Auditing

- A risk neutral manager produces output

$$q = \theta + a$$

where $\theta \in \{\theta_1, \theta_2\}$ with $\theta_1 < \theta_2$, $\Pr(\theta_1) = 1/2$. Cost of Effort is $\psi(a) = a^2/2$.

- First Best: Choose type contingent (a_i, T_i) to

$$Max \ \frac{1}{2}(\theta_1 + a_1 - T_1) + \frac{1}{2}(\theta_2 + a_2 - T_2) \text{ subject to}$$

$$T_i - \psi(a) \geq 0$$

- First Best Solution

$$a_i = 1 \text{ and } T_i = \frac{1}{2}$$

Auditing

- Second Best: When θ_i is private information, we habe an additional binding ICC for the efficient type

$$T_2 - \frac{a_2^2}{2} \geq T_1 - \frac{(Max(0, a_1 - \Delta\theta))^2}{2}$$

- For simplicity assume that $\Delta\theta = \theta_2 - \theta_1 = 1$. Since $a_1 \leq 1$, this assumption implies type θ_2 can produce q_1 with zero effort (simplifies IC).

$$T_2 - \frac{a_2^2}{2} \geq T_1$$

Auditing

- Second Best:

$$Max\ \frac{1}{2}(\theta_1 + a_1 - T_1) + \frac{1}{2}(\theta_2 + a_2 - T_2) \text{ subject to}$$

$$T_1 - \frac{a_1^2}{2} \geq 0 \text{ (IR1)}$$

$$T_2 - \frac{a_2^2}{2} \geq T_1 \text{ (IC2)}$$

Second Best Solution

$$a_1^* = 1/2 \quad a_2^* = 1$$

Auditing

Supervision

- An auditor observes an imperfect signal (hard information)

$$\Pr(y_i \mid \theta_i) = \delta > \frac{1}{2} > 1 - \delta = \Pr(y_j \mid \theta_i)$$

  The manager also observes this signal.

Auditing

Honest But Costly Auditor

- Manager learns θ_i
- Principal offers contracts to the manager and the auditor.
- Manager chooses a_i
- An output contingent audit is triggered with the probability specified in the auditor's contract (after q_i is observed.
- The objective of the principal is to reduce the informational rent left to the efficient manager. The optimal contract imposes a punishment when output is low but the auditor's signal indicates the manager is the efficient type.

Auditing

- Let γ be the probability that an audit is triggered when output is low, and let K denote the punishment on the manager if the auditor observes y_2.
- Also assume that z is the cost of sending the auditor and K^* is the maximim penalty that can be imposed.

Auditing

- The principal's problem

$$Max\ \frac{1}{2}(\theta_1+a_1-T_1+\gamma[(1-\delta)K-z])+\frac{1}{2}(\theta_2+a_2-T_2) \text{ subject to}$$

$$T_1-\frac{a_1^2}{2}-\gamma(1-\delta)K\geq 0 \qquad \text{(IR1)}$$

$$T_2-\frac{a_2^2}{2}\geq 0 \qquad \text{(IR2)}$$

$$T_2-\frac{a_2^2}{2}\geq T_1-\gamma\delta K \qquad \text{(IC2)}$$

- The trade-off is between z and rent reduction! Three new effects: (i) new auditing cost γz, (ii) type 1 error (low type wrongly punished, but this washes out by a corresponding increase in T_1 and (iii) auditing reduces efficient type's rent.

Auditing

- Combine (IR1) and (IC2) to obtain

$$T_2 - \frac{a_2^2}{2} \geq \frac{a_1^2}{2} - \underbrace{\gamma(2\delta - 1)K}_{\text{reduction in high type's rent}}$$

- Optimal contract for a given z has the following properties:
  - When K^* is low and/or when the signal has low accuracy, there is no audit.
  - When K^* is high enough, given the linearity of the problem $\gamma = 1$ (always audit when low output is observed)
  - Audit reduves efficiengt type's rent but as long as rent remains positive we have $a_1^* = 1/2$ (underprovision of effort by the inefficient type).

Auditing

Cheap But Corruptible Auditor

- Now collusion can take place between the supervisor and the manager once low output is observed and y_2 is observed.
- First difference: To avoid collusion, the principal must reward the auditor for revealing y_2 with w such that

$$w \geq K.$$

- Second difference: Audit is free, principal pays no cost z. But the principal still faces an expected audit cost

$$\gamma w(1-\delta)/2$$

- The contracting problem is similar except now $w(1-\delta)$ replaces z and $w \geq K$ is imposed.

Cheap But Corruptible Auditor

- Note that now the principal's audit cost is increasing in the agent's punishment K.
- Principal now faces the following trade-off between auditing costs and informational rent reduction. In expected terms, the auditior gets

$$\frac{\gamma K(1-\delta)}{2}$$

which results in a reduction of manager's rent by

$$\frac{\gamma K(2\delta-1)}{2} \qquad \text{(as before)}$$

Therefore, principal calls for an audit if and only if

$$\delta \geq \frac{2}{3}$$

Auditing

- One key point is that now maximum deterrence is no longer strictly optimal. (Becker's principal of maximum deterrence)
- When the efficient manager's information rent has been eliminated and first best effort levels are reached, furtther rise in K brings no additional benefit as the benefits of higher K is entirely offset by a higher w to prevent collusion.