Example 1: Finding Mixed Strategy Nash Equilibrium

Find all the pure and mixed strategy equilibria of the following game by constructing the best response correspondences of the players:

	L	R
T	$2, 1$	$0, 2$
B	$1, 2$	$3, 0$

Answer: First let us consider best responses to pure strategies

$BR_1(L) = T$	$BR_2(T) = R$
$BR_1(R) = B$	$BR_2(B) = L$

So the game has NO pure strategy Nash Equilibrium.

Mixed Strategies: Consider a mixed strategy in which

- P1 chooses T with probability p and B with probability $1 - p$
- P2 chooses L with probability q and R with probability $1 - q$

Given player 2's mixed strategy $(q, 1 - q)$,

if P1 plays T:	expected payoff	$2q + (1 - q)0 = 2q$
if P1 plays B:	expected payoff	$q + (1 - q)3 = 3 - 2q$

For P1 to be indifferent between T and B in equilibrium, we must have

$$2q = 3 - 2q \Rightarrow q = \frac{3}{4}$$

$$BR_1 = \left\{ \begin{array}{ll} \text{T} & \text{if } q > \frac{3}{4} \\ \text{B} & \text{if } q < \frac{3}{4} \\ \text{indifferent between T and B} & \text{if } q = \frac{3}{4} \end{array} \right\}$$

Similarly, given P1's mixed strategy $(p, 1 - p)$, we have for player 2:

if P2 plays L:	expected payoff	$p + (1 - p)2 = 2 - p$
if P2 plays R	expected payoff	$2p + (1 - p)0 = 2p$

For P2 to be indifferent between L and R, we must have

$$2 - p = 2p \Rightarrow p = \frac{2}{3}$$

$$BR_2((p, 1 - p)) = \left\{ \begin{array}{ll} \text{L} & \text{if } p < \frac{2}{3} \\ \text{R} & \text{if } p > \frac{2}{3} \\ \text{indifferent between R and L} & \text{if } p = \frac{2}{3} \end{array} \right\}$$

Hence in the (unique) mixed strategy NE

- Player 1 plays T with probability $\frac{2}{3}$ and B with probability $\frac{1}{3}$.
- Player 2 plays L with probability $\frac{3}{4}$ and R with probability $\frac{1}{4}$.

Example 2: **Finding Mixed Strategy Nash Equilibrium**

Find all the pure and mixed strategy equilibria of the following game by constructing the best response correspondences of the players:

	Opera(O)	Fight (F)
Opera(O)	$2, 1$	$0, 0$
Fight (F)	$0, 0$	$1, 2$

Answer: First let us consider best responses to pure strategies

$BR_1(O) = O$	$BR_2(O) = O$
$BR_1(F) = F$	$BR_2(F) = F$

So the game has TWO pure strategy Nash Equilibria (Opera,Opera) and (Fight, Fight).

Mixed Strategies: Consider a mixed strategy in which

- P1 chooses O with probability p and F with probability $1 - p$
- P2 chooses O with probability q and F with probability $1 - q$

Given P2's mixed strategy $(q, 1 - q)$, we have for player 1:

if P1 plays 0:	expected payoff	$2q + (1 - q)0 = 2q$
if P1 plays F:	expected payoff	$q0 + (1 - q)1 = 1 - q$

For player 1 to be indifferent between Opera and Fight in equilibrium, we must have

$$2q = 1 - q \Rightarrow q = \frac{1}{3}$$

$$BR_1((q, 1 - q)) = \left\{ \begin{array}{ll} \text{Opera} & \text{if } q > \frac{1}{3} \\ \text{Fight} & \text{if } q < \frac{1}{3} \\ \text{indifferent between Opera and Fight} & \text{if } q = \frac{1}{3} \end{array} \right\}$$

Similarly, given player 1's mixed strategy $(p, 1 - p)$,

if P2 plays 0:	expected payoff	$p + (1 - p)0 = p$
if P1 plays F:	expected payoff	$p0 + (1 - p)2 = 2 - 2p$

For P2 to be indifferent between Opera and Fight in equilibrium, we must have

$$p = 2 - 2p \Rightarrow p = \frac{2}{3}$$

$$BR_2((p, 1 - p)) = \left\{ \begin{array}{ll} \text{Opera} & \text{if } p > \frac{2}{3} \\ \text{Fight} & \text{if } p > \frac{2}{3} \\ \text{indifferent between Opera and Fight} & \text{if } p = \frac{2}{3} \end{array} \right\}$$

Hence in the (unique) mixed strategy NE

- Player 1 plays Opera with probability $\frac{2}{3}$ and Fight with probability $\frac{1}{3}$.
- Player 2 plays Opera with probability $\frac{1}{3}$ and Fight with probability $\frac{2}{3}$.