Simple Economics of Adverse Selection

Buyer and Seller

- Consider a transaction between a buyer and a seller where the seller does not know perfectly how much a buyer is willing to pay for a good. Suppose also that the seller sets the terms of the contract.
- Buyer's Preferences

$$u(q, T, \theta) = \theta v(q) - T \text{ where } \theta \in \{\theta_L, \theta_H\} \text{ and } Pr(\theta_L) = \beta$$

$$v' > 0 \text{ and } v'' < 0$$

- Seller

$$\pi = T - cq$$

First Best–Seller knows Buyer's Type

"**First Best Problem**"

- For each $i \in \{L, H\}$, the seller solves

$$\text{Choose } q_i \text{ and } T_i \text{ to Maximize } T_i - cq_i$$

subject to

$$\theta_i v(q_i) - T_i \geq \bar{u}$$

Solution:

$$\theta_i v'(q_i^*) = c$$

Standard Screening Problem–Second Best

"**Second Best Problem**"

- The seller chooses q_L, q_H, T_L and T_H to

$$\text{Maximize } \beta[T_L - cq_L] + (1-\beta)[T_H - cq_H]$$

subject to

$$\theta_L v(q_L) - T_L \geq 0 \quad \text{(IRL)}$$

$$\theta_H v(q_H) - T_H \geq 0 \quad \text{(IRH)}$$

$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H \quad \text{(ICL)}$$

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \quad \text{(ICH)}$$

Observations

- Observation 1: IRH does not bind. (IRL and ICH together imply $\theta_H v(q_H) - T_H > 0$.
- Observation 2a: ICH is violated at the first best solution.
- Observation 2b: ICL holds at the first best solution.

Relaxed Problem

- Accordingly, we can solve the relaxed problem with IRL and ICH both binding.
- The seller chooses q_L, q_H, T_L and T_H to

$$\text{Maximize } \beta[T_L - cq_L] + (1-\beta)[T_H - cq_H]$$

subject to

$$\theta_L v(q_L) - T_L \geq 0 \qquad \text{(IRL)}$$

$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L \qquad \text{(ICH)}$$

Second-Best Solution

- Substitute for

$$T_L = \theta_L v(q_L)$$

$$T_H = \theta_H[v(q_H) - v(q_L)] + \theta_L v(q_L)$$

Second-Best Solution

- Solution:

$$\theta_H v'(q_H^{SB}) = c \text{ (same as FB)}$$

$$\theta_L v'(q_L^*) = \frac{c}{1 - \left(\frac{1-\beta}{\beta}\frac{\theta_H - \theta_L}{\theta_L}\right)} > c \Rightarrow q_L^{SB} < q_L^{FB}$$

- Finally we need to check that at q_L^{SB} and q_H^{SB} the omitted ICL is satisfied.

Properties of the Solution

- The second best optimal consumption for type θ_H is the same as first best optimal consumption, but that of type θ_L is lower (efficiency at the top).
- While type θ_H obtains a strictly positive informational rent, type θ_L obtains no surplus.
- The consumption distortion of type θ_L is the result of the seller's attempt to reduce the informational rent of type θ_H.

APPLICATION: Regulation

- Natural monopoly with exogenous cost parameter $\theta \in \{\theta_L, \theta_H\}$. Define

$$\Delta\theta \equiv \theta_H - \theta_L > 0$$

- Firm's cost of producing the good is

$$c = \theta - e$$

Cost of effort is given by

$$\psi(e) = [Max(0, e)]^2/2$$

- Regulator wants the good to be produced against the lowest possible payment

$$p = s + c$$

Firm's payoff is

$$p - c - \psi(e) = s - \psi(e)$$

Full Observability (First Best)

- If the regulator observes θ, then it tries to achive

$$\text{Minimize } s + c = s + \theta - e$$

subject to

$$s - [Max(0, e)]^2/2 \geq 0$$

which yields

$$e^* = 1 \text{ and } s^* = 1/2$$

Interpretation: Regulator offers a fixed total payment p which induces the firm to minimize $c + \psi(e)$.

Regulation

- When the regulator cannot observe θ, it has prior beliefs $Pr(\theta_L) = \beta$
- Contracts: (s_i, c_i) chosen by θ_i who then sets $e_i = \theta_i - c_i$.
- Incentive problem exists because the efficient type might try to mimick the inefficient type to secure a higher subsidy used as a compensation for effort.
- The regulator chooses these contracts to minimize

$$\beta(s_L + \theta_L \ - e_L) + (1 - \beta)(s_H + \theta_H \ - e_H)$$

Since $\theta_i' s$ are exogenous, this becomes choosing the contracts to minimize

$$\beta(s_L - e_L) + (1 - \beta)(s_H - e_H)$$

Regulator's Problem

- Choose s_L and e_L to minimize $\beta(s_L - e_L) + (1-\beta)(s_H - e_H)$ subject to

$$s_L - [Max(0, e_L)]^2/2 \geq 0 \quad \text{(IRL)}$$

$$s_H - [Max(0, e_H)]^2/2 \geq 0 \quad \text{(IRH)}$$

$$s_H - [Max(0, e_H)]^2/2 \geq s_L - [Max(0, e_L + \Delta\theta)]^2/2 \quad \text{(ICH)}$$

$$s_L - [Max(0, e_L)]^2/2 \geq s_H - [Max(0, e_H - \Delta\theta)]^2/2 \quad \text{(ICL)}$$

Reduced Problem (after observing only IRH and ICL bind)

- Choose s_L and e_L to minimize $\beta(s_L - e_L) + (1-\beta)(s_H - e_H)$ subject to

$$s_H - [Max(0, e_H)]^2/2 \geq 0 \quad \text{(IRH)}$$

$$s_L - [Max(0, e_L)]^2/2 \geq s_H - [Max(0, e_H - \Delta\theta)]^2/2 \quad \text{(ICL)}$$

- Solution:

$$\begin{aligned} e_L^{SB} &= 1 \\ e_H^{SB} &= 1 - \left(\frac{\beta}{1-\beta}\right)\Delta\theta < 1 \end{aligned}$$

Underprovision of effort for the inefficient type, informational rent for the efficient type. Again inefficient type's effort is distorted down to reduce the informational rent of the efficient type.

Application: Optimal Income Taxation

- Income is given by

$$q = \theta e \text{ where } \theta \in \{\theta_L, \theta_H\} \text{ and } \theta_L < \theta_H$$

$$\Pr(\theta = \theta_L) = \beta$$

- Individual Utility

$$u(q - t - \psi(e))$$

  where t is tax or subsidy and $\psi(e)$ is convex effort cost function.

- Government Budget Constraint

$$\beta t_L + (1 - \beta) t_H = 0$$

First Best (Type is Known)

- Government chooses t_L, t_H, e_L and e_H to maximize

$$\beta u(\theta_L e_L - t_L - \psi(e_L)) + (1-\beta)\beta u(\theta_H e_H - t_H - \psi(e_H))$$

subject to

$$\beta t_L + (1-\beta) t_H = 0 \Rightarrow t_H = -\frac{\beta t_L}{(1-\beta)}$$

- First best is given by

$$\begin{aligned} \theta_L &= \psi'(e_L) \\ \theta_H &= \psi'(e_H) \\ u'(\theta_L e_L - t_L - \psi(e_L)) &= u'(\theta_H e_H - t_H - \psi(e_H)) \end{aligned}$$

Type Unknown (Second Best)

- Under adverse selection, G cannot observe θ_L and θ_H, but imposes t_i conditional on observed income q_i.

Optimal Taxation

- ICC for low type

$$q_H = \theta_H e_H \overset{\text{mimicking by } \theta_L \text{ requires}}{\Rightarrow} \theta_L e_L' = \theta_H e_H \Rightarrow e_L' = \frac{\theta_H e_H}{\theta_L}$$

$$u(\theta_L e_L - t_L - \psi(e_L)) \geq u(\theta_L \left(\frac{\theta_H e_H}{\theta_L}\right) - t_H - \psi(\frac{\theta_H e_H}{\theta_L}))$$

$$\Rightarrow u(\theta_L e_L - t_L - \psi(e_L)) \geq u(\theta_H e_H - t_H - \psi(\frac{\theta_H e_H}{\theta_L})) \quad \text{(ICL)}$$

Optimal Taxation

- ICC for high type

$$q_L = \theta_L e_L \overset{\text{mimicking by } \theta_H \text{ requires}}{\Rightarrow} \theta_H e'_H = \theta_L e_L \Rightarrow e'_H = \frac{\theta_L e_L}{\theta_H}$$

$$u(\theta_H e_H - t_H - \psi(e_H)) \geq u(\theta_H \left(\frac{\theta_L e_L}{\theta_H}\right) - t_L - \psi(\frac{\theta_L e_L}{\theta_H}))$$

$$\Rightarrow u(\theta_H e_H - t_H - \psi(e_H)) \geq u(\theta_L e_L - t_L - \psi(\frac{\theta_L e_L}{\theta_H})) \quad \text{(ICH)}$$

Optimal Taxation

- Let's identify which ICC is violated at the first best solution

First Best: $\theta_L e_L^{FB} - t_L^{FB} - \psi(e_L^{FB}) = \theta_H e_H^{FB} - t_H^{FB} - \psi(e_H^{FB})$

- Consider ICH

$$u(\theta_H e_H^{FB} - t_H^{FB} - \psi(e_H^{FB})) \overset{?}{\geq} u(\theta_L e_L^{FB} - t_L^{FB} - \psi(\frac{\theta_L e_L^{FB}}{\theta_H})) \text{ (NO!)}$$

- Consider ICL

$$u(\theta_L e_L^{FB} - t_L^{FB} - \psi(e_L^{FB})) \overset{?}{\geq} u(\theta_H e_H^{FB} - t_H^{FB} - \psi(\frac{\theta_H e_H^{FB}}{\theta_L}))\text{(YES!)}$$

Optimal Taxation

- ICH must be binding at the second best, along with the goverment budget constraint.
- Government chooses t_L, t_H, e_L and e_H to maximize

$$\beta u(\theta_L e_L - t_L - \psi(e_L)) + (1-\beta)\beta u(\theta_H e_H - t_H - \psi(e_H))$$

subject to

$$\beta t_L + (1-\beta)t_H = 0 \Rightarrow t_H = -\frac{\beta t_L}{(1-\beta)} \qquad \text{(GBB)}$$

$$\theta_H e_H - t_H - \psi(e_H) = \theta_L e_L - t_L - \psi(\frac{\theta_L e_L}{\theta_H}) \qquad \text{(ICH)}$$

SECOND BEST

- Using the binding constraints to eliminate t_L and t_H from the maximand, and maximizing with respect to e_H and e_L

$$\begin{aligned} \theta_H &= \psi'(e_H) \text{ (no distortion, same as FB)} \\ \psi'(e_L) &= \theta_L - (1-\beta)\gamma[\psi'(e_L) - \frac{\theta_L}{\theta_H}\psi'(\frac{\theta_L e_L}{\theta_H})] \end{aligned}$$

where

$$\gamma \equiv \frac{u'_L - u'_H}{\beta u'_L + (1-\beta)u'_H} > 0$$

- Again, the second best involves underprovision of effort by the inefficient type to reduce the informational rent of the efficient type.

Optimal Taxation

- The reason why it is second best efficient to underprovide effort here is that a lower e_L limits the welfare difference between high and low productivity individuals (from the binding ICH)

$$(\theta_H e_H - t_H - \psi(e_H)) - \theta_L e_L - t_L - \psi(e_L) = \left(\psi(e_L) - \psi(\frac{\theta_L e_L}{\theta_H})\right)$$