ECO 5341 Sequential Two-Stage Games

Saltuk Ozerturk (SMU)

General Structure

- Stage 1: P1 moves first and chooses an action x to maximize own profit function
$$\pi_1(x, y)$$
- Stage 2: After observing x, P2 chooses an action y to maximize own profit function
$$\pi_2(x, y).$$
- Important: P1 anticipates how P2 will respond to x in the second stage.

How to solve two-stage games

- **Step 1:** Go to the last stage.
  - Given x, differentiate $\pi_2(x, y)$ with respect to y and equate derivative to zero.
  - Solve for P2's best response function $y^*(x)$

How to solve two-stage games

- **Step 2:** Go to the first stage.
  - P1 anticipates P2's best response $y^*(x)$ and incorporates it into its own objective function. Write $\pi_1(x, y^*(x))$ instead of $\pi_1(x, y)$
  - Differentiate $\pi_1(x, y^*(x))$ with respect to x, equate derivative to zero and find x^*.
  - Finally find $y^*(x^*)$.

Wages and Employment in a Unionized Firm

- Consider a monopolist firm in a product market. The firm faces an inverse demand function

$$P = 200 - q$$

where q is the firm's output and P is the price.

Labor Demand by the Firm

- Producing one unit of output requires one unit of labor. Hence, we have

$$q = L$$

where L is the total labor input the firm uses.

Wages and Employment in a Unionized Firm

- Each unit of labor costs the firm a wage w. To produce $q = L$ units, the firm must pay wL.
- The firm's profit function from producing q units is given by

$$\pi(q) = Pq - \text{labor cost}$$

- Since $q = L$ we have $P = 200 - L$ and labor cost is wL, the profits of the firm becomes

$$\pi(L, w) = (200 - L)\, L - wL$$

The Union

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- The Union's payoff increasing both in wage w (chosen by the Union) and by L (chosen by the Firm).

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- The Union's payoff increasing both in wage w (chosen by the Union) and by L (chosen by the Firm).
- The Union chooses w to maximize

$$U(w, L) = (w - w_m)L$$

where w_m is the minimum wage that the workers can secure themselves in an alternative employment.

Wages and Employment in a Unionized Firm

- **Sequence of moves in the game**
- In the first stage, the Union chooses w to maximize

$$U(w, L) = (w - w_m)L = wL - w_m L$$

- In the second stage, after observing w set by the Union, the Firm chooses L to maximize profits given by

$$\pi(L, w) = (200 - L)L - wL$$

Backward Induction Equilibrium

- Let's first solve the Firm's best response function after the Union sets w.
- Given the wage w , the Firm chooses L to maximize profits given by

$$\begin{aligned} \pi(L, w) &= (200 - L)L - wL \\ &\Rightarrow \pi(L, w) = 200L - L^2 - wL \end{aligned}$$

- Differentiate $\pi(L, w)$ with respect to L and obtain the first order condition

$$200 - 2L - w = 0 \ \Rightarrow L^*(w) = 100 - \frac{w}{2}$$

The Union's Problem

- In the first stage, anticipating the Firm's subsequent best response $L^*(w)$, the Union chooses w to maximize

$$\begin{aligned} U(w, L^*(w)) &= (w - w_m)L^*(w) \\ &\Rightarrow U(w, L^*(w)) = (w - w_m)\left(100 - \frac{w}{2}\right) \\ &\Rightarrow U(w, L^*(w)) = 100w - \frac{w^2}{2} - 100w_m + \frac{w_m w}{2} \end{aligned}$$

Wages and Employment in a Unionized Firm

- The first order condition is

$$100 - w + \frac{w_m}{2} = 0 \Rightarrow w^* = 100 + \frac{w_m}{2}$$

- Now let's go back to the firm's best response in stage 2 to find L^*. We have

$$w^* = 100 + \frac{w_m}{2}$$

and

$$L^*(w) = 100 - \frac{w}{2}$$

Hence

$$L^*(w^*) = 100 - \frac{1}{2}\left(100 + \frac{w_m}{2}\right)$$

$$\Rightarrow L^* = 50 - \frac{w_m}{4}$$

- The equilibrium employment L^* decreases as minimum wage w_m increases.

Sequential Price Competition with Differentiated Goods

- Two firms with profit functions (recall from earlier lecture):

$$\pi_1(p_1, p_2) = p_1(10 - p_1 + p_2)$$

$$\pi_2(p_1, p_2) = p_2(10 - p_2 + p_1)$$

  - Both firms want to maximize profits. Firm 1 moves first and chooses p_1. In the second stage, Firm 2 moves and chooses p_2.

Sequential Price Competition

Second Stage Problem

- After observing p_1, Firm 2 chooses p_2 to maximize

$$\begin{aligned} \pi_2(p_1, p_2) &= p_2(10 - p_2 + p_1) \\ &\Rightarrow \pi_2(p_1, p_2) = 10p_2 - p_2^2 + p_2 p_1 \end{aligned}$$

- First order derivative with respect to p_2 yields

$$\begin{aligned} 10 - 2p_2 + p_1 &= 0 \\ &\Rightarrow p_2^*(p_1) = 5 + \frac{1}{2}p_1 \end{aligned}$$

Firm 2 sets a higher price as rival price p_1 increases.

Price Competition

First Stage Problem

- Anticipating $p_2^*(p_1)$, Firm 1 chooses p_1 to maximize

$$\pi_1\left(p_1, p_2^*(p_1)\right) = p_1(10 - p_1 + p_2^*(p_1))$$

$$= p_1\left[10 - p_1 + \left(5 + \frac{1}{2}p_1\right)\right]$$

$$= 15p_1 - \frac{1}{2}p_1^2$$

- First order derivative with respect to p_1 yields the first order condition

$$15 - p_1 = 0 \Rightarrow p_1^* = 15$$

- Since

$$p_2^*(p_1) = 5 + \frac{1}{2} p_1$$

we obtain

$$p_2^* = 12.5$$

ECO 5341 Stackelberg Model of Duopoly

Saltuk Ozerturk (SMU)

Stackelberg Duopoly

- Suppose that two firms (Firm 1 and Firm 2) face an industry demand

$$P = 150 - Q$$

  where

$$Q = q_1 + q_2$$

  is the total industry output. Both firms have the same unit production cost $c = 30$.
- Assume that first Firm 1 moves and chooses q_1. In the second stage, after observing q_1, Firm 2 moves and chooses q_2.

Deriving Firm 2's best response in the second stage

- Given q_1 chosen by Firm 1, Firm 2 chooses q_2 to maximize

$$\pi_1 (q_1, q_2) = (150 - q_1 - q_2)q_2 - 30q_2$$

First order condition:

$$150 - 2q_2 - q_1 - 30 = 0$$

which yields the best response function:

$$q_2^*(q_1) = 60 - \frac{q_1}{2}$$

Deriving Firm 1's optimal choice of q_1

- Note that Firm 1 perfectly knows how Firm 2 will respond to any q_1 that it chooses. Firm 1 chooses q_1 to maximize

$$\pi_1(q_1, q_2) = (150 - q_1 - q_2^*(q_1))q_1 - 30q_1$$

subject to

$$q_2^*(q_1) = 60 - \frac{q_1}{2}$$

- Firm 1 chooses q_1 to maximize

$$\pi_1(q_1, q_2) = (150 - q_1 - q_2^*(q_1))q_1 - 30q_1$$

which becomes

$$\pi_1(q_1, q_2) = (150 - q_1 - \left(60 - \frac{q_1}{2}\right))q_1 - 30q_1$$

$$= 90q_1 - \frac{q_1^2}{2} - 30q_1$$

and yields the first order condition

$$90 - q_1^S - 30 = 0 \Rightarrow q_1^S = 60$$

$$\Rightarrow q_2^S = 60 - \frac{q_1^S}{2} \Rightarrow q_2^S = 60 - \frac{60}{2} \Rightarrow q_2^S = 30$$

Comparison of Cournot Duopoly NE and Stackelberg NE

	q_1	q_2	P	π_1^*	π_2^*
Cournot NE	40	40	70	1600	1600
Stackelberg NE	60	30	60	1800	900

- Firm1 who moves first enjoys a first mover advantage. Behaves more aggressively and secures more profits π_1^* than what it achieves in Cournot NE.