ECO 5341 Price Competition

Saltuk Ozerturk (SMU)

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Bertrand Price Competition with Homogenous Products

- Consider a market with two firms, Firm 1 and Firm 2. Both firms produce homogenous (identical) products at a unit cost $c = 0$ (for simplicity).
- Two firms are competing by simultaneously setting prices of an identical product to place on the market.

Bertrand Price Competition

- Firms' products are viewed identically to consumers — all consumers buy from the firm with a lower price.
- When the firms charge the same price, the firms split the market and each firm captures exactly half of the market demand.

Bertrand Price Competition

- Suppose firm i sets price $p_i \in [0, \infty)$ when the rival firm j sets a price $p_j \in [0, \infty)$. Then the demand q_i for Firm i's product is given by

$$q_i(p_i, p_j) = \begin{cases} 1 - p_i & \text{if} \quad p_i < p_j \\ \frac{1-p_i}{2} & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

Bertrand Price Competition

- Strategic (Normal) Form of the game:
  - Players: Two Firms $N = \{1, 2\}$
  - Strategies: Firm $i \in N$ chooses price $p_i \in [0, +\infty)$.

Bertrand Price Competition

- Payoffs of the firms:

$$\pi_1(p_1, p_2) = \begin{cases} p_1(1-p_1) & \text{if} \quad p_1 < p_2 \\ \dfrac{p_1(1-p_1)}{2} & \text{if} \quad p_1 = p_2 \\ 0 & \text{if} \quad p_1 > p_2 \end{cases}$$

$$\pi_2(p_1, p_2) = \begin{cases} p_2(1-p_2) & \text{if} \quad p_2 < p_1 \\ \dfrac{p_2(1-p_2)}{2} & \text{if} \quad p_1 = p_2 \\ 0 & \text{if} \quad p_2 > p_1 \end{cases}$$

Bertrand Price Competition

Is there any Nash Equilibria with $p_1 \neq p_2$?

- Suppose, without loss of generality, there is a NE in which firm 1 sets $p_1 = a$ and $p_2 = b$ where

$$c = 0 < p_2 = b < p_1 = a < 1$$

- For this to be a NE, each firm must be best responding to each other.
- In this proposed equilibrium we have

$$\pi_1(a, b) = 0$$

since $p_2 = b < p_1 = a$.

Any Nash Equilibria with $p_1 \neq p_2$ (continued)

- But note that, instead of setting $p_1 = a$ and receiving 0, F1 can set $p_1 = b$ and get

$$\pi_1 (b, b) = \frac{b (1 - b)}{2} > 0$$

- Hence we cannot have a NE in which firm 1 (or firm 2) sets a higher price than its rival.

Is there any Nash Equilibria with $p_1 = p_2 > 0$?

- Suppose, there is a NE in which firms set $p_1 = p_2 = a$ where

$$0 < a < 1$$

  For this to be a NE, each firm must be best responding to each other.

- In this proposed equilibrium we have

$$\pi_1(a, a) = \frac{a(1-a)}{2} > 0$$

- But note that, instead of setting $p_1 = a$, F1 can set $p_1 = a - \varepsilon$ where $\varepsilon > 0$ and receive

$$\pi_1 (a - \varepsilon, b) = (a - \varepsilon)(1 - a + \varepsilon) > \frac{a(1 - a)}{2}$$

- Hence no NE in which **two firms both set same price strictly higher than marginal cost c=0**

Is $p_1 = p_2 = 0$ **a Nash Equilibrium**?

- If both firms set a price equal to their marginal cost, they share the market but receive a zero profit.

$$\pi_1(0,0) = \frac{0(1-0)}{2} = 0$$
$$\pi_2(0,0) = \frac{0(1-0)}{2} = 0$$

Is $p_1 = p_2 = 0$ **a Nash Equilibrium**?

- Is there a profitable devitation for any of the two firms. The answer is no.
- If a firm sets a lower price than 0, this firm will sell each unit a loss and make negative profits.
- If the same firm deviates and sets a higher price than c=0, it will not be able to sell anything and continue to receive zero profit.
- Therefore $p_1 = p_2 = c$ **is the unique Nash Equilibrium**.

Price Competition with Differentiated Products

- Suppose two firms produce differentiated products at a unit cost $c = 0$.
- The firms are competing by simultaneously setting prices
- Firms' products are viewed as imperfect substitutes by consumers.

Price Competition

- Suppose firm i sets price $p_i \in [0, \infty)$ when the rival firm j sets a price $p_j \in [0, \infty)$. Then the demand q_i for Firm i's product is given by

$$q_i(p_i, p_j) = 10 - \alpha p_i + p_j$$

which implies

$$q_1(p_1, p_2) = 10 - \alpha p_1 + p_2$$

$$q_2(p_1, p_2) = 10 - \alpha p_2 + p_1$$

- We assume that $\alpha > 1$ so that own-price effect is larger than the cross-price effect.

Price Competition

- Strategic (Normal) Form of the game:
  - Players: Two Firms $N = \{1, 2\}$
  - Strategies: Firm $i \in N$ chooses price $p_i \in [0, +\infty)$.
  - Payoffs of the firms:

$$\pi_1 (p_1, p_2) = p_1 q_1 (p_1, p_2) = p_1(10 - \alpha p_1 + p_2)$$

$$\pi_2 (p_1, p_2) = p_2 q_2 (p_1, p_2) = p_1(10 - \alpha p_2 + p_1)$$

  - Both firms want to maximize profits.

Deriving the Best Response Function of Firm 1

- Given any p_2 by Firm 2, Firm 1 chooses p_1 to maximize

$$\begin{aligned} \pi_1(p_1, p_2) &= p_1(10 - \alpha p_1 + p_2) \\ &\Rightarrow \pi_1(p_1, p_2) = 10p_1 - \alpha p_1^2 + p_2 p_1 \end{aligned}$$

First order derivative with respect to p_1 yields the first order condition

$$\begin{aligned} 10 - 2\alpha p_1 + p_2 &= 0 \\ &\Rightarrow p_1^*(p_2) = \frac{5}{\alpha} + \frac{1}{2\alpha} p_2 \end{aligned}$$

That is, Firm 1 sets a higher price as the rival firm's price p_2 increases and lower price as α increases.

Deriving the Best Response Function of Firm 2

- Given any p_1 by Firm 1, Firm 2 chooses p_2 to maximize

$$\begin{aligned}\pi_2(p_1, p_2) &= p_2(10 - \alpha p_2 + p_1) \\ &\Rightarrow \pi_2(p_1, p_2) = 10p_2 - \alpha p_2^2 + p_1 p_2\end{aligned}$$

First order derivative with respect to p_2 yields the first order condition

$$\begin{aligned}10 - 2\alpha p_2 + p_1 &= 0 \\ &\Rightarrow p_2^*(p_1) = \frac{5}{\alpha} + \frac{1}{2\alpha} p_1\end{aligned}$$

That is, Firm 2 sets a higher price as the rival firm's price p_1 increases and lower price as α increases.

- The Nash equilibrium pair (p_1^*, p_2^*) solves the system of equations described by the best responses.

$$p_1^*(p_2^*) = \frac{5}{\alpha} + \frac{1}{2\alpha} p_2^*$$

$$p_2^*(p_1^*) = \frac{5}{\alpha} + \frac{1}{2\alpha} p_1^*$$

$$\Rightarrow p_1^* = \frac{5}{\alpha} + \frac{5}{2\alpha^2} + \left(\frac{1}{4\alpha^2}\right) p_1^*$$

$$\Rightarrow \left(\frac{4\alpha^2 - 1}{4\alpha^2}\right) p_1^* = \frac{10\alpha + 5}{2\alpha^2}$$

$$\Rightarrow \frac{(2\alpha - 1)(2\alpha + 1)}{4\alpha^2} p_1^* = \frac{5(2\alpha + 1)}{2\alpha^2}$$

$$\Rightarrow p_1^* = p_2^* = \frac{10}{2\alpha - 1}$$

- Note that the Nash Equilibrium Price pair

$$p_1^* = p_2^* = \frac{10}{2\alpha - 1}$$

is drastically different than the unique Nash Equilibrium of the Bertrand duopoly with identical products.

Price Competition

- With identical products, the NE was $p_1^* = p_2^* = c$ which would imply here that

$$p_1^* = p_2^* = 0$$

since we assumed that $c = 0$.

- Note that with

$$p_1^* = p_2^* = \frac{10}{2\alpha - 1}$$

as α approaches to infinity we again have $p_1^* = p_2^* = 0$.Why?