ECO 5341 Public Good Provision and Tragedy of the Commons

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- Since Hume (1739), political philosophers and economists have recognized that if citizens respond only tp private incentives, public goods will be underprovided and public resources overutilized

Public Good

Under-provision of a public good

- Consider two citizens who derive utility from a public good (like a public hospital).
- If public good has quality

$$Q = q_1 + q_2$$

after constributions by citizens given by q_1 and q_2, then citizen i's utility is

$$u_i(q_1, q_2) = \underbrace{(q_1 + q_2)}_{Q}^{\frac{1}{2}} - cq_1$$

- In a NE, Citizen 1 chooses q_1 to maximize

$$u_1(q_1, q_2) = (q_1 + q_2)^{\frac{1}{2}} - cq_1$$

$$\text{FOC} \quad : \quad \frac{1}{2\left(q_1^* + q_2\right)^{\frac{1}{2}}} - c = 0$$

$$\Rightarrow \quad q_1^*(q_2) = \frac{1}{4c^2} - q_2$$

Public Good

- Citizen 2 chooses q_2 to maximize

$$u_2(q_1, q_2) = (q_1 + q_2)^{\frac{1}{2}} - cq_2$$

$$\text{FOC} \quad : \quad \frac{1}{2\left(q_1 + q_2^*\right)^{\frac{1}{2}}} - c = 0$$

$$\Rightarrow \quad q_2^*(q_1) = \frac{1}{4c^2} - q_1$$

Public Good

- Hence the NE level of public good is

$$\underbrace{(q_1^* + q_2^*)}_{Q^{NE}} = \frac{1}{4c^2}$$

$$\text{with } q_1^* = q_2^* = \frac{1}{8c^2}$$

Social Planner's Solution

- Suppose there is a benevolent social planner (SP) who chooses Q to maximize the total welfare of two citizens given by.

$$
\begin{aligned}
& u_1(q_1, q_2) + u_2(q_1, q_2) \\
= \; & \underbrace{(q_1 + q_2)}_{Q}^{\frac{1}{2}} - cq_1 + \underbrace{(q_1 + q_2)}_{Q}^{\frac{1}{2}} - cq_2 \\
= \; & 2Q^{\frac{1}{2}} - cQ
\end{aligned}
$$

Social Planner's Solution

- Social Planner chooses Q to maximize

$$2Q^{\frac{1}{2}} - cQ$$

$$\text{FOC: } 2\frac{1}{2}\frac{1}{(Q^S)^{1/2}} - c = 0 \Rightarrow Q^s = \frac{1}{c^2}$$

- In a NE there is always under-provision of the public good compared to the social optimum.

$$Q^s = \frac{1}{c^2} > Q^{NE} = \frac{1}{4c^2}$$

- Free riding!

Tragedy of the Commons

- Consider 2 farmers in a village who graze their goats in the village green,
- Let g_i denote the number of goats that farmer i owns. Thus

$$G = g_1 + g_2$$

- Value of each goat to a farmer is $V(G)$ where

$$V'(G) < 0 \text{ and } V''(G) < 0$$

  That is, as G incrases V decreases.
- Each goat costs $\$c$ to graze.

Commons

- F1 chooses g_1 to maximize

$$\pi_1(g_1, g_2) = V(G)g_1 - cg_1$$

$$\text{FOC:} \quad V'(g_1^* + g_2)g_1^* + V(g_1^* + g_2) - c = 0$$

- F2 chooses g_2 to maximize

$$\text{FOC:} \quad V'(g_1 + g_2^*)g_2^* + V(g_1 + g_2^*) - c = 0$$

- NE (g_1^*, g_2^*) must satisfy

$$V'(\underbrace{G^*}_{g_1^*+g_2^*})g_1^* + V(\underbrace{G^*}_{g_1^*+g_2^*}) - c = 0 \qquad (1)$$

and

$$V'(\underbrace{G^*}_{g_1^*+g_2^*})g_2^* + V(\underbrace{G^*}_{g_1^*+g_2^*}) - c = 0 \qquad (2)$$

- Summing (1) and (2) side by side yields

$$V'(G^*)\,(g_1^* + g_2^*) + 2V(G^*) - 2c = 0$$

NE level G^* solves

$$\frac{1}{2}V'(G^*)G^* + V(G^*) - c = 0$$

Social Optimum

- Suppose there is a social planner (SP) who chooses G to maximize the total welfare of two farmers.
- SP chooses socially optimum level G to maximize

$$
\begin{aligned}
& \pi_1(g_1, g_2) + \pi_1(g_1, g_2) \\
= \; & V(G)g_1 - cg_1 + V(G)g_2 - cg_2 \\
= \; & V(G)(g_1 + g_2) - c(g_1 + g_2) \\
= \; & V(G)G - cG \quad (\text{since } g_1 + g_2 = G)
\end{aligned}
$$

Social Optimum

- SP chooses G to maximize

$$V(G)G - cG$$

FOC: Social optimum G^s solves

$$V'(G^s)G^s + V(G) - c = 0$$

Social Optimum

- Compare the NE level

$$\frac{1}{2}V'(G^*)G^* + V(G^*) - c = 0$$

with social planer's solution

$$V'(G^s)G^s + V(G^s) - c = 0$$

Since $V''(G) < 0$ we have

$$G^s < G^*.$$

- In a NE the common resource always over-utilized compared to the social optimum

Social Optimum

- Example $c = 10$ and

$V(G) = 60 - G^2$	$V'(G) = -2G$	$V''(G) = -2$

NE solves

$$\begin{aligned} \frac{1}{2}V'(G^*)G^* + V(G^*) - c &= 0 \\ \frac{1}{2}(-2G^*) * G^* + 60 - G^{*2} - 10 &= 0 \\ G^* &= \sqrt{\frac{50}{2}} = 5 \end{aligned}$$

Social Optimum

- Social planer's solution is given by

$$
\begin{aligned}
-(2G^s * G^s) + 60 - (G^s)^2 - 10 &= 0 \\
3(G^s)^2 &= 50 \\
G^s &= \sqrt{\frac{50}{3}} < G^* = 5
\end{aligned}
$$