as

$$\Gamma \equiv \text{Var} [E[\tilde{v}|s]|p_\lambda^*] = \text{Var} [\tilde{s}|p_\lambda^*] \text{ and } \tilde{Z} \equiv \frac{E[\tilde{v}|s] - p}{\sqrt{\Gamma}}.$$

Conditional on the conjecture  $p_\lambda^*$ ,  $p$  is non-stochastic by definition and  $E[\tilde{v}|s]$  is normally distributed. Hence, conditional on  $p_\lambda^*$ ,  $(\tilde{Z}|p_\lambda^*)^2$  has a non-central chi-square distribution (see Hogg and Craig 1978, 288). Then, for  $t > 0$  the moment-generating function for  $(\tilde{Z}|p_\lambda^*)^2$  can be written as

$$E[\exp(-t(\tilde{Z})^2)|p_{\lambda}^*] = -\frac{1}{\sqrt{1+2t}} \exp\left[\frac{-(E[(\tilde{Z}|p_{\lambda}^*)^2]t)}{1+2t}\right]. \tag{A4}$$

But  $E[E[\tilde{v}|s]|p_{\lambda}^*] = E[\tilde{v}|p_{\lambda}^*]$ ; hence, we have

$$E[\tilde{Z}|p_{\lambda}^*] = \frac{E[\tilde{v}|p_{\lambda}^*] - p}{\sqrt{\Gamma}}.$$

Furthermore, since  $\tilde{v} = \tilde{s} + \tilde{\varepsilon}$ , it follows that  $\text{Var}[\tilde{v}|p_{\lambda}^*] = \hat{h}^{-1} + \Gamma$ . Therefore, setting  $t = [\Gamma(\hat{h} + 2ak)]/2$ , one can evaluate (37) and rewrite the conditional expected utility of the informed in (36) as

$$-(1 + \frac{2ak}{\hat{h}})^{-\frac{1}{2}} \times [1 + \Gamma(\hat{h} + 2ak)]^{-\frac{1}{2}} \times \exp\left[-\frac{(E[\tilde{v}|p_{\lambda}^*] - p)^2}{2\text{Var}[\tilde{v}|p_{\lambda}^*]} + af\right]. \tag{A5}$$

In equilibrium, the unconditional expected utilities of both types of investors must be the same. Equate the uninformed conditional utility in (A2) to the last expression in (A5) and take expectations of both sides. Solving for  $f$  now yields

$$f = \frac{1}{2a} \log\left[\left(1 + \frac{2ak}{\hat{h}}\right)(1 + \text{Var}[\tilde{s}|p_{\lambda}^*](\hat{h} + 2ak))\right].$$

Note that  $\text{Var}[\tilde{s}|p_{\lambda}^*] = \text{Var}[\tilde{v}|p_{\lambda}^*] - \hat{h}^{-1}$ . Using (18) and (19), one gets the expression for the fee  $f$  in the proposition.

*A.6. Proof of proposition 6*

To obtain the equilibrium conditions (25) and (26), maximize the seller’s expected profits with respect to  $\lambda$  and  $k$ . For the comparative statics analysis with respect to  $\sigma_x^2$ , totally differentiate the equilibrium system (25) and (26). Suppressing arguments, this yields

$$\begin{aligned} [\lambda f_{kk} - V_{kk}]dk + [f_k + \lambda f_{k\lambda} - V_{k\lambda}]d\lambda + [\lambda f_{k\sigma_x^2}]d\sigma_x^2 &= 0. \\ [f_k + \lambda f_{\lambda k} - V_{\lambda k}]dk + [2f_{\lambda} + \lambda f_{\lambda\lambda} - V_{\lambda\lambda}]d\lambda + [f_{\sigma_x^2} + \lambda f_{\lambda\sigma_x^2}]d\sigma_x^2 &= 0. \end{aligned}$$

In equilibrium  $\lambda f_k = V_k$ , which implies that, evaluated at the solution, we have  $f_k + \lambda f_{k\lambda} - V_{k\lambda} = 0$ . Therefore, evaluated at the solution,

$$\frac{\partial k}{\partial \sigma_x^2|(\lambda^*, k^*)} = -\frac{\lambda f_{k\sigma_x^2}}{\lambda f_{kk} - V_{kk}} > 0, \tag{A6}$$

which follows, since  $f_{k\sigma_x^2} > 0$  and  $\lambda f_{kk} - V_{kk} < 0$  by the second-order conditions for a maximum. Similarly, the comparative statics exercise for  $\lambda$  yields

$$\frac{\partial \lambda}{\partial \sigma_x^2|_{(\lambda^*, k^*)}} = - \frac{\overbrace{(f_{\sigma_x^2} + \lambda f_{\lambda \sigma_x^2})}^+}{\underbrace{(2f_{\lambda} + \lambda f_{\lambda \lambda} - V_{\lambda \lambda})}_-} > 0. \tag{A7}$$

Since  $\hat{h}$  is an increasing function of  $k$  and  $\lambda$ , it is straightforward to obtain

$$\frac{\partial \hat{h}}{\partial \sigma_x^2} = \frac{\partial \hat{h}}{\partial k} \frac{\partial k}{\partial \sigma_x^2} + \frac{\partial \hat{h}}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma_x^2} > 0. \tag{A8}$$

*A.7. Proof of proposition 7*

The derivation follows very similar steps to the derivation in the proof of proposition 3, the only exception being that  $\tilde{z} \equiv \tilde{v} - \hat{s} - (D_I/2k)$ ; note that  $(\tilde{z} | s) \sim N(s - \hat{s} - (D_I/2k), h^{-1})$ .

*A.8. Proof of proposition 8*

Proof follows steps similar to those used in the derivation in the proof of proposition 4. The additional element is that now the total informed demand contains the seller's position  $D_s$  as well, which is obtained by setting  $\hat{s} = s$  in (28).

*A.9. Proof of proposition 9*

To check whether truthful disclosure is a Perfect Bayesian Equilibrium when the seller's position is unobservable, fix the clients' beliefs such that they believe that the seller is truthful. Then, the portfolio demand of a client is

$$D_i^* = (h + 2ak) \left( \frac{\hat{s} - p}{a} \right). \tag{A9}$$

Using this along with the seller's optimal demand given in (28) and the uninformed demand in (17), one can write the market-clearing condition and solve for the equilibrium price distribution as  $p(\hat{s}, x^s) = \beta \hat{s} - \gamma x^s$  with  $(\gamma/\beta) = (a/\lambda h)$ . Maximizing (32) by substituting  $\tilde{p} = \beta \hat{s} - \gamma x^s$  yields

$$\hat{s} = \left[ \frac{2\lambda k b \gamma^2 \sigma_x^2 - \beta}{1 - 2\beta + 2\lambda k b \gamma^2 \sigma_x^2} \right] s,$$

which proves that truthful disclosure is *not* a Perfect Bayesian Equilibrium.

Now, consider the alternative case, where the seller's trading position is observable to her clients. This time, for any disclosure  $\hat{s}$ , clients perfectly infer the true  $s$ . In this case, the portfolio demand of a client is

$$D_i^* = (h + 2ak) \left( \frac{s - p}{a} \right) - 2k(s - \hat{s}). \tag{A10}$$

Using this along with the seller's optimal demand given in (28) and the uninformed demand in (17), one can write the market-clearing condition and solve for the equilibrium price distribution as  $p(\hat{s}, x^s) = \beta s - \gamma x^s$  with



$(\gamma/\beta) = (a/\lambda h)$ . Notice that this is the same equilibrium as in proposition 8 when the seller is truthful. Therefore, when the seller's trading position is observable and the clients infer the true  $s$ , equilibrium is completely independent from the seller's disclosure. Accordingly, the problem of optimal disclosure choice in (32) reduces to choosing  $\hat{s}$  to minimize  $s^2 - 2s\hat{s}$ , which yields  $\hat{s} = s$ .

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