ECO 5341 Collusion in Infinitely Repeated Cournot Competition

Saltuk Ozerturk (SMU)
Remember our Cournot Competition Example

- Two firms (Firm 1 and Firm 2) face an industry demand

\[ P = 150 - Q \]

where

\[ Q = q_1 + q_2 \]

is the total industry output.

- Both firms have the same unit production cost \( c = 30 \).
- The firms are competing by simultaneously setting their quantities to maximize own profits.
Cournot Nash Equilibrium Pair $q_1^c$ and $q_2^c$ solve

\[ q_1^c(q_2^c) = 60 - \frac{q_2^c}{2} \]
\[ q_2^c(q_1^c) = 60 - \frac{q_1^c}{2} \]

which yields

\[ q_1^c = q_2^c = 40 \]
Cournot Profits of Each Firm

\[ \pi_1^c (q_1^c, q_2^c) = (150 - q_1^c - q_2^c)q_1^c - 30q_1^c \]
\[ \Rightarrow \pi_1 (q_1, q_2) = (150 - 80) \times 40 - 30 \times 40 = 1600 \]

\[ \pi_2^c (q_1^c, q_2^c) = (150 - q_1^c - q_2^c)q_2^c - 30q_2^c \]
\[ \Rightarrow \pi_2 (q_1, q_2) = (150 - 80) \times 40 - 30 \times 40 = 1600 \]
Monopoly Output and Price in this market is

\[ q^m = 60 \]

\[ P^m = 150 - Q = 150 - 60 = 90 \]

which yields a monopoly profit

\[ \pi^m = (P^m - c) q^m = (90 - 30) \times 60 = 3600 \]
Collusion outcome $q_1 = q_2 = q^m/2 = 30$ is not a Nash Equilibrium in the One-Shot Stage Game.

Can we sustain the collusion outcome in the infinitely repeated Cournot game?
Trigger Strategy for Firm i

- Start with Collusion quantity $q_i = q^m/2 = 30$.
- Continue with $q_i = q^m/2 = 30$ as long as everyone has chosen $q^m/2 = 30$.
- Choose the Cournot quantity $q_i^c = 40$ otherwise.
- Is the above trigger strategy a SPE?
Trigger Strategy says the following.

- Each firm starts with the collusion quantity $q_i = q^m/2 = 30$.
- As long as no one deviates from the collusion quantity $q_i = q^m/2 = 30$ continue to produce at $q_i = q^m/2 = 30$. This collusion phase will give a profit of $1800$ to each firm.
- If any of the two firms deviate from $q_i = q^m/2 = 30$ and produce some other quantity, then start producing $q_i^c = 40$ forever. Note that when firm $i$ produces $q_i^c = 40$, the other firm’s best response is also $q_j^c = 40$. In this case, both firms will receive their Cournot profits of $1600$ until infinity.
First we need to describe the best one-shot deviation. What is the best quantity that a deviating firm would produce when the rival is still producing at $q^m/2 = 30$?

Suppose after any history of collusion (that is after a history in which both firms produced at $q^m/2 = 30$), Firm 1 is going to deviate. What is the best deviation? Remember Firm 1’s best response

$$q_1^*(q_2) = 60 - \frac{30}{2} = 45$$

Hence Firm 1’s best deviation is $q_1^D = 45$
Let’s find Firm 1’s deviation profit for the period that it deviates to $q_1^D = 45$. Recall that when deviation happens, in that period Firm 2 still produces $q_2 = q^m/2 = 30$. Hence we have

$$\pi_1^D = (150 - 45 - 30) \times 45 - (45 \times 30) = 45 \times 45 = 2025$$

Hence when Firm 1 deviates, she faces the following profit stream

$$(2025, 1600, 1600, 1600, 1600, 1600, \ldots)$$

If Firm 1 does not deviate and sticks with $q_1 = q^m/2 = 30$, she receives

$$(1800, 1800, 1800, 1800, 1800, 1800, \ldots)$$
When Firm 1 deviates, she faces the following profit stream

\[(2025, 1600, 1600, 1600, 1600, 1600, 1600, ... )\]

which she values at

\[
(2025 + 1600\delta + 1600\delta^2 + 1600\delta^3 + .... )
\]

\[= 2025 + 1600\delta(1 + \delta + \delta^2 + ..... )\]

\[= 2025 + \frac{1600\delta}{1 - \delta}\]
If Firm 1 does not deviate and sticks with $q_1 = q^m/2 = 30$, she receives

$$(1800, 1800, 1800, 1800, 1800, 1800, \ldots)$$

which she values at

$$(1800 + 1800\delta + 1800\delta^2 + 1800\delta^3 + \ldots)$$

$$= 1800(1 + \delta + \delta^2 + \ldots)$$

$$= \frac{1800}{1 - \delta}.$$
No profitable one-shot deviation property requires

\[
\frac{1800}{1 - \delta} \geq 2025 + \frac{1600\delta}{1 - \delta}
\]

\[\Rightarrow \delta \geq \frac{9}{17}.\]

Conclusion. The trigger strategy is a SPE for \(\delta \geq \frac{9}{17}\). Collusion can be sustained if the Firms are patient enough.