Multiple Tasks and Effort Substitution

- Single Agent (with CARA preferences) performs two tasks

$$
\begin{aligned}
q_1 &= a_1 + \varepsilon_1 \\
q_2 &= a_2 + \varepsilon_2
\end{aligned}
$$

where ε_1 and ε_2 are independent with $\varepsilon_i \sim N(0, \sigma_i^2)$

Multiple Tasks and Effort Substitution

- Cost of effort

$$\varphi(a_1, a_2) = \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) + \delta a_1 a_2$$

where

$$0 \leq \delta \leq \sqrt{c_1 c_2}$$

Whenever $\delta \geq 0$, raising effort in one task increases the marginal cost of effort on the other task.

Multiple Tasks and Effort Substitution

- Linear Compensation:

$$w = t + s_1 q_1 + s_2 q_2$$

- Agent's Effort Choice involves choosing (a_1, a_2) to

$$Max \ t + s_1 a_1 + s_2 a_2 - \frac{1}{2}(c_1 a_1^2 + c_2 a_2^2) - \delta a_1 a_2 - \frac{n}{2}(\sum s_i^2 \sigma_i^2)$$

which yields

$$a_i = \frac{s_i - \delta a_j}{c_i}$$

Multiple Tasks and Effort Substitution

- Solution:

$$a_1^*(s_1, s_2) = \frac{s_1 c_2 - \delta s_2}{c_1 c_2 - \delta^2} \quad a_2^*(s_1, s_2) = \frac{s_1 c_2 - \delta s_2}{c_1 c_2 - \delta^2}$$

Multiple Tasks and Effort Substitution

- The principal's problem is choosing (t, s_1, s_2) to

$$Max\ (1 - s_1)a_1^*(s_1, s_2) + (1 - s_2)a_2^*(s_1, s_2) - t$$

subject to

$$t + s_1 a_1^* + s_2 a_2^* - \frac{1}{2}(c_1 a_1^{*2} + c_2 a_2^{*2}) - \delta a_1^* a_2^* - \frac{n}{2}(\sum s_i^2 \sigma_i^2) \geq 0$$

Multiple Tasks and Effort Substitution

- The solution is

$$s_i^* = \frac{1 + (c_j - \delta)n\sigma_j^2}{1 + n(c_j\sigma_j^2 + c_i\sigma_i^2) + n^2\sigma_i^2\sigma_j^2(c_i c_j - \delta^2)}$$

- Special Case ($\delta = 0$: No effort substitution problem)

$$s_i^* = \frac{1}{1 + nc_i\sigma_i^2} \text{ (standard solution)}$$

- Key observation is the complemantarity between $s_i^{*\prime}s$ due to effort substitution.

$$\frac{\partial s_1^*}{\partial \sigma_2^2} < 0$$

Conflicting Tasks

- An agent/salesperson tries to sell two products made by the same manufacturer, product 1 and product 2.
- Sales of product i is $q_i \in \{0, 1\}$ andthe q_i's are independently distributed as

$$\Pr(q_i = 1) = \alpha + \rho a_i - \gamma a_j$$

where the effort to promote product i denoted by $a_i \in \{0, 1\}$. One unit of promotion effort costs the agent $c > 0$. We assume that expending effort to promote both products is efficient, that is

$$\rho - \gamma > c$$

Conflicting Tasks

Solution with Single Agent

- Assume that agent is risk neutral, wages cannot be negative and single agent promotes both products,

$$\begin{aligned} w_{10} &= w_{01} = w_1 \quad \text{(wage for one sale)} \\ w_{00} &= 0 \quad \text{(wage for no sale)} \\ & \bar{w} \quad \text{(wage for 2 sales)} \end{aligned}$$

Define the sales probability of a product when $a_1 = a_2 = 1$

$$\varphi = \alpha + \rho - \gamma.$$

Conflicting Tasks

- The Principal's Problem is choosing w_1 and $\bar{w}$ to

$$\text{Minimize} \quad \varphi^2 \bar{w} + 2\varphi(1-\varphi)w_1$$

subject to

$$\begin{aligned} &\varphi^2 \bar{w} + 2\varphi(1-\varphi)w_1 - 2c \geq \\ &(\varphi+\gamma)(\varphi-\rho)\bar{w} + \\ &[(\varphi+\gamma)+(\varphi-\rho)-2(\varphi+\gamma)(\varphi-\rho)]w_1 - c \end{aligned} \tag{1}$$

$$\varphi^2 \bar{w} + 2\varphi(1-\varphi)w_1 - 2c \geq \alpha^2 \bar{w} + 2\alpha(1-\alpha)w_1 \tag{2}$$

(1) ensures agent prefers two efforts to one, (2) ensures agent prefers 2 efforts to no effort.

Conflicting Tasks

- It turns out that the optimal incentive scheme has $w_1 = 0$ and the binding IC is the second one.which reduces to

$$\varphi^2 \bar{w} - 2c \geq \alpha^2 \bar{w} \tag{3}$$

At the optimum we have

$$\bar{w} = \frac{2c}{\varphi^2 - \alpha^2}$$

which leaves the single agent a rent equal to $\alpha^2 \bar{w}$ or

$$\frac{2\alpha^2 c}{\varphi^2 - \alpha^2}$$

Conflicting Tasks

The Principal now hires two agents and asks them each to promote one product. We can concentrate on a single agent and by symmetry, the other agent will get the same incentive scheme.

- The cost minimizing incentive scheme that induces $a_1 = a_2 = 1$ is given by

$$\text{Minimize } \varphi^2 \bar{w} + \varphi(1-\varphi)w_{10}$$

subject to

$$\varphi^2 \bar{w} + \varphi(1-\varphi)w_{10} - c \geq (\varphi - \rho)[(\varphi + \gamma)\bar{w} + (1 - \varphi - \gamma)w_{10}]$$

Each agent only undertakes one task, so there is a single IC. Not exerting effort reduces one agent's success probability by $(\varphi - \rho)$ while increasing the rival's from φ to $\varphi + \gamma$.

Conflicting Tasks

- Solution has

$$\bar{w} = 0 \text{ and } w_{10} > 0$$

The failure of other agent is rewarded, since it is itself an indication that the agent has exerted high effort. The optimum is

$$w_{10} = \frac{c}{\varphi(1-\varphi) - (\varphi - \rho)(1 - \varphi - \gamma)}$$

which leaves each agent a rent equal to

$$\frac{(\varphi - \rho)(1 - \varphi - \gamma)c}{\varphi(1-\varphi) - (\varphi - \rho)(1 - \varphi - \gamma)}$$

Conflicting Tasks

- To see which arrangement (single agent or two agents) is better we need to compare the rents left to agents.
- With some algebra, one can show that hiring a single agent is the better solution if and only if

$$\frac{2(\alpha+\rho-\gamma)^2}{\alpha^2} > \frac{(\alpha+\rho-\gamma)(1-\alpha-\rho+\gamma)}{(\alpha-\gamma)(1-\alpha-\rho)}$$

  LHS is decreasing in γ which measures the intensity of the conflict between tasks. RHS is increasing in γ. For $\gamma = 0$, single agent is optimal. As γ approaches to α RHS tends to $+\infty$ and two agents are optimal.