Review Problem for Quiz 1

Consider an overlapping generations economy, where when young, a consumer has
\[ y = 240 \]
and has no endowment when old. The population is constant, i.e. we have
\[ N_t = N_{t-1} \Rightarrow \frac{N_t}{N_{t-1}} = n = 1 \]
The slope of a consumer’s indifference curve is
\[ -\frac{c_2}{3c_1} \]
where \( c_1 \) is consumption when young and \( c_2 \) is consumption when old.

The stock of money supply is growing at a rate \( z = 1.2 \), i.e.,
\[ \frac{M_t}{M_{t-1}} = z = 1.2 \]

a) Find the golden rule allocation \((c_{1,GR}, c_{2,GR})\) that a social planner would choose.

**ANSWER:** The social planner’s budget constraint is
\[ N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y \]
which can be rewritten as
\[ c_{1,t} + \frac{N_{t-1}}{N_t} c_{2,t} \leq y \rightarrow c_{1,t} + c_{2,t} \leq 240 \]
\[ \rightarrow c_{1,t} + c_{2,t} \leq 240 \]
with stationarity, this becomes
\[ c_1 + c_2 \leq 240 \]
Note that the slope of the budget line is
\[ -1. \]
Golden rule allocation is defined by two conditions:

(i) It must be on the budget line, i.e.,

\[ c_1^{GR} + c_2^{GR} = 240 \]  \hspace{1cm} (1)

(ii) The slope of the indifference curve at the golden rule allocation must be equal to the slope of the budget line, i.e.,

\[ -1 = -\frac{c_2^{GR}}{3c_1^{GR}} \]

\[ \Rightarrow 3c_1^{GR} = c_2^{GR} \]  \hspace{1cm} (2)

Combining (1) and (2), we get

\[ c_1^{GR} + 3c_1^{GR} = 240 \]

\[ \Rightarrow 4c_1^{GR} = 240 \]

\[ \Rightarrow c_1^{GR} = 60 \text{ and } c_2^{GR} = 180. \]

b) Suppose a consumer of generation \( t \) can buy \( m_t \) units money when young and use the money to finance consumption when old. Let the value of money at period \( t \) be denoted by \( v_t \). Write down two budget constraints when young and old and combine them to obtain a lifetime budget constraint

**ANSWER:** Budget constraint when young

\[ c_{1,t} + v_t m_t \leq 240 \]  \hspace{1cm} (3)

Budget constraint when old

\[ c_{2,t+1} \leq v_{t+1} m_t \]  \hspace{1cm} (4)

Rewrite the young age budget constraint as

\[ \frac{c_{1,t}}{v_t} + m_t \leq \frac{240}{v_t} \]  \hspace{1cm} (5)

and rewrite the old age budget constraint as

\[ \frac{c_{2,t+1}}{v_{t+1}} \leq m_t \]  \hspace{1cm} (6)
Summing up these last two inequalities, we obtain the lifetime budget constraint of generation $t$ as

$$\frac{c_{1,t}}{v_t} + \frac{c_{2,t+1}}{v_{t+1}} \leq \frac{240}{v_t}$$

$$\implies c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 240$$

c) Find the equilibrium rate of return on money $\frac{v_{t+1}}{v_t}$ by equating demand for money at period $t$ to supply of money at period $t$ and assuming stationarity.

**ANSWER:** Demand for money at time $t$ is

$$N_t(y - c_{1,t})$$

Supply of money at time $t$ is

$$v_t M_t$$

Equating supply and demand, we get

$$v_t M_t = N_t(y - c_{1,t}) \implies v_t = \frac{N_t(y - c_{1,t})}{M_t}$$

Similarly for period $t+1$, we have

$$v_{t+1} = \frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}}$$

As a result, we obtain

$$\frac{v_{t+1}}{v_t} = \frac{N_{t+1}(y - c_{1,t+1})}{N_t(y - c_{1,t})} \frac{M_t}{M_{t+1}} = \frac{N_{t+1}}{N_t} \frac{M_t}{M_{t+1}} (y - c_{1,t+1})$$

But with stationarity, we have $c_{1,t+1} = c_{1,t}$ and we have constant population, i.e.

$$\frac{N_{t+1}}{N_t} = 1$$

Therefore the above expression simplifies to

$$\frac{v_{t+1}}{v_t} = \frac{M_t}{M_{t+1}} = \frac{1}{z} = \frac{1}{1.20}$$

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d) Revisit the individual lifetime budget constraint you found in part (b) and rewrite it using the equilibrium rate of return on money you found in part (c). How does the individual lifetime budget constraint differ from the social planner’s budget constraint in (a). Is the competitive monetary equilibrium allocation different from the golden rule allocation?

**Answer:** The individual lifetime budget constraint we found in part (b) was

\[ c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} \leq 240 \]

Imposing

\[ \frac{v_{t+1}}{v_t} = \frac{1}{1.20} \]

this becomes

\[ c_{1,t} + 1.2c_{2,t+1} \leq 240 \]

But note that with stationarity, this becomes

\[ c_1 + 1.2c_2 \leq 240 \]

which is DIFFERENT than social planner’s budget constraint which was

\[ c_1 + c_2 \leq 240. \]

Therefore we arrive at the conclusion that the competitive monetary equilibrium allocation IS NOT THE SAME AS the golden rule allocation when money supply is growing.

e) Find the competitive monetary equilibrium allocation \((c_1^*, c_2^*)\).

**Answer:** The competitive monetary equilibrium allocation \((c_1^*, c_2^*)\) satisfies two equations

\[ c_1^* + 1.2c_2^* = 240 \]

and

\[ \frac{1}{1.2} = -\frac{c_2^*}{3c_1^*} \Rightarrow 3c_1^* = 1.2c_2^* \]

Combining these two equations, we get

\[ c_1^* + 3c_1^* = 240 \]

\[ \Rightarrow c_1^* = 60 \text{ and } c_2^* = 150. \]