# Examples of Games with Continuum of Actions

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BUYER-SELLER GAME

- A buyer and a seller simultaneously submit a price, which can be any non-negative number.
- Trade takes place when the price $p_b$ chosen by the buyer is at least high as the price $p_s$ chosen by the seller, i.e., $p_b \geq p_s$

BUYER-SELLER GAME (continued)

- If trade occurs, payoff of the buyer is his value $v = 10$ minus the price $p_b$ he pays and payoff of seller is the price $p_b$ she receives minus her cost $c = 5$.
- If trade does not occur, both players receive zero payoff.

# Buyer-Seller Game

Payoff to Buyer and Seller

$$u_b(p_b, p_s) = \begin{cases} 10 - p_b & \text{if } p_b \geq p_s \\ 0 & \text{otherwise.} \end{cases}$$

$$u_s(p_b, p_s) = \begin{cases} p_b - 5 & \text{if } p_b \geq p_s \\ 0 & \text{otherwise.} \end{cases}$$

# Buyer-Seller Game

## Problem

*Is the strategy profile* $p_b = 7$ *and* $p_s = 6$ *a NE. Explain formally.*

## Solution

*No it is not a NE, since*

$$u_b(7,6) = 3 < u_b(6,6) = 4$$

*Hence* $p_b = 7$ *is not a BR to* $p_s = 6$*.*

# Buyer-Seller Game (continued)

**Problem**

*Is the strategy profile $p_b = 6$ and $p_s = 6$ a NE. Explain formally.*

**Solution**

*Yes it is a NE since*

| | | |
|---|---|---|
| $u_b(6,6) = 4$ | $u_b(6+\varepsilon, 6) = 4 - \varepsilon$ | $u_b(6-\varepsilon, 6) = 0$ |

*Hence $p_b = 6$ is a BR to $p_s = 6$. Furthermore,*

| | | |
|---|---|---|
| $u_s(6,6) = 1$ | $u_s(6, 6+\varepsilon) = 0$ | $u_s(6, 6-\varepsilon) = 1$ |

*Hence $p_s = 6$ is a BR to $p_b = 6$.*

# Buyer-Seller Game (continued)

## Problem

*Are there any NE where* $p_b = p_b = x$ *where* $5 \leq x \leq 10$?

## Solution

$$u_b(x, x) = 10 - x \geq 0$$

| | |
|---|---|
| $u_b(x + \varepsilon, x) = 10 - x - \varepsilon$ | $u_b(x - \varepsilon, x) = 0$ |

*Hence* $p_b = x$ *is a BR to* $p_s = x$. *Furthermore,*

| | | |
|---|---|---|
| $u_s(x, x) = x - 5 \geq 0$ | $u_s(x, x + \varepsilon) = 0$ | $u_s(x, x - \varepsilon) = x - 5$ |

*Hence* $p_s = x$ *is a BR to* $p_b = x$. *Therefore, any* $p_b = p_b = x$ *where* $5 \leq x \leq 10$ *is a NE*

# Gunslingers

- Consider the following duel. Each of two gunslingers independently chooses when to draw their weapon and fire.
- The one who fires first wins the duel, which has a value 50, while the other loses, which has a cost 10.

# Gunslingers continued

- However, if the one who draws first does so before 10 seconds has passed he suffers a reputation cost of 40.
- If they fire at the same time, they win with equal probabilities.
- Assume that the players can choose any nonnegative real number as their strategy.

# Gunslingers

- Denote P1's strategy as $t_1$ and denote P2's strategy as $t_2$.

$$u_1(t_1, t_2) = \begin{cases} 50 & \text{if } 10 \leq t_1 < t_2 \\ 20 & \text{if } 10 \leq t_1 = t_2 \\ 10 & \text{if } t_1 < 10 \text{ and } t_1 < t_2 \\ -20 & \text{if } t_1 = t_2 < 10 \\ -10 & \text{if } t_1 > t_2 > 10. \end{cases}$$

$$u_2(t_1, t_2) = \begin{cases} 50 & \text{if } 10 \leq t_2 < t_1 \\ 20 & \text{if } 10 \leq t_1 = t_2 \\ 10 & \text{if } t_2 < 10 \text{ and } t_2 < t_1 \\ -20 & \text{if } t_1 = t_2 < 10 \\ -10 & \text{if } t_2 > t_1 > 10. \end{cases}$$

# Gunslingers

## Problem

*Is* $t_1 = 30$ *and* $t_2 = 30$ *a Nash Equilibrium? Explain formally.*

## Solution

*Note that*

$$u_1(30, 30) = 20 < u_1(29, 30) = 50$$

*Hence this is not a NE.*

# Gunslingers

**Problem**

*Are there any NE where $t_1 = t_2 < 10$? Explain formally.*

**Solution**

*Consider any profile $t_1 = t_2 = a < 10$. We have*

$$u_1(a, a) = -20 < u_1(a - \varepsilon, a) = 10$$

*Hence there is no such NE.*

# Gunslingers

## Problem

*Are there any NE where* $10 < t_1 < t_2$*? Explain formally.*

## Solution

*Consider any profile* $10 < t_1 = a < t_2 = b$*. We have*

$$u_2(a, b) = -10 < u_2(a, a) = 20$$

*Hence there is no such NE.*

# Gunslingers

## Problem

*Is $t_1 = t_2 = 10$ a Nash Equilibrium? Explain formally.*

## Solution

*Yes $t_1 = t_2 = 10$ is a Nash Equilibrium. We have*

$$u_1(10, 10) = 20 \quad u_1(10 + \varepsilon, 10) = -10 \quad u_1(10 - \varepsilon, 10) = 10$$

*Hence $t_1 = 10$ is BR to $t_2 = 10$. Furthermore,*

$$u_2(10, 10) = 20 \quad u_2(10, 10 + \varepsilon) = -10 \quad u_2(10, 10 - \varepsilon) = 10$$

*Hence $t_2 = 10$ is BR to $t_1 = 10$.*