STAT 6350: Analysis of Lifetime Data   Exam

Date: 13th Nov. 2007 (Tuesday) 3:30 p.m. - 5:00 p.m.

1. (25%) An investigator, performing an animal study designed to evaluate the effects of vegetable and vegetable-fiber on mammary carcinogenesis risk, randomly assigned female Sprague-Dawley rats to five dietary groups (control diet, control diet plus vegetable mixture 1, control diet plus vegetable mixture 2, control diet plus vegetable-fiber mixture 1 and control diet plus vegetable-fiber mixture 2). Mammary tumors were induced by a single oral dose (5mg dissolved in 1.0 ml. corn oil) of 7,12-dimethylbenz(a)anthracene (DMBA) administered by intragastric intubation when the animals were 7 weeks old.

   Starting 6 weeks after DMBA administration, each rat was examined once weekly for 14 weeks (post DMBA administration). Of interest is the estimation of the distribution of the time in days until onset of the first palpable tumor. Describe in detail the types of censoring and/or truncation that is represented by the following rats:

   (a) A rat who had a palpable tumor at the first examination at six weeks after intubation with DMBA.

   (b) A rat that survived the study without having any tumors.

   (c) A rat which did not have a tumor at week 12 but who had a tumor at week 13.

   (d) A rat who died (without tumor present) before the first examination at six weeks after intubation with DMBA.

   (e) A rat who died without tumor present at day 37 after intubation with DMBA.

2. (35%) 19 units were placed on test and the experimenter removed 3 surviving units from the test at the time of the third and the fifth observed failures. The experiment was terminated when eight failures are observed and the remaining 5 surviving units are censored. The eight ordered observed failures are 0.19, 0.78, 0.96, 1.31, 2.78, 4.85, 6.50, 7.35 hours. Assume that the lifetime of the units are independent random variables with constant hazard rate $1/\theta$.

   (a) What is the maximum likelihood estimate of θ, say $\hat{\theta}$?

   (b) What is the variance of the maximum likelihood estimate of θ?

   (c) Construct an **exact** 95% confidence interval for θ.

   (d) What is the maximum likelihood estimate of $S(5)$, the probability of surviving more than 5 hours?

   (e) What is the maximum likelihood estimate of $t_{0.9}$ (the 90% quantile), $\hat{t}_{0.9}$? Estimate the variance of $\hat{t}_{0.9}$.

   (f) Construct an approximate 95% confidence interval for $t_{0.9}$ based on $Z_{\ln(\hat{t}_{0.9})} \dot{\sim} NOR(0,1)$.

   (g) The experimenter plotted the data on a Weibull probability paper (Figure 1) and claim that the Weibull model should be used to fit this dataset instead of a distribution with constant hazard. Comment on this claim based on the Weibull probability plot in Figure 1.

3. (40%) The data was collected on 15 bone marrow transplant patients with Hodgkin's disease (HOD). All patients were given an autogeneic transplant where their own marrow was cleansed and returned to them after a high dose of chemotherapy.

Table 1: Time to death or relapse (in days) for patients with bone marrow transplants

Days	Status	Days	Status
30	Death	307	Censored
36	Censored	406	Death
41	Death	446	Censored
52	Death	484	Death
62	Censored	748	Death
108	Death	1290	Censored
132	Death	1345	Censored
180	Censored		

For the purpose of nonparametric estimation, we have the following results for Kaplan-Meier estimates of $F(t)$:

t_i	n_i	d_i	$\hat{p}_i$	$1-\hat{p}_i$	$\hat{S}(t)$	$\hat{F}(t)$
30	15	1	0.06667	0.93333	0.93333	0.06667
41	13	1	0.07692	0.92308	0.86154	0.13846
52	12	1	0.08333	0.91667	0.78974	0.21026
108	10	1	0.10000	0.90000	0.71077	0.28923
132	9	1	0.11111	0.88889	0.63179	0.36821
406	6	1	0.16667	0.83333	0.52650	0.47350
484	4	1	0.25000	0.75000	0.39487	0.60513
748	3	1	0.33333	0.66667	0.26325	0.73675

(a) Compute an estimate of $Var[\hat{S}(41)]$ and construct an 95% nonparametric confidence interval for $S(41)$ based on $Z_{\ln[-\ln \hat{S}(t)]} \dot{\sim} NOR(0,1)$.

(b) Compute an estimate of $Var[\hat{F}(52)]$ and construct an 95% nonparametric confidence interval for $F(52)$ based on $Z_{\text{logit}[\hat{F}(t)]} \dot{\sim} NOR(0,1)$.

(c) In Figure 2, we presented the Weibull probability plot for the bone marrow transplant data, what are the graphical estimates of the Weibull parameters η and β? (Note that the Weibull distribution has c.d.f. $F(t;\eta,\beta) = 1-\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], t>0, \beta>0, \eta>0.$)

(d) In Figure 3, we presented the lognormal probability plot for the bone marrow transplant data, what are the graphical estimates of the lognormal parameters μ and σ?

(e) Use the estimates in part (c) to compute parametric estimates of $S(41)$ and $F(52)$.

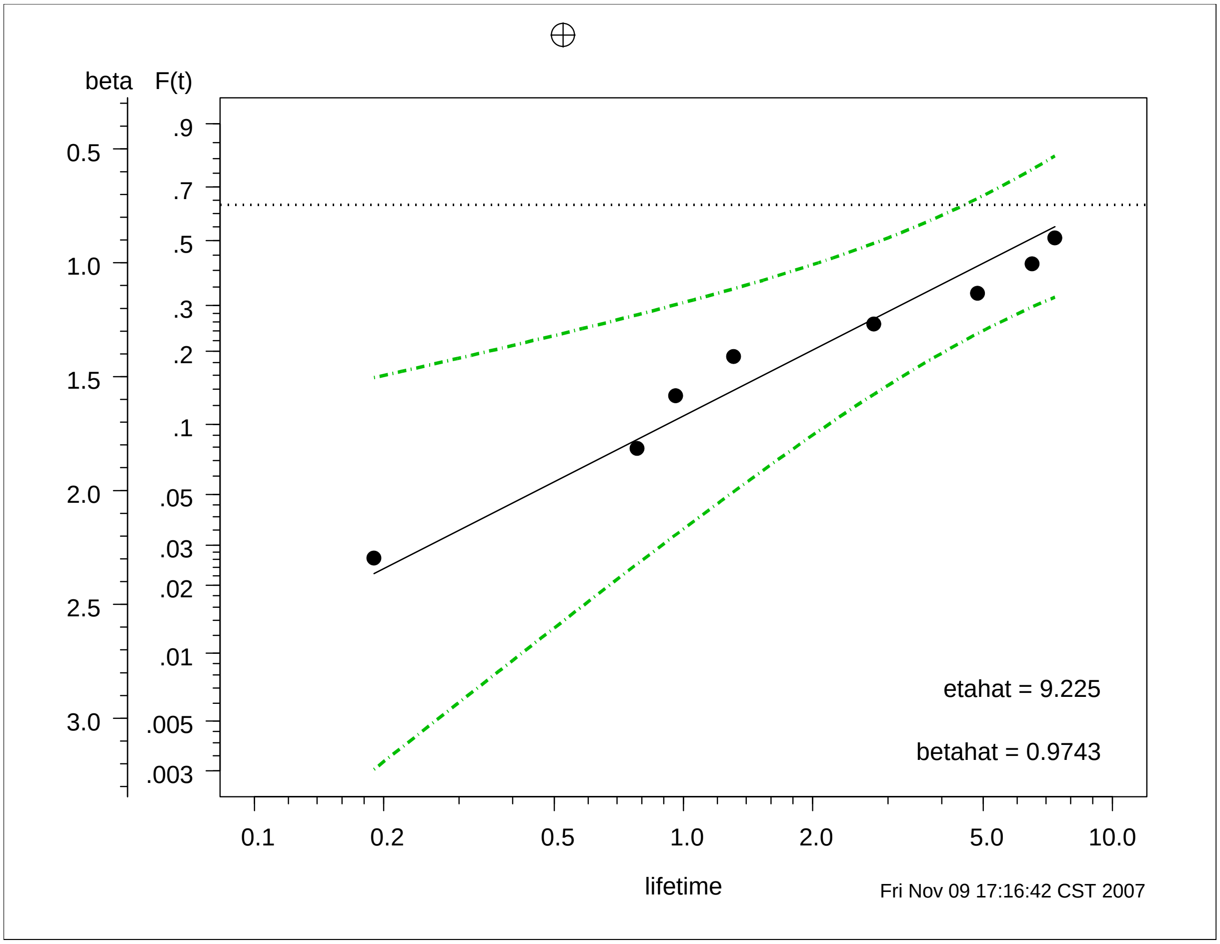

Figure 1. Weibull probability plot for Question 2(g)

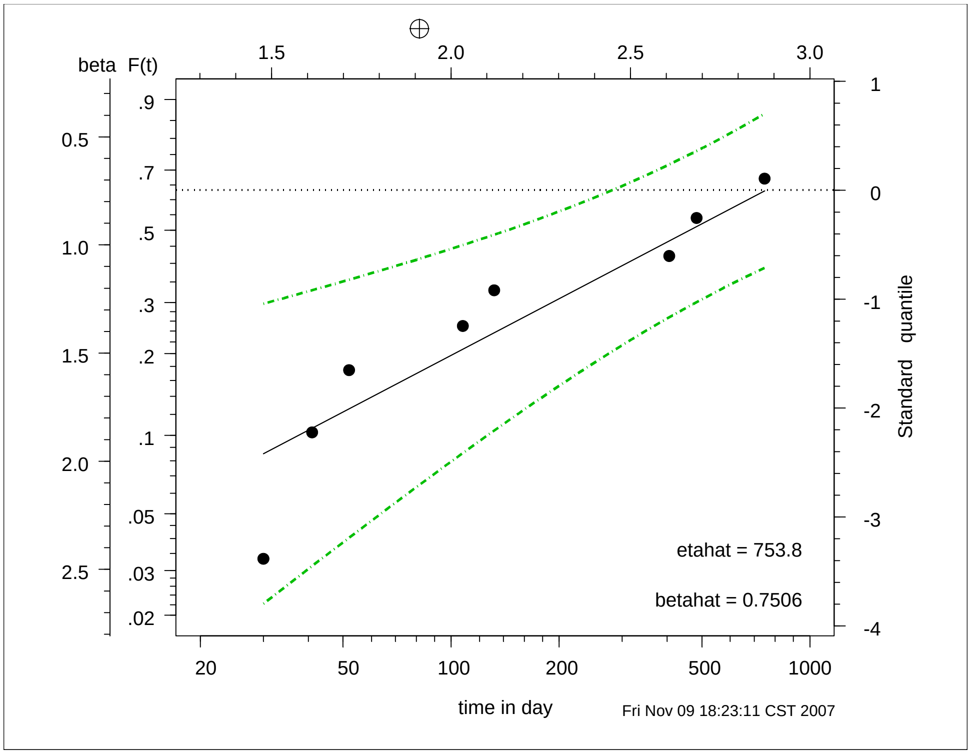

Figure 2. Weibull probability plot for the bone marrow transplant data.

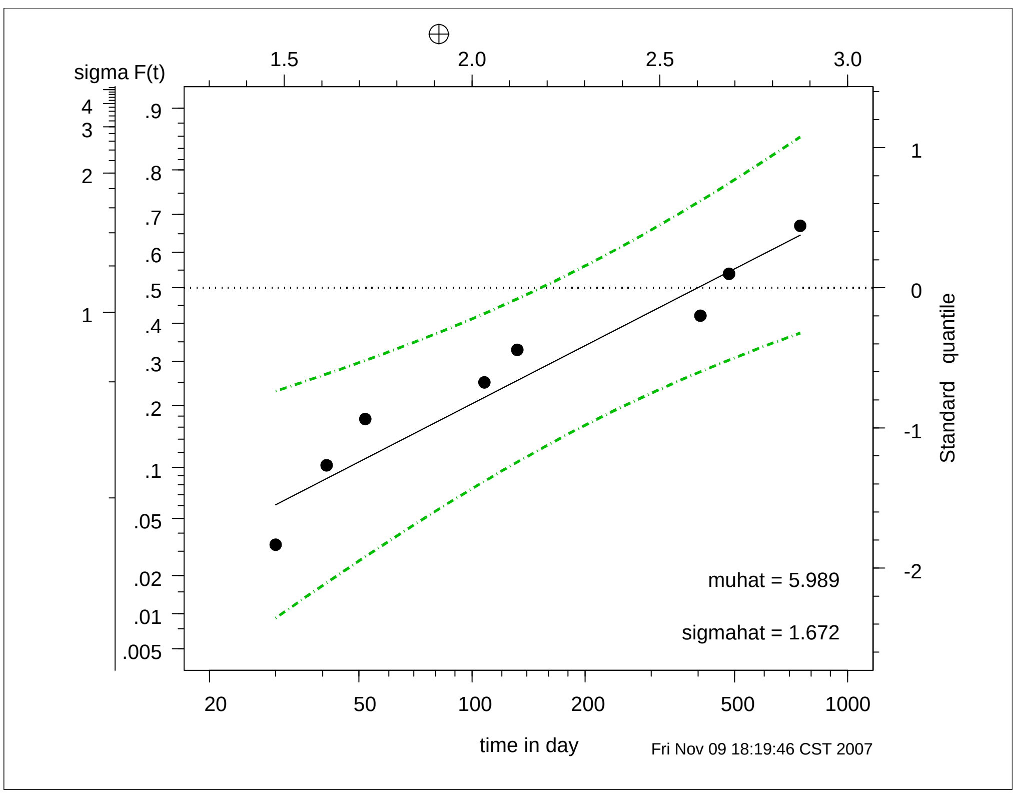

Figure 3. Lognormal probability plot for the bone marrow transplant data.