ECO 5341 Cournot Competition and Collusion

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Cournot Quantity Competition

- Suppose that two firms (Firm 1 and Firm 2) face an industry demand
$$P = 150 - Q$$
where
$$Q = q_1 + q_2$$
is the total industry output.
- Both firms have the same unit production cost $c = 30$.
- The firms are competing by simultaneously setting their quantities to maximize own profits.

Deriving Firm 1's best response

- For any given q_2, Firm 1 chooses q_1 to maximize

$$\pi_1 (q_1, q_2) = (150 - q_1 - q_2)q_1 - 30q_1$$

First order condition:

$$150 - 2q_1 - q_2 - 30 = 0$$

which yields the best response function:

$$q_1^*(q_2) = 60 - \frac{q_2}{2}$$

- - **Deriving Firm 2's best response**
- For any given q_1, Firm 2 chooses q_2 to maximize

$$\pi_2(q_1, q_2) = (150 - q_1 - q_2)q_2 - 30q_2$$

First order condition:

$$150 - 2q_2 - q_1 - 30 = 0$$

which yields the best response function:

$$q_2^*(q_1) = 60 - \frac{q_1}{2}$$

Cournot NE Pair Solves q_1^c and q_2^c solve

$$q_1^c(q_2^c) = 60 - \frac{q_2^c}{2}$$

$$q_2^c(q_1^c) = 60 - \frac{q_1^c}{2}$$

which yields

$$q_1^c = q_2^c = 40$$

Cournot Equilibrium continued

- Cournot NE market price is given by

$$P^c = 150 - Q = 150 - (q_1^c + q_2^c) = 70$$

Cournot Competition

- Cournot Profits of Each Firm

$$\begin{aligned}\pi_1^c(q_1^c, q_2^c) &= P^c q_1^c - 30q_1^c \\ &= (70 * 40) - (30 * 40) \\ &= 1600\end{aligned}$$

$$\begin{aligned}\pi_2^c(q_1^c, q_2^c) &= P^c q_2^c - 30q_2^c \\ &= (70 * 40) - (30 * 40) \\ &= 1600\end{aligned}$$

Monopoly

- Monopoly Output and Price in this market is (how do I know?)

$$q^m = 60$$

$$\begin{aligned} P^m &= 150 - Q \\ &= 150 - 60 \\ &= 90 \end{aligned}$$

which yields a monopoly profit

$$\pi^m = (P^m - c)\, q^m = (90 - 30) * 60 = 3600$$

Monopoly

- In principle, the two firms could decide to collude and act like a monopolist.
- By colluding, each can agree to produce half of the monopoly output 60

$$q_1 = q_2 = \frac{q^m}{2} = 30$$

and sell at the monopoly price

$$P^m = 90$$

making a profit of 1800 each, which is better than Cournot profit of 1600.

Collusion outcome $q_1 = q_2 = \frac{q^m}{2} = 30$ is not a Nash Equilibrium.

- Suppose Firm 2 produces at

$$q_2 = 30.$$

- What is Firm 1's best response to $q_2 = 30$? No!

$$q_1^*(q_2) = 60 - \frac{q_2}{2} = 60 - \frac{30}{2} = 45$$

A Finite (Baby) version of Collusion Game. This is Exercise 1.5 in the textbook (page 49)

- Suppose each of the two firms can either produce $q^m/2 = 30$ or the Cournot equilibrium quantity $q^c = 40$. No other quantity is feasible. The game matrix look like this (you should verify all the payoffs)

	$q^m/2 = 30$	$q^c = 40$
$q^m/2 = 30$	$(1800, 1800)$	$(1500, 2000)$
$q^c = 40$	$(2000, 1500)$	$(1600, 1600)$

- But note that this game is a Prisoners' Dilemma! $q^c = 40$ is a strictly dominant action for both players.

Farmers

- Consider two farmers, Farmer 1 and Farmer 2 who choose the number of palm trees x_i to plant in a common green field.
- If F1 plants x_1 and F2 plants x_2, then there will a total number of

  $$X = x_1 + x_2$$

  trees.
- When a total number of X trees are planted, each tree can fetch a price P given by

  $$P = 600 - X$$

  Note that the more palm trees planted, the lower the price each tree can fetch.
- Each palm tree costs the farmer $c = \$60$ to plant.
- Therefore the profit of each farmer is given by

  $$\pi_1 (x_1, x_2) = (600 - x_1 - x_2)x_1 - 60x_1$$

  $$\pi_2 (x_1, x_2) = (600 - x_1 - x_2)x_2 - 60x_2$$
- Each farmer i simultaneusly chooses own x_i to maximize own profits π_i. That is, F1 chooses x_1 to maximize $\pi_1 (x_1, x_2)$ and F2 chooses x_2 to maximize $\pi_2 (x_1, x_2)$.
- **Question:** Derive F1's best response $x_1^*(x_2)$ and F2's best response $x_2^*(x_1)$. Find the equilibrium number of trees (x_1^*, x_2^*) planted. What is the equilibrium profit of each farmer in this equilibrium?

Answer: Choose x_1 to Maximize

$$\pi_1 (x_1, x_2) = (600 - x_1 - x_2)x_1 - 60x_1$$

FOC is

$$600 - 2x_1 - x_2 - 60 = 0$$

which gives us

$$x_1^*(x_2) = 270 - \frac{x_2}{2}$$

Similarly, choosing x_2 to maximize

$$\pi_2\left(x_1, x_2\right) = (600 - x_1 - x_2)x_2 - 60x_2$$

gives us the FOC

$$600 - 2x_2 - x_1 - 60 = 0$$

and thus

$$x_2^*(x_1) = 270 - \frac{x_1}{2}$$

The NE pair (x_1^*, x_2^*) solves

$$x_1^* = 270 - \frac{1}{2}x_2^*$$

$$x_1^* = 270 - \frac{1}{2}\underbrace{\left[270 - \frac{1}{2}x_1^*\right]}_{x_2^*}$$

$$x_1^* = 270 - \frac{270}{2} + \frac{1}{4}x_1^*$$

$$\frac{3x_1^*}{4} = 135 \Rightarrow x_1^* = 180$$

Hence

$$\begin{aligned} x_2^*(x_1^*) &= 270 - \frac{x_1^*}{2} \\ &= 270 - \frac{180}{2} \\ &\rightarrow x_2^* = 180 \end{aligned}$$

We have

$$X = x_1^* + x_2^* = 180 + 180 = 360$$

$$P^* = 600 - \underbrace{360}_{X} = 240$$

$$\begin{aligned} \pi_1^* &= \pi_2^* \\ &= \left(P - c\right)x \\ &= \left(240 - 60\right)180 \\ &= 32,400 \end{aligned}$$