ECO 5341 Infinitely Repeated Games

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Reconsider the Prisoners' Dilemma

		Player 2 Cooperate (C)	Player 2 Defect (D)
Player 1	Cooperate (C)	2, 2	-1, 3
	Defect (D)	3, -1	0, 0

- In the one-shot version, the unique NE is (D,D).
- Can we sustain the outcome (C,C) if this game is "infinitely" repeated?
- There is no final period. The game is repeated every period.
- Each player has a discount factor $\delta \in (0, 1)$.

Payoffs

- Each player has a discount factor $\delta \in (0, 1)$.
- If starting today, a player receives an infinite sequence of payoffs

$$u_1, u_2, u_3, u_4 \ldots\ldots\ldots\ldots$$

- The present value of this payoff sequence is

$$(u_1 + \delta u_2 + \delta^2 u_3 + \delta^3 u_4 + \ldots\ldots\ldots\ldots\ldots)$$

- Example: Period payoffs are equal to 2.

$$(2 + \delta 2 + \delta^2 2 + \delta^3 2 + \ldots\ldots\ldots\ldots\ldots)$$

$$= 2(1 + \delta + \delta^2 + \delta^3 + \ldots\ldots\ldots\ldots\ldots)$$

$$= \frac{2}{(1 - \delta)}$$

Equilibria of Infinitely Repeated Games

- There is no end period of the game. We cannot apply a backward induction type algorithm.
- We use One-Shot-Deviation Property to check whether a strategy profile is a SPE.

One-Shot-Deviation Property

- A strategy profile is a SPE of an infinitely repeated game if and only if no player can gain by changing her action after any history, keeping both the strategies of the other players and the remainder of her own strategy constant.
  - Take a history, for each player check if she has a profitable one-shot deviation (OSD). Do that for arch possible history.
  - If no player has a profitable OSD after any history, you have a SPE.

	Cooperate (C)	Defect (D)
Cooperate (C)	2, 2	-1, 3
Defect (D)	3, -1	0, 0

- **Trigger Strategy**
  - Cooperate (play C) in the very first stage.
  - Cooperate and play C as long as everyone has always cooperated.
  - Defect and play D otherwise
- Is the above strategy a SPE?
- Suppose you think the other player will play according to the above rule. Do you have an incentive to deviate from it?

- In an infinitely repeated game, a strategy specifies what a player does in period t after every possible history h_t.A history h_t in period t is a collection of all outcomes in the prior $t-1$ stages.
- Examples of possible histories at time t

$$h_t = \underbrace{(C,C)}_{t=1},\underbrace{(C,C)}_{t=2},\underbrace{(C,C)}_{t=3},\underbrace{(C,C)}_{t=4},\underbrace{(C,C)}_{t=5}.......\underbrace{(C,C)}_{t-1}$$

$$h_t = \underbrace{(C,C)}_{t=1},\underbrace{(C,C)}_{t=2},\underbrace{(C,C)}_{t=3},\underbrace{(C,C)}_{t=4},\underbrace{(C,D)}_{t=5}.......\underbrace{(D,D)}_{t-1}$$

	Cooperate (C)	Defect (D)
Cooperate (C)	2, 2	-1, 3
Defect (D)	3, -1	0, 0

- There are two types of histories to consider for a trigger strategy .
  - Cooperative phase: Nobody has ever defected at any point in the past

$$h_t = \underbrace{(C,C)}_{t=1}, \underbrace{(C,C)}_{t=2}, \underbrace{(C,C)}_{t=3}, \underbrace{(C,C)}_{t=4}, \underbrace{(C,C)}_{t=5} \ldots\ldots \underbrace{(C,C)}_{t-1}$$

- After such a history, Trigger Strategy says plays C in period t.
  - Punishment phase: Somebody has played D at some point

$$h_t = \underbrace{(C, C)}_{t=1}, \underbrace{(C, C)}_{t=2}, \underbrace{(C, C)}_{t=3}, \underbrace{(C, D)}_{t=4}, \underbrace{(D, D)}_{t=5} .... \underbrace{(D, D)}_{t-1}$$

After such a history, Trigger Strategy says plays D in period t.

Infinitely Repeated Games

- To check whether Trigger Strategy (TS) is a SPE, we need to check if there is any profitable One-Shot Deviation (OSD) for any player after each possible history. Recall again that there are two possible histories at time t.
- So we check if there is a profitable OSD from TS after a cooperative history and also after a history that includes defection by any player.

- **Cooperative history: What do to after a history**

$$h_t = \underbrace{(C,C)}_{t=1}, \underbrace{(C,C)}_{t=2}, \underbrace{(C,C)}_{t=3}, \underbrace{(C,C)}_{t=4}, \underbrace{(C,C)}_{t=5} \dots\dots \underbrace{(C,C)}_{t-1}$$

- If you follow the TS and cooperate, you expect to receive 2 forever. Hence the payoff is

$$(2 + 2\delta + 2\delta^2 + 2\delta^3 + \dots\dots\dots\dots) = \frac{2}{(1-\delta)}$$

- If you defect, you get 3 today and 0 from tomorrow onwards. Hence the payoff is

$$(3 + 0\delta + 0\delta^2 + 0\delta^3 + \dots\dots\dots\dots) = 3$$

Following the TS and playing C is optimal if

$$\frac{2}{(1-\delta)} \geq 3 \Rightarrow \delta \geq \frac{1}{3}.$$

	Cooperate (C)	Defect (D)
Cooperate (C)	2, 2	-1, 3
Defect (D)	3, -1	0, 0

- **History with Defection: What do to after a history**

$$h_t = \underbrace{(C,C)}_{t=1}, \underbrace{(C,C)}_{t=2}, \underbrace{(C,C)}_{t=3}, \underbrace{(C,C)}_{t=4}, \underbrace{(C,C)}_{t=5} \ldots\ldots \underbrace{(C,D)}_{t-1}$$

- Defecting gets you 0. If you cooperate, you get -1 today and at most 0 starting from tomorrow (Why?).
- Hence after observing any history with defection, following TS and playing D is optimal.

- Conclusion: Trigger Strategy is a SPE if and only if $\delta \geq \frac{1}{3}$. Cooperation can be sustained if players are patient enough and future interaction is likely enough.

Another example

	Cooperate (C)	Defect (D)
Cooperate (C)	4, 4	0, 5
Defect (D)	5, 0	1, 1

Show that Trigger Strategy is a SPE if and only if $\delta \geq \frac{1}{4}$ (see page 90-92 in textbook)

ECO 5341 Collusion in Infinitely Repeated Cournot Competition

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Remember our Cournot Competition Example

- Two firms (Firm 1 and Firm 2) face an industry demand

$$P = 150 - Q$$

where

$$Q = q_1 + q_2$$

is the total industry output.

- Both firms have the same unit production cost $c = 30$.
- The firms are competing by simultaneously setting their quantities to maximize own profits.

- **Cournot Nash Equilibrium Pair** q_1^c and q_2^c solve

$$q_1^c(q_2^c) = 60 - \frac{q_2^c}{2}$$

$$q_2^c(q_1^c) = 60 - \frac{q_1^c}{2}$$

which yields

$$q_1^c = q_2^c = 40$$

- Cournot Profits of Each Firm

$$
\begin{aligned}
\pi_1^c(q_1^c, q_2^c) &= (150 - q_1^c - q_2^c)q_1^c - 30q_1^c \\
&\Rightarrow \pi_1(q_1, q_2) = (150 - 80) * 40 - 30 * 40 = 1600
\end{aligned}
$$

$$
\begin{aligned}
\pi_2^c(q_1^c, q_2^c) &= (150 - q_1^c - q_2^c)q_2^c - 30q_2^c \\
&\Rightarrow \pi_2(q_1, q_2) = (150 - 80) * 40 - 30 * 40 = 1600
\end{aligned}
$$

Infinitely Repeated Cournot

- Monopoly Output and Price in this market is

$$q^m = 60$$

$$P^m = 150 - Q = 150 - 60 = 90$$

which yields a monopoly profit

$$\pi^m = (P^m - c)\, q^m = (90 - 30) * 60 = 3600$$

Infinitely Repeated Cournot

- Collusion outcome $q_1 = q_2 = q^m/2 = 30$ is not a Nash Equilibrium in the One-Shot Stage Game
- Can we sustain the collusion outcome in the infinitely repeated Cournot game?

Trigger Strategy for Firm i

- Start with Collusion quantity $q_i = q^m/2 = 30$.
- Continue with $q_i = q^m/2 = 30$ as long as everyone has chosen $q^m/2 = 30$.
- Choose the Cournot quantity $q_i^c = 40$ otherwise.
- Is the above trigger strategy a SPE?

Infinitely Repeated Cournot

Trigger Strategy says the following.

- Each firm starts with the collusion quantity $q_i = q^m/2 = 30$.
- As long as no one deviates from the collusion quantity $q_i = q^m/2 = 30$ continue to produce at $q_i = q^m/2 = 30$. This collusion phase will give a profit of \$1800 to each firm.
- If any of the two firms deviate from $q_i = q^m/2 = 30$ and produce some other quentity, then start producing $q_i^c = 40$ forever. Note that when firm i produces $q_i^c = 40$, the other firm's best response is also $q_j^c = 40$. In this case, both firms will receive their Cournot profits of \$1600 until infinity.

Infinitely Repeated Cournot

- First we need to describe the best one-shot deviation. What is the best quantity that a deviating firm would produce when the rival is still producing at $q^m/2 = 30$?
- Suppose after any history of collusion (that is after a history in which both firms produced at $q^m/2 = 30$), Firm 1 is going to deviate. What is the best deviation? Remember Firm 1's best response

$$q_1^*(q_2) = 60 - \frac{30}{2} = 45$$

- Hence Firm 1's best deviation is $q_1^D = 45$

Infinitely Repeated Cournot

- Let's find Firm 1's deviation profit for the period that it deviates to $q_1^D = 45$. Recall that when deviation happens, in that period Firm 2 still produces $q_2 = q^m/2 = 30$. hence we have

$$\pi_1^D = (150 - 45 - 30) * 45 - (45 * 30) = 45 * 45 = 2025$$

- Hence when Firm 1 deviates, she faces the following profit stream

$$(2025, 1600, 1600, 1600, 1600, 1600...........)$$

- If Firm 1 does not deviate and sticks with $q_1 = q^m/2 = 30$, she receives

$$(1800, 1800, 1800, 1800, 1800, 1800...........)$$

- When Firm 1 deviates, she faces the following profit stream

$$(2025, 1600, 1600, 1600, 1600, 1600...)$$

which she values at

$$\begin{aligned} & (2025 + 1600\delta + 1600\delta^2 + 1600\delta^3 + ...) \\ = \ & 2025 + 1600\delta(1 + \delta + \delta^2 + ...) \\ = \ & 2025 + \frac{1600\delta}{1-\delta} \end{aligned}$$

Infinitely Repeated Cournot

- If Firm 1 does not deviate and sticks with $q_1 = q^m/2 = 30$, she receives

$$(1800, 1800, 1800, 1800, 1800, 1800...........)$$

which she values at

$$\begin{aligned} & (1800 + 1800\delta + 1800\delta^2 + 1800\delta^3 + ...) \\ = & \; 1800(1 + \delta + \delta^2 + ...) \\ = & \; \frac{1800}{1-\delta}. \end{aligned}$$

Infinitely Repeated Cournot

- No profitable one-shot deviation property requires

$$\frac{1800}{1-\delta} \geq 2025 + \frac{1600\delta}{1-\delta}$$

$$\Rightarrow \delta \geq \frac{9}{17}.$$

- Conclusion. The trigger strategy is a SPE for $\delta \geq \frac{9}{17}$. Collusion can be sustained if the Firms are patient enough.