The Contribution of Economic Fundamentals to Movements in Exchange Rates

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Abstract
Starting from the asset pricing approach of Engel and West, we examine the degree to which fundamentals can explain exchange rate fluctuations. We show that it is not possible to obtain sharp inferences about the relative contribution of fundamentals using only data on observed monetary fundamentals--money minus output differentials across countries--and exchange rates. We use additional data on interest rate and price differentials along with the implications of the monetary model of exchange rates to decompose exchange rate fluctuations. In general, we find that money demand shifts, along with observed monetary fundamentals, are an important contributor to exchange rate fluctuations.
1. Introduction

The well-known paper by Meese and Rogoff (1983) showed that a simple random walk model for exchange rates can beat various time series and structural models in terms of out-of-sample forecasting performance. Although some of the subsequent literature on exchange rate predictability find evidence in favor of beating the random walk benchmark, most of those results do not hold up to scrutiny. The extant literature has found the linkage between the nominal exchange rate and fundamentals to be weak (Cheung, Chinn, and Pascual 2005; Sarno 2005). This weak linkage has become known as the “exchange rate disconnect puzzle”.

Engel and West (2005) took a new line of attack in this analysis and demonstrate that this so-called disconnect between fundamentals and nominal exchange rates can be reconciled within a rational expectations model. The Engel and West (2005) model implies that the exchange rate is the present discounted value of expected economic fundamentals. Specifically,

\[
s_t = f_t + E_t \left[ \sum_{j=1}^{\infty} \psi^j \cdot \Delta f_{t+j} \right] + R_t, \tag{1.1}
\]

where \( s_t \) is the spot exchange rate, \( f_t \) is the current value of observed fundamentals (for example money growth and output growth differentials), and \( \psi \) is the discount factor. The term \( R_t \) includes current and expected future values of unobserved fundamentals (risk premia, money demand shocks, etc) as well as perhaps “nonfundamental” determinants of exchange rate movements.

The “exchange rate disconnect puzzle” reflects the fact that fluctuations in \( s_t - f_t \) can be “large” and persistent, while the promise of the present value approach is that this disconnect can be explained by the expectations of future fundamentals. The potential empirical success of the Engel and West model hinges on two major assumptions. First, fundamentals are non-stationary.
Second, the factor used to discount future fundamentals is “large” (between 0.9 and unity). Nonstationary fundamentals impart nonstationarity to exchange rates while a large discount factor gives greater weight to expectations of future fundamentals relative to current fundamentals. As a result, current fundamentals are only weakly related to exchange rates as exchange rates appear to follow an approximate random walk. The first assumption of nonstationary fundamentals has been supported by empirical work (Engel and West, 2005; Engel, Mark, and West, 2007), however, only recently has there been direct evidence in support for the second assumption of a large discount factor (Sarno and Sojli, 2009).

The key research question that still remains is to what extent can expectations of future fundamentals explain exchange rate movements? The challenge in evaluating the present value model is that not only the expected future fundamentals are not observed but other economic fundamentals, i.e. the $R_t^i$ in equation (1.1), are also not observed. Indeed, Engel and West (2005) acknowledge that the kind of decompositions based on forecasting observed fundamentals such as those applied to stock prices (see Campbell and Shiller (1988)) is made difficult by the presence of unobserved fundamentals.¹

In this paper, we use a simple monetary model of exchange rates to specify explicitly the relationship between economic fundamentals and exchange rates. To sharpen our focus on expectations about future fundamentals, we use a state-space model to conveniently model the relationship between observed fundamentals and the unobserved predictable components of fundamentals. We integrate the state-space model into the present value model of the exchange

¹ An active literature has developed based on the insights in the Engel and West (2005). Several papers try to infer the importance of unobserved fundamentals in high frequency data by using on information order flows (Evans (2010) and Rime, Sarno, and Sojli (2010)). Nason and Rogers (2008) generalize the Engel and West present value model of the exchange rate to a dynamic stochastic general equilibrium model and estimate a resulting unobserved components model using quarterly US and Canada data.
rate to show the links between the predictable component of fundamentals and exchange rate fluctuations. We use annual data on the pound to the dollar exchange rate, money, output, prices, and interest rates for the UK and US from 1880 to 2010. The directly observed fundamental in our model is money supply differentials between the UK and US minus output differentials between the UK and the US. This variable has been the primary focus of the literature’s examination of fundamentals’ contribution to exchange rate movements. For example, Mark (1995) evaluates the ability of this variable to forecast the future exchange rate movements for a set of countries including the United States, Canada, Germany, Japan, and Switzerland since the end of the Bretton Woods regime. Rapach and Wohar (2002) construct this variable for 14 industrial countries covering a period of more than a century and study the cointegration relationship between the exchange rates and the fundamentals. Mark and Sul (2001) further demonstrate that the panel data techniques are able to find more evidence of predictability of this variable to the future exchange rate movements. More recently, Cerra and Saxena (2010) conduct a comprehensive study of a very large dataset consisting of 98 countries and find more evidence that this fundamental variable helps to forecast the future exchange rate movements.

We show that a state-space model using only two observables—money supply minus output differential and the exchange rate—has difficulty inferring the relative importance of expected future fundamentals. Using Bayesian model averaging across different specifications of the state-space model, we show that the posterior distribution of the contribution of observed fundamentals to the variance of exchange rates is bimodal, with roughly equal weight placed on close to a zero contribution and on close to a 100% contribution. The reason for this great uncertainty about the relative contributions of observed fundamentals is that in the data the
predictable component of changes in observed fundamentals is relatively small compared to the unpredictable component--most of the information about future fundamentals is contained in exchange rates rather than observable fundamentals. This makes identification of the separate contribution of expectations of future observed fundamentals problematic.

To solve this identification problem, we bring additional information to bear on the analysis which helps to move us away from the polar cases of 0% and 100% variance decomposition. First we incorporate data on interest rate and price differentials by including two additional observation equations in our state-space model—one corresponding to a relative money demand equation and the other to deviations from covered interest parity. These additional observation equations provide information about previously unobserved fundamentals—money demand shifters and uncovered interest parity risk premium. Another source of information is prior information about key parameters in the state-space model. Specifically, prior information about the half-life of deviations from purchasing power parity help to identify expected future deviations from purchasing power parity while prior information about the interest semi-elasticity of money demand helps determines the value of the discount factor. These additional sources of information can result in sharper inference about the relative contribution of the various fundamentals. Specifically, we find that monetary fundamentals, especially money demand shifters, explain the bulk of exchange rate movements. Fluctuations in the risk premium play a lesser role.

Our findings have important implications for exchange rate models that relate the exchange rate fluctuations to the economic fundamentals such as the output and monetary factors. Our results indicate that these economic fundamentals, either directly observed or indirectly inferred, contribute to exchange rate movements in a substantial way. The large
literature that finds it difficult for economic models to produce a better out-of-sample forecast for the exchange rate than a random walk may simply be due to the predictable component of fundamentals being small relative to the unpredictable component. That is, a simple regression cannot detect the small signals buried under the volatile noise.

The rest of the paper is organized as follows. In section 2, we outline the simple monetary model of exchange rates used by Engel and West (2005) to show how the spot exchange rate can be written as a function of expectations of future fundamentals, some of which are observed and some of which are unobserved. In section 3, we develop a state-space model to describe the dynamics of the predictable component of observed fundamentals and embed it in the simple rational expectations monetary model of exchange rates. In section 4, we demonstrate using Bayesian model averaging that there is substantial uncertainty about the quantitative contribution of observed fundamentals to exchange rate movements. In section 5, we use additional information to obtain tighter inferences about relative contributions of observed and unobserved fundamentals. Section 6 conducts a sensitivity analysis to alternative model specifications and choice of priors. In Section 7, we provide additional evidence that the factor that is a major contributor to exchange rate movements is truly associated with money demand shifters. Section 8 concludes.

2. The Monetary Exchange Rate Model

We start with the classical monetary model as below (all variables are in logarithm except for the interest rates, and asterisk denotes foreign variable):

\[ m_t - p_t = \phi y_t - \lambda i_t + v_t^{md} \] (2.1)

\[ m_t^* - p_t^* = \phi y_t^* - \lambda i_t^* + v_t^{md} \] (2.2)
The variable definitions used in our model are the natural log of money supply \( (m) \), natural log of price level \( (p) \), and nominal interest rate \( (i) \). Variables with asterisk represent the foreign country\(^2\). The terms \( y^{md}_t \) and \( y^{*md}_t \) represent unobserved variables that shift money demand.

We integrate into the model a generalized Uncovered Interest Parity (UIP) condition that allows for a time-varying risk premium, \( r^{ui}_t \), since in general the UIP fails and the equilibrium model would imply a non-trivial risk premium (see Engel (1996)):

\[
i_t - i^{*}_t = E_t s_{t+1} - s_t + r^{ui}_t, \tag{2.3}
\]

where \( s_t \) is the natural log of the exchange rate. To complete model, we add the Purchasing Power Parity (PPP) relationship:

\[
s_t = p_t - p^{*}_t + r^{ppp}_t. \tag{2.4}
\]

Since the PPP in general only holds in the long run (Rogoff (1996)), the variable \( r^{ppp}_t \) picks up these deviations from PPP.

Combining equations (2.1) through (2.4), one can derive a stochastic difference equation that describes how the exchange rate would depend on observed monetary fundamentals and an unobserved remainder. The algebra can be manipulated so as to express the exchange change rate determination in terms of its deviation from observed fundamentals, similar to the stock price decomposition by Campbell and Shiller (1988):

\[
s_t - f_t = \psi \cdot E_t[s_{t+1} - f_{t+1}] + \psi \cdot E_t[\Delta f_{t+1}] + R_t, \tag{2.5}
\]

where, \( f_t \equiv (m_t - m^{*}_t) - \phi(y_t - y^{*}_t) \) is the observed monetary fundamental, and \( \psi = \frac{\lambda}{1 + \lambda} \) is the so-called discount factor.\(^3\) In the following estimation exercise, we set \( \phi = 1 \) as in Rapach and

\(^2\) We treat UK as the home country. The exchange rate is quoted as pounds per dollar.
Wohar (2002) and Mark (1995). As we have data on money and output, \( f_t \) is typically observable. The unobserved term, \( R_t \), consists of the unobserved money demand shifter as well as deviations from both uncovered interest rate parity and purchasing power parity:

\[
R_t = \psi r_t^{uip} + (1 - \psi) r_t^{ppp} - (1 - \psi) r_t^{md} \quad \text{where} \quad r_t^{md} = v_t^{md} - v_t^{md*}.
\]

Engel and West (2005, 2006) show that this asset price formulation for the exchange rates is very general and can be derived from a variety of monetary policy models including the Taylor rule and may include more fundamental information under that type of monetary policy rule.

We can iterate eq. (2.5) forward. Under the assumption of no explosive solution, the model can be solved as below:

\[
s_t - f_t = E_t \left[ \sum_{j=0}^{\infty} \psi^j \cdot \Delta f_{t+j} \right] + E_t \left[ \sum_{j=0}^{\infty} \psi^j \cdot R_{t+j} \right],
\]

where, \( s_t - f_t \) is the deviation of the current exchange rate from its current observed monetary fundamental. Eq. (2.6) is similar to the present discounted value formula for the exchange rate derived in Engel and West (2005, 2006), and this equation states that any deviation of current exchange rate from its observed fundamentals should reflect the variation of the present discounted value of agent’s expected future economic fundamentals.

### 3. Decomposing the contribution of observed fundamentals and unobserved shocks

One obstacle in evaluating the above exchange rate model is that what matters in explaining current deviation of exchange rate from its fundamentals are agent’s expectations of future fundamentals but these expectations are not directly observable. The state-space model

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3 Note here that it is the UIP equation that links interest rate differentials to the exchange rate change and hence gives the result that the discount factor is a function of the interest semi-elasticity of money demand.
offers a convenient framework in which we can model the expectations as latent factors and allow them to have flexible dynamics. In doing this, we can extract agent’s expectations using the Kalman filter and decompose the current deviation of exchange rate from its fundamentals into the contributions of expectations of future observed fundamentals and current and future unobserved remainder. Furthermore, instead of assuming that the discount factor is close to unity as in Engel and West (2005, 2006), we can directly estimate the discount factor and provide further statistical evidence for the Engel and West model.

Given equation (2.6), the exchange rate relative to current observed fundamentals, 

\[ s_t - f_t, \]

depends on expectations of \( \Delta f_{t+i} \) and expectations of \( r_{t+i} = \frac{R_{t+i}}{\psi} \). Denote the expectations by \( E_t[\Delta f_{t+i}] = g_t \) and \( E_t[r_{t+i}] = \mu_t \). The realized variables are just the sum of their conditional expectation and a realized unforecastable shock:

\[
\Delta f_t = g_{t-1} + \varepsilon_t^f, \tag{3.1}
\]

\[
r_t = \mu_{t-1} + \varepsilon_t^r. \tag{3.2}
\]

The realized shocks (or forecast errors) \( \varepsilon_t^f \) and \( \varepsilon_t^r \) are white noise. At the same time, the predictable components of \( \Delta f_t \) and \( r_t \) follow autoregressive processes:

\[
(1 - \phi_g(L)L)g_t = \varepsilon_t^g, \tag{3.3}
\]

\[
(1 - \phi_\mu(L)L)\mu_t = \varepsilon_t^\mu, \tag{3.4}
\]

where \( \varepsilon_t^g \) and \( \varepsilon_t^\mu \) are expectation (or “news”) shocks and lag polynomials \( \phi_g(L) = \sum_{i=1}^{k} \phi_{g,i} L^{-i} \)

and \( \phi_\mu(L) = \sum_{i=1}^{k} \phi_{\mu,i} L^{-i} \) describe the dynamics of the predictable components. The four shocks

\[
[\varepsilon_t^g \varepsilon_t^\mu \varepsilon_t^f \varepsilon_t^r]
\]

are allowed to be contemporaneously correlated but they are serially
uncorrelated. Note the implied univariate time series models for $\Delta f_i$ and $r_i$ are just ARMA(k,max(1,k)) models, where k is the number of autoregressive terms in $\phi_k(L)$ and $\phi_\mu(L)$. Recall that while economic agents know the values of $g_i$ and $\mu_i$, the econometrician does not and must infer them from observable variables.

Using equation (2.6) and evaluating expectations, we can write the exchange rate relative to observed current monetary fundamental as:

$$r_{t+1} - f_{t+1} = B_1(L)g_t + B_2(L)\mu_t + \psi\mu_{t-1} + \psi\varepsilon_t^r,$$  \hspace{1cm} (3.5)

where $E_t\left[\sum_{j=1}^\infty \psi^j \cdot \Delta f_{t+j}\right] = B_1(L)g_t$, $E_t\left[\sum_{j=1}^\infty \psi^{j+1} \cdot r_{t+j}\right] = B_2(L)\mu_t$, and $\psi r_t = \psi\mu_{t-1} + \psi\varepsilon_t^r$. The contribution of expectations of observed fundamentals, $E_t\left[\sum_{j=1}^\infty \psi^j \cdot \Delta f_{t+j}\right]$, will in fact depend on current (and possibly lagged) values of the unobserved component $g_i$. The coefficients of the lag polynomials, $B_1(L)$ and $B_2(L)$, depend on the values of $\psi$, $\phi_k(L)$, and $\phi_\mu(L)$. In general, the larger the value of $\psi$ and the more persistent is $g_i$, the larger are the coefficients of $B_1(L)$.

Given the observed monetary fundamentals, $\Delta f_i$, we can write the model in a state-space form with the measurement equation:

$$\begin{bmatrix} \Delta f_t \\ s_t - f_t \end{bmatrix} = \begin{bmatrix} L & 0 & 1 & 0 \\ 0 & \mu_i & 0 & 0 \end{bmatrix} \begin{bmatrix} g_t \\ \mu_t \\ \varepsilon_t^r \\ \varepsilon_t^r \end{bmatrix},$$  \hspace{1cm} (3.6)

and the transition equation:
Given the parameters of the state-space model, we can determine the relative contribution of monetary fundamentals through the effect of \( g_t \) on \( s_t - f_t \) in equation (3.6).

As none of the state variables is directly observed and must be inferred from the two observed time series, identification of the state-space model will depend on the dynamics of the observed time series and its variance/covariance matrix.\(^4\) Balke and Wohar (2002) and Ma and Wohar (2012) show that inference about the relative contribution of the unobserved components may be very weak if the observed fundamental does not provide a lot of direct information about the relative size of the predictable component of fundamentals. For example, if the variance of innovations to the predictable components, \( \sigma_g^2 \), is small relative to the variance of the unpredictable component, \( \sigma_f^2 \), then observations of \( \Delta f_t \) provide very little information about \( g_t \), leaving only \( s_t - f_t \) to infer both \( g_t \) and \( \mu_t \).\(^5\)

One can see this by rewriting the state-space model as a VARMA:

\[
\begin{bmatrix}
1 - \phi_g(L) & 0 & 0 & 0 \\
0 & 1 - \phi_g(L) & 0 & 0 \\
0 & 0 & 1 - \phi_g(L) & 0 \\
0 & 0 & 0 & 1 - \phi_g(L)
\end{bmatrix}
\begin{bmatrix}
\Delta f_t \\
\mu_t \\
\varepsilon_t^f \\
\varepsilon_t^g
\end{bmatrix}
= \begin{bmatrix}
\varepsilon_t^\mu \\
\varepsilon_t^f \\
\varepsilon_t^f \\
\varepsilon_t^g
\end{bmatrix}
\]

(3.7)

\[
\begin{align*}
1 - \phi_g(L)L & \quad 0 \\
0 & \quad (1 - \phi_g(L)L)(1 - \phi_{\mu}(L)L)
\end{align*}
\begin{bmatrix}
\Delta f_t \\
s_t - f_t
\end{bmatrix}
=
\begin{align*}
L & \quad 0 \\
B_1(L)(1 - \phi_{\mu}(L)L) & \quad (B_2(L) + \psi)(1 - \phi_g(L)L)
\end{align*}
\begin{bmatrix}
\varepsilon_t^\mu \\
\varepsilon_t^f \\
\varepsilon_t^f \\
\varepsilon_t^g
\end{bmatrix}
\]

(3.8)

\[\text{See the Supplementary Appendix for a more detailed discussion of identification of the above state-space model.}\]

\[\text{Ma and Nelson (2012) show that for a large class of models that a small signal-to-noise ratio indicates that the model is weakly identified and, as a result, the uncertainty about the parameter estimates will be large.}\]
If \( \sigma^2_g / \sigma^2_f \) is “small”, then observations of \( \Delta f \) are not sufficient to identify \( \phi_g(L) \)--this polynomial is nearly canceled out in the \( \Delta f \) equation. As there could be numerous combinations of \( \phi_g(L) \) and \( \phi_{\mu}(L) \) that would yield the same autoregressive dynamics for \( s_i - f_i \), whether there is sufficient information to identify \( \phi_g(L) \) and \( \phi_{\mu}(L) \) would depend on the moving average dynamics of \( s_i - f_i \).

Taking annual observations of the nominal exchange rate, relative money supply, and relative real GDP between the US and UK and UK and US interest rates from 1880 to 2010 (see Supplementary Appendix for details of data construction), the top panel of Figure 1 plots the log UK-US exchange rate, \( s \), and the log level of the observed monetary fundamental, \( f \), while the lower panel of Figure 1 plots the realized fundamentals growth (\( \Delta f \)) along with the deviation of current exchange rate from the observed monetary fundamentals (\( s - f \)). In the data, \( s - f \) is quite persistent and volatile while the realized fundamentals growth is much less persistent and less volatile. Figure 1 suggests that the persistent component \( g \) is likely to be “small” relative to \( \Delta f \), which in turn implies that it may be difficult to separately identify \( g \) and \( \mu \) from data on \( s - f \) alone.

4. Implications of weak identification for exchange rate decompositions.

As suggested above, most of the information about the predictable component of the observed monetary fundamentals might actually be in the exchange rate, \( s - f \), rather than in observed monetary fundamentals growth itself, \( \Delta f \). This suggests that the model is weakly identified as essentially a single data series (\( s - f \)) is used to identify two components (\( g \) and
To demonstrate the extent to which this identification problem holds in practice, we consider five alternative, non-nested models. Each of these five models gives rise to an ARMA(4,4) model for \( s_t - f_t \) but will imply very different exchange rate decompositions. Specifically, we consider the following AR specifications for \( \phi_s(L)L \) and \( \phi_\mu(L)L \) respectively:

(i) AR(2) and AR(2), (ii) AR(4) and AR(0), (iii) AR(3) and AR(1), (iv) AR(1) and AR(3), (v) AR(0) and AR(4). For all five models, we assume that the innovations in the four components \( [\varepsilon_i^s \varepsilon_i^\mu \varepsilon_i^f \varepsilon_i^\mu]' \) are correlated with one another.

We take a Bayesian approach to account for uncertainty about the specification of the underlying state-space model. To evaluate these alternative models, we first estimate the posterior distribution of the parameters for each of the five competing models. The posterior distribution of the parameters given the data and a particular model, we will denote as

\[ P(\theta_m | Y, M = m) \],

where \( M \) is the set of models, \( h(m) \) is the prior probability of model \( m \),

\[ B(m) = \int L(Y_T, \theta_m) h(\theta_m) \, d\theta_m \],

\( L(Y_T, \theta_m) \) is the likelihood, and \( h(\theta_m) \) is the prior density of the parameters. As we are interested in determining the relative contribution of fundamentals to exchange rate decomposition we calculate the variance decomposition, \( V(\theta_m, m) \), implied by a given model \( m \) and parameter vector, \( \theta_m \). Given the posterior distribution of the parameters, we can obtain the posterior distribution of the variance decomposition for a given model,
Finally, we can employ Bayesian model averaging to account for model uncertainty on the posterior distribution of the variance decomposition.

Given the nonlinear (in parameters) structure of the model, there is no closed form solution for the posterior distribution given standard priors; therefore, we use a Metropolis-Hastings Markov Chain Monte Carlo (MH-MCMC) to approximate the posterior distribution of the parameters given the model (see the Supplemental Appendix for details). We can easily construct the posterior distribution of the variance decomposition from the results of the MH-MCMC as well the posterior probabilities for the five models given the data. In this section, we consider the case of very diffuse priors so that the likelihood function is the principal determinant of the posterior distribution.\(^6\) The posterior distribution is based on 500,000 draws from the MH-MCMC after a burn-in period of 500,000 draws.

To save space we do not present all the histograms of the posterior distribution of parameters (these are available upon request). Figure 2 displays for all five models the posterior distribution of the contributions of the predictable component of the fundamentals, \(g\), to the variance of deviations of the exchange rate from observed monetary fundamentals, \(s_f - s\). For the Model 1, the posterior distribution of the observed fundamental’s contribution to exchange rate variance is concentrated around 100 percent but does show a small secondary mode close to zero. Model 2, in which \(\phi_g(L)\) is an AR(4) and \(\phi_f(L)\) is an AR(0), also implies that the observed fundamentals explains nearly all the variance of exchange rates. Models 4 and 5, on

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\(^6\) Formally, for each of the models the individual autoregressive parameters have a prior distribution of joint truncated normal \(N(0,100)\), the prior distribution \(\psi\) is \(U(0,1)\), the variances of innovations in Eq. (3.7) are distributed \(U(0,1000)\) while the co-variances of innovations in Eq. (3.7) are distributed \(U(-1000,1000)\). Draws in autoregressive parameters that imply nonstationarity are rejected as are draws where the variance-covariance matrix of innovations in Eq. (3.7) is not positive definite. These prior distributions ensure that for this model and data, the acceptance in the Metropolis-Hastings sampler depends only on the likelihoods; thus, when comparing models the likelihoods are going to be decisive.
the other hand, imply that observed monetary fundamentals explain almost none of the variance of exchange rates ($\phi_g(L)$ is an AR(1) and $\phi_\mu(L)$ is an AR(3) for Model 4 and AR(0) and AR(4), respectively, for Model 5). Model 3 ($\phi_g(L)$ is an AR(3) and $\phi_\mu(L)$ is an AR(1)), suggests a bimodal distribution with probability mass concentrated on the extremes.

Which of the five models does the data prefer? From the MCMC posterior distribution, we can construct the posterior distribution of the log likelihoods, $log(L(Y_T, \theta_m))$, for each model. While Model 1 has the highest posterior probability, it is not substantially higher than Model 5 and even Models 2 through 4 have non-negligible posterior probabilities. In fact, the cumulative distribution of the posterior distribution of log likelihoods for the five models (not shown to conserve space) are quite close to one another and benchmark model does not stochastically dominate all the other models.

In the lower right panel of Figure 2 we plot the histogram for the observed monetary fundamental’s variance decomposition of $s_t - f_t$ once we account for model uncertainty. Here the variance decomposition for each model is weighted by its posterior probability. Taking into account of model uncertainty suggests a bimodal distribution for the contribution of observed monetary fundamentals on the variance of $s_t - f_t$, with the probability mass concentrated on either a zero contribution or 100 percent contribution. Using data on exchange rates and observed monetary fundamentals alone is not sufficient to determine to what extent exchange rates are driven by monetary fundamentals.

5. Incorporating additional information to the analysis

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7 The posterior probabilities of the five models are 0.38, 0.09, 0.11, 0.15, and 0.27, respectively.
8 The figure that plots all five cumulative distributions of the posterior distribution is available upon request.
As we demonstrated above, the basic model that only includes the exchange rate and the observed monetary fundamentals as observables does not generate sharp inferences about the relative contribution of observed fundamentals to exchange rate fluctuations. In other words, the two data series $s_t - f_t$ and $\Delta f_t$ do not have sufficient information to pin down the relative contribution of the fundamentals. In principle, to resolve this issue one has to provide more information. In this section, we use a combination of additional data, the insights implied by the simple monetary model, and prior information about key parameters to sharpen inferences about the sources of exchange rate fluctuations. In particular, the information we bring to bear is about the remainder term, $R_t$, rather than directly about $\Delta f_t$.

5.1. An expanded decomposition of exchange rates

The monetary model discussed in section 2 suggests that the we can break the remainder into its constituent parts: an unobserved money demand shifter where $r_t^{md} = v_t^{md} - v_t^{md^*}$, deviations from uncovered interest parity $r_t^{up}$, and deviations from purchasing power parity $r_t^{ppp}$. The monetary model suggests that these along with observed monetary fundamentals will in turn help determine exchange rate movements. The monetary model also suggests that price differentials ($p_t - p_t^*$) and interest rate differentials are related to $r_t^{md}$, $r_t^{up}$, and $r_t^{ppp}$.

Using notation similar to that in section 3, we assume each of the four components of exchange rates consists of a predictable and unpredictable component:

$$\Delta f_t = g_{t-1} + \varepsilon_{t}^{f}$$

$$r_t^{ppp} = \mu_{t-1}^{ppp} + \varepsilon_{t}^{ppp}$$

$$r_t^{up} = \mu_{t-1}^{up} + \varepsilon_{t}^{up}$$

$$r_t^{md} = v_t^{md} - v_t^{md^*} = (1 - \phi_y(L) L) g_t = \varepsilon_{t}^{g}$$

$$r_t^{ppp} = (1 - \phi_{ppp}(L)L) \mu_t^{ppp} = \varepsilon_{t}^{r_{ppp}}$$

$$r_t^{up} = (1 - \phi_{up}(L)L) \mu_t^{up} = \varepsilon_{t}^{r_{up}}$$

$$r_t^{md} = v_t^{md} - v_t^{md^*} = (1 - \phi_y(L) L) g_t = \varepsilon_{t}^{g}$$

$$r_t^{ppp} = (1 - \phi_{ppp}(L)L) \mu_t^{ppp} = \varepsilon_{t}^{r_{ppp}}$$

$$r_t^{up} = (1 - \phi_{up}(L)L) \mu_t^{up} = \varepsilon_{t}^{r_{up}}$$

$$(5.1)$$

$$(5.2)$$

$$(5.3)$$
\[ r^\text{md}_t = \mu^\text{md}_{t+1} + \varepsilon^\text{md}_t \quad (1 - \phi^\text{md}_L)\mu^\text{md}_t = \varepsilon^\mu^\text{md}_t \]  

(5.4)

We specify \( \phi^\text{md}(L)L, \phi^\text{ppp}(L)L, \phi^\text{w}(L)L, \phi^\text{md}(L)L \) to be AR(1)'s to keep the model parsimonious and also to keep a similar ARMA(4,4) structure for the reduced form for \( s_t - f_t \) that was present in the simple model of section 3. The state vector is:

\[ S_t = \begin{bmatrix} g_t & g_{t-1} & \mu^\text{w,ppp}_t & \mu^\text{w,ppp}_{t-1} & \mu^\text{w,ppp}_t & \mu^\text{w,ppp}_{t-1} & \varepsilon^f_t & \varepsilon^\text{w,ppp}_t & \varepsilon^\text{w,ppp}_t & \varepsilon^\text{w,ppp}_t \end{bmatrix} \]

and the transition equation is

\[ S_t = FS_{t-1} + V_t \]  

(5.5)

where

\[ V_t = \begin{bmatrix} \varepsilon^s_t & 0 & \varepsilon^\text{w,ppp}_t & 0 & \varepsilon^f_t & \varepsilon^\text{w,ppp}_t & \varepsilon^\text{w,ppp}_t & \varepsilon^\text{w,ppp}_t \end{bmatrix}. \]

We denote the variance-covariance matrix of \( V_t \) by \( Q \). The Supplemental Appendix describes the \( F \) matrix in the state equation in detail.

We use data on interest rate and price level differentials to construct two additional observations equations. Our observation vector now consists of four variables: relative velocities or relative money demand in the two countries \( (m_t - y_t - p_t - (m^*_t - y^*_t - p^*_t) = f_t - (p_t - p^*_t)) \), interest rate differentials \( (i_t - i^*_t) \), along with the growth rate in observed monetary fundamentals \( (\Delta f_t) \), and the exchange rate relative to current observed monetary fundamental \( (s_t - f_t) \).

Using the simple monetary model in section 2, the relative money demand equation is given by:

\[ f_t - (p_t - p^*_t) = -\frac{\mu}{\gamma^\text{w}}(i_t - i^*_t) + r^\text{md}_t. \]  

(5.6)

The uncovered interest rate condition implies:

\[ i_t - i^*_t = E_t(s_{t+1} - f_{t+1}) + E_t\Delta f_{t+1} - (s_t - f_t) + r^\text{w,pp}_t. \]  

(5.7)
Recall that the growth rate of observed monetary fundamentals is

$$\Delta f_t = g_{t-1} + \epsilon^f_t,$$

(5.8)

and the exchange rate equation is given by

$$s_t - f_t = E_s \left[ \sum_{j=1}^{\infty} \psi^j \cdot \Delta f_{t+j} \right] + E_s \left[ \sum_{j=0}^{\infty} \psi^{j+1} \cdot r_{t+j} \right],$$

(5.9)

where \( \psi \cdot r_t = \psi r_t^{up} + (1 - \psi) r_t^{ppp} - (1 - \psi) r_t^{md} \). We do not include deviations from PPP in the observation vector as it is just a linear combination of two of the other observation variables:

\( f_t - (p_t - p_t^*) \) and \( s_t - f_t \).

Taking expectations and writing equations (5.6)-(5.9) in terms of the state variables, \( S_t \), yields the following measurement equation:

$$
\left[
\begin{array}{c}
\Delta f_t \\
\Delta i_t \\
s_t - f_t
\end{array}
\right] = \left[
\begin{array}{c}
-H_{sf} H_{sf} + H_{sf} F - H_{up} H_{up} + H_{md} \\
-H_{sf} H_{sf} + H_{up} F - H_{up} H_{up} + H_{md} \\
H_{sf} H_{sf} + H_{md}
\end{array}
\right] S_t,
$$

(5.10)

where the loading matrices \( H \)'s are described in detail in the Supplemental Appendix. Equations (5.5) and (5.10) describe the state-space model which includes observations on relative money demand, interest rate differentials, growth rate of observed monetary fundamentals, and the exchange rate minus fundamentals.

We also can bring prior information to bear on the estimation of some of the key model parameters. While we do not use observations on PPP deviations, there is, however, a large literature on PPP deviations that we can draw upon to provide information about \( \mu_t^{ppp} \). We assume a prior distribution for \( \phi^{ppp} \) (see equation 5.2) so that the half-life for PPP deviations is
similar to that found in the literature (see Rogoff, 1996). Specifically, we assume a Beta(10,2) distribution for the prior distribution of \( \phi_{ppp} \).

One of the other key parameters in the model is the discount factor: \( \psi = \frac{\lambda}{1+\lambda} \) where \( \lambda \) is the interest semi-elasticity of money demand. As there is a large literature on the estimation of money demand, we use this literature to help determine a prior distribution for \( \lambda \). Specifically, we set the prior distribution of \( \lambda \) to be a Gamma with mode equal to 10 and standard deviation of 8.66.\(^9\) We chose this prior based on studies of long-run money demand that typically estimate the semi-elasticity of money to be in the range from around 5 to 20.\(^{10}\) Bilson (1978) estimates \( \lambda \) to be 15 in the monetary model, whereas Frankel (1979) finds \( \lambda \) to be equal to 7.25 while Stock and Watson (1993, 802, table 2, panel I) give a value of \( \lambda \) around 10. A more recent study by Haug and Tam (2007) suggest values ranging from around 10 to 20.\(^{11}\)

### 5.2. Estimation Results

Figure 3 displays posterior distributions of \( \lambda \) and \( \psi \) and contrasts them with their prior distributions. As before we use a Metropolis-Hastings MCMC to draw 500,000 draws from the posterior distribution (with a burn-in sample of 500,000 draws). The posterior distribution for \( \lambda \) suggests a semi-elasticity of around 17, somewhat higher than the mode of the prior distribution, but the posterior distribution is substantially tighter than the prior distribution. The posterior distribution for the discount factor, \( \psi \), suggests a value around .95 not far from the

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\(^9\) Formally, we set prior distribution for \( \lambda \) to be a Gamma(3,5).

\(^{10}\) All of the variables are scaled up by 100 so that the appropriate scale for \( \lambda \) is around ten. See the discussion in Engel and West (2005).

\(^{11}\) Prior distributions on the other autoregressive parameters are fairly diffuse: \( N(0.5,(1.5)^2) \), truncated at -1 and 1. The variances in Q are distributed \( U(0,1000) \) while the co-variances in Q are distributed \( U(-1000,1000) \). Draws for which the Q matrix is not positive definite are rejected.
mode of the prior, but again substantially tighter. Figure 3 also displays posterior distributions of autoregressive parameters for the four predictable components: \( g_t, \mu_{uip}^t, \mu_{ppp}^t, \) and \( \mu_{md}^t \). The posterior distribution of the autoregressive parameters suggest that all the unobserved predictable components are fairly persistent\(^{12}\).

Figure 4 displays the posterior distribution of variance decomposition of \( (s_t - f_t)\).\(^{13}\) We can see that the contributions of the risk premium \( (r_{uip}^t)\) and of deviations from PPP \( (r_{ppp}^t)\) are small. The greatest contribution comes from the predictable component of the money demand shifter, \( \mu_{md}^t \), while the contribution of predictable component of directly observed monetary fundamentals, \( g_t \), is more modest. Note that the contribution of the covariance between \( g_t \) and \( \mu_{md}^t \) tends to be negative. However, the posterior distribution for the joint contribution of the predictable components of monetary fundamentals, \( g_t \) and \( \mu_{md}^t \), is very large and centered closely around 100 percent. Thus, the exchange rate disconnect in our context appears to be due to fluctuations in money demand that are typically not accounted for in the so-called fundamentals.

Figure 5 displays historical decompositions of \( s_t - f_t \). The historical decompositions display the contribution of each of the states over the entire sample. The historical decompositions reported in Figure 5 are based on the 5\(^{th}\), 95\(^{th}\), and 50\(^{th}\) percentiles of the sample distribution. Note because the \( Q \) matrix (variance-covariance matrix in the transition equation) in the state-space model is not diagonal, the states are in general correlated with one another and the historical decomposition are, in general, not orthogonal.

\(^{12}\) Mark and Wu (1998) also find that the expected risk premium is persistent using a VAR to calculate expectations.

\(^{13}\) These values can be above 100% due to the fact that some of the covariances are negative.
The top left panel displays the historical decomposition of $s_t - f_t$ due to the monetary fundamentals, both the predictable component of $\Delta f_t$ and unobserved money demand shifters, $r_{it}^{md}$. The individual contributions of deviations from uncovered interest rate parity, $r_{it}^{uip}$, and deviations from PPP, $r_{it}^{ppp}$, are also presented in Figure 5. From Figure 5, one observes that most of movement in the UK/US exchange rate, especially in the later part of the sample, appear to be due to monetary fundamentals, and in particular to money demand shifters, $r_{it}^{md}$. The risk premium, $r_{it}^{uip}$, and deviations from PPP, $r_{it}^{ppp}$, while not as important as $r_{it}^{md}$ also contribute to exchange rate fluctuations. In particular, the results suggest that monetary factors ($f_t$ and $r_{it}^{md}$) explain the long swings of the exchange rate while deviations from parity conditions appear to explain some of the short-run movements in exchange rates. The lower right graph in Figure 5 shows the total contribution of all the factors to movements of $s_t - f_t$. This graph is a check to see that the contributions of the factors add up to the exchange rate itself.\(^\text{14}\)

In summary, an expanded four-observation variable state-space model where we model each of the components in the remainder term, $r_t$, suggests that the unobservable money demand shifters ($r_{it}^{md}$) explain a large fraction of the exchange rate fluctuations, with the contribution of $\Delta f_t$ being much smaller. This is, in fact, consistent with the previous literature that concludes that the connection between the observed monetary fundamentals, here $f_t$, and exchange rate is weak. Our interpretation is that it is actually unobserved fundamentals, specifically relative to the predictable component.

\(^{14}\) In the Supplementary Appendix to this paper, we present additional historical decompositions for the level of exchange rate ($s_t$) and for $\Delta f_t$. Factors underlying $f_t$ and $r_{it}^{md}$ explain nearly all of the low frequency movements in the exchange rate. On the other hand, the predictable component, $g_t$, explain only a small fraction of the fluctuations in $\Delta f_t$. This is consistent with the results in section 4, where the predictable component of $\Delta f_t$ was small relative to the unpredictable component.
money demand shifters, \( r^{md} \), that are responsible for a large fraction of exchange rate fluctuations.\(^{15}\)

**6. Sensitivity Analysis**

To examine whether the inferences derived in the previous section hold up to alternative choices about model specification and prior distributions, in this section we consider several changes in the benchmark model of section 5. This includes: changing the vector of observable variables, allowing diffuse priors for the discount factor, modeling money demand shifters as nonstationary, allowing the parameters in the state-space model to differ across fixed and flexible exchange rate regimes, and including an idiosyncratic factor in the exchange rate equation to capture the possibility of exchange rate fluctuations unrelated to fundamentals.

**6.1. Changing the vector of observables.**

In the benchmark model we presented and estimated in section 5.1 and 5.2, we included \( s_t - \bar{f}_t \) and \( f_t - (p_t - p_t^*) \) as observables along with \( i_t - i_t^* \) and \( \Delta f_t \). To be sure that the results are not driven by which variables are included as observables, we drop \( f_t - (p_t - p_t^*) \) from the observation vector and include deviations from PPP \( (s_t - (p_t - p_t^*)) \) as an observable.\(^{16}\)

We find that the results from this specification are qualitatively similar to the results from the benchmark model. The posterior distributions of the underlying parameters are similar and

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\(^{15}\) These results are reminiscent of West (1987) who shows that the volatility of deutschmark-dollar exchange rates can be reconciled with the volatility of fundamentals if one allows for shocks to money demand and deviations to PPP. Here we actually include observable information about money demand to help identify the magnitude of money demand shocks.

\(^{16}\) See the Supplemental Appendix for derivation of the state-space model that includes deviations from PPP as an observable.
more importantly, the variance decompositions for $s_t - f_t$ are similar across models (see Supplementary Appendix). Figure 6 presents the posterior distributions of the variance decomposition for the case where we replace $f_t - (p_t - p_{t*})$ with PPP deviations. Like the benchmark model, the variance in relative money demand are the single most important contributor to the variance of $s_t - f_t$, while the posterior distribution of the joint contribution of $\Delta f_t$ and $r_{md}$ is also highly concentrated around 100%. Compared to the benchmark model, fluctuations in the UIP risk premium are more important here, typically contributing more than the variance in the predictable component of $\Delta f_t$. In summary, using direct observations of deviations from PPP instead of observations on $f_t - (p_t - p_{t*})$ does not change the conclusion that fluctuations in money demand shifter is the single most important contributor to fluctuations in $s_t - f_t$.\footnote{We also considered the case where we include $f_t - (p_t - p_{t*})$ and $s_t - (p_t - p_{t*})$ as observables but do not include $s_t - f_t$ as an observable. For this case, the results were also qualitatively similar to the benchmark model with relative money demand shocks being the most important contributor to the variance of $s_t - f_t$ (see Supplemental Appendix).}

\section*{6.2 Diffuse priors}

To see if the results were sensitive to our priors about money demand, we consider the alternative prior where the prior distribution of the discount factor, $\psi$, is substantially more diffuse than the benchmark case.\footnote{In particular, we set the prior distribution for $\psi$ to be $U(0,1)$.} Note that a diffuse prior for $\psi$ actually implies a tight prior for $\lambda$. 

\begin{thebibliography}{99}

\end{thebibliography}
Figure 7 displays the prior and posterior distributions of $\lambda$, $\psi$, and the autoregressive parameters when we assume a diffuse prior for $\psi$. Despite the flat prior for $\psi$, the posterior distribution for the discount factor is substantially more concentrated than the prior and is centered on the value of 0.9. The posterior distribution of $\lambda$ is slightly shifted to the left relative to the benchmark model and centered around the value of ten. The posterior distributions of the autoregressive parameters are similar to those of the benchmark model. The slightly lower values for the discount factor notwithstanding, the variance decomposition for the diffuse prior case is very similar to the benchmark model (see Supplementary Appendix Figure S-5). Once again, most of the variance of $s_i - f_i$ is explained by the variance of the predictable component of $r_i^{md}$.

We also considered a model where the prior distribution for semi-elasticity of money demand with respect to the interest rate was diffuse (see Supplementary Appendix for the figures). Once again, despite the prior distribution quite different from benchmark model, the posterior distribution is qualitatively similar. Likewise, the variance decompositions for the model with diffuse priors for $\lambda$ are similar to those in the benchmark model (see Supplementary Appendix Figure S-7). It appears that diffuse priors on either $\lambda$ or on $\psi$ yield results that are not qualitatively different from those in the benchmark model.

### 6.3 Nonstationary money demand shifter.

The previous models suggest that there is substantial persistence in the relative money demand variable, $f_i - (p_i - p_i^*)$. The benchmark model assumed that $r_i^{md}$ is stationary. To

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19 For the diffuse $\lambda$ case, we set the prior distribution of $\lambda$ to a Gamma(1.1,100). Recall that $\psi = \lambda/1 + \lambda$. A diffuse distribution for $\lambda$ implies a relatively tight prior for $\psi$ close to one.
determine whether the results hold up when $r_t^{md}$ is nonstationary, we modify the above model by positing that: $\Delta r_t^{md} = \mu_{t-1}^{md} + \varepsilon_t^{md}$ with $(1 - \phi_{md} L) \mu_t^{md} = \varepsilon_t^{\mu,md}$, where $\mu_t^{md}$ is stationary. We can rewrite the exchange rate equation as:

$$s_t - f_t = -r_t^{md} + E \left[ \sum_{j=1}^{\infty} \gamma r^j \cdot (\Delta f_{t+j} - \Delta r_{t+j}^{md}) \right] + E \left[ \sum_{j=0}^{\infty} \gamma^j \cdot (\psi_{r_{t+j}}^{mp} + (1 - \psi_r) r_{t+j}^{mp}) \right]. \quad (6.1)$$

This equation has a nice interpretation. One can think of $s_t - f_t + r_t^{md}$ as the deviation of the exchange rate from its current monetary fundamentals, where the monetary fundamentals include relative money supplies ($m_t - m_t^*$) less relative money demands ($y_t - y_t^* + r_t^{md}$). The bracket terms in equation (6.1) are just expectations of future growth rates of monetary fundamentals and current and expected values of deviations from PPP and UIP.\textsuperscript{20}

Because $r_t^{md}$ is nonstationary, we cannot do the standard variance decomposition exercise, so instead we present the historical decomposition of the exchange rate in Figure 8.\textsuperscript{21} From Figure 8, it is clear that the most of the long-run movements in the exchange rate are due to the monetary fundamentals. Note, in fact, that the vast majority of long-run swings in $s_t - f_t$ can be captured by current value of $r_t^{md}$. Expectations of future $\Delta f_t$ as well as the uncovered interest parity factor play only a modest role in changes in $s_t - f_t$ while expectations of future $\Delta r_t^{md}$ contribute to some of the higher frequency movements in $s_t - f_t$.

### 6.4 Fixed versus flexible exchange rate regimes.

\textsuperscript{20} See the Supplementary Appendix for the state-space representation of this model.

\textsuperscript{21} The distribution for the semi-elasticity and the discount factor are similar to the previous models. The mode (and mean) of the posterior distribution $\phi_{md}$ is smaller than in the previous models, as $\phi_{md}$ reflects the persistence of $\Delta r_t^{md}$ rather than $r_t^{md}$. 

24
As the benchmark model combines periods of fixed and floating exchange rate regimes, it might be important to account for different exchange rates regimes. In particular we want to see if the distinction between fixed and flexible exchange rate regimes is important empirically, as Engel and Kim (1999) point out. To investigate this issue we estimated a model in which the interest semi-elasticity of money demand, the autoregressive parameters in the state-space model, and the overall scale of the variance/covariance matrix differ across fixed versus flexible regimes. We estimate the model with nonstationary money demand as there are apparent trends in both observations of \( f_t - (p_t - p_t^*) \) and \( s_t - f_t \) during the recent floating exchange rate period.

Comparing fixed versus flexible exchange rate periods, the posterior distributions of the semi-elasticity of money and the discount factor are lower under fixed exchange rates than under flexible exchange rates (see Figures S-9 and S-10 in the Supplementary Appendix). The posterior distributions for the semi-elasticity of money as well as the discount factors in both fixed and flexible exchange rate regimes are substantially lower than in the benchmark model with the fixed exchange rate regime parameters being, in turn, substantially smaller than the flexible exchange rate parameters. All of the variances are substantially smaller in fixed exchange rate period compared with flexible exchange rate periods. This finding is largely consistent with Engel and Kim (1999) who also report that the volatile regime of the exchange rate between U.S. and U.K. appears to correspond to the floating periods.

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22 Given the large number of covariances in the Q matrix (36 parameters) and the limited number of observations in both the fixed and flexible exchange rate period, we restricted the variance/covariance matrix Q to change across regimes by setting \( Q_i = P L_i P' \) where P is a lower triangular matrix with ones on the diagonal and is constant across regimes while \( L_i \) is a diagonal matrix whose elements can vary across regimes. This allows the Q matrix to change across regimes but limits the number of variance parameters that vary across regimes to eight.

23 For some of the variances, the posterior distribution for flexible exchange periods is very diffuse suggesting that the limited number of observations prevents the model from estimating these variances with precision.
Figure 9 displays the historical decomposition of \( s_t - f_t \) for the model in which we allow the parameters to change across fixed and flexible exchange rate regimes. Like the previous results, most of the variation in \( s_t - f_t \) is due to fluctuations in \( r_t^{md} \). Interestingly, deviations from PPP play a more important role in this model than they typically played in the previous models. This appears to be especially the case during periods of fixed exchange rates where expectations about future changes in \( f_t \) and \( r_t^{md} \) were estimated to have very small effects on fluctuations in exchanges.

### 6.5 Idiosyncratic factor in the Exchange Rate Observation Equation

Thus far, we have modeled the remainder term in equation (2.6) as reflecting unobserved fundamentals. However, this remainder might also reflect nonfundamental fluctuations in \( s_t - f_t \). To evaluate whether nonfundamental factors have played a role in fluctuations in \( s_t - f_t \), we include an idiosyncratic factor in the observation equation for \( s_t - f_t \). We assume the idiosyncratic factor affects only \( s_t - f_t \) and is uncorrelated with the other state variables in the model. We consider both the case where this idiosyncratic factor is stationary and the case where it is nonstationary (see Supplementary Appendix for details) and estimate both models by Bayesian methods. For both of these models, the other factors are assumed to be stationary (similar to the benchmark model). Figure 10 displays posterior distribution of the historical decompositions for the model with a nonstationary idiosyncratic component in \( s_t - f_t \). From Figure 10, it is clear that this idiosyncratic factor contributes little to fluctuations in \( s_t - f_t \);  

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24 The model with a stationary idiosyncratic component yields similar results.
lending support for our interpretation of the remainder term as reflecting unobserved fundamentals.

7. Discussion and interpretation

The above results suggest that within the context of the simple monetary model, relative money demand shocks can account for a large portion of exchange rate fluctuations and that taken as whole monetary fundamentals \( f_t - \tau_{md}^r = m_t - m_t^* - (y_t - y_t^*) - \tau_{md}^r \) account for a vast majority of exchange rate fluctuations. Two questions naturally arise. First, does the money demand factor estimated above, \( \tau_{md}^r \), reflect actual money demand or is a “remainder” that is used to fit \( s_t - f_t \) and as such has no real economic interpretation? Second, what role does the asset pricing approach play in the apparent reconciliation of exchange rates to monetary fundamentals (both observed and unobserved)?

7.1 Is \( \tau_{md}^r \) really a money demand shifter?

Recall that what we call the relative demand factor, \( \tau_{md}^r \), is derived from equation (5.6) reproduced for convenience below:

\[
f_t - (p_t - p_t^*) = -\frac{\nu}{1-\nu}(i_t - i_t^*) + \tau_{md}^r
\]  

(7.1)

where \( \frac{\nu}{1-\nu} = \lambda \) is the interest semi-elasticity of money demand. As both \( f_t - (p_t - p_t^*) \) and \( (i_t - i_t^*) \) are directly observed, if we knew the value of \( \lambda \) then we could back out \( \tau_{md}^r \) completely independent of any data on exchange rates. In this case, it would seem clear that \( \tau_{md}^r \) reflects relative money demand across the two countries. However, in our empirical analysis we
estimate \( \lambda \) and, in fact, use our model to also “fit” \((i_t - i_t^*)\), so it is conceptually possible that \( r_t^{md} \) is used to “fit” exchange rates rather than relative money demand. To what extent is \( r_t^{md} \) actually being used to help capture movements in \( f_t - (p_t - p_t^*) \) as opposed to \( s_t - f_t \)?

Figure 1 displays the historical decomposition for \( f_t - (p_t - p_t^*) \) for the model with nonstationary money demand and that allows for differences between fixed and flexible exchange rate regimes—the other models yield very similar decompositions. From Figure 1, \( r_t^{md} \) explains most of the movements in \( f_t - (p_t - p_t^*) \). The other factors affect \( f_t - (p_t - p_t^*) \) indirectly through their effect on \(-w_{1, \varphi}(i_t - i_t^*)\) but this indirect effect is small compared to the direct effect of \( r_t^{md} \) on \( f_t - (p_t - p_t^*) \). It is clear that the model is using \( r_t^{md} \) to “fit” \( f_t - (p_t - p_t^*) \) which does not include any direct data on exchange rates.

As it turns out, \( r_t^{md} \) is useful for “fitting” both \( f_t - (p_t - p_t^*) \) and \( s_t - f_t \). In the data, observed monetary fundamentals, \( f_t \), move so much relative to the exchange rate and price differentials that \( f_t - (p_t - p_t^*) \) and \( s_t - f_t \) are strongly negatively correlated. This is particularly true in the recent flexible exchange rate period. Here the relative money supply in the UK versus the US has risen dramatically relative to the exchange rate and price differentials. Fluctuations in \( r_t^{md} \) are useful in reconciling this strong negative correlation.

Why \( r_t^{md} \) and not the other factors? Deviations from PPP and UIP are small relative to movements in \( f_t - (p_t - p_t^*) \) and \( s_t - f_t \). Therefore, it is the presence of \( r_t^{md} \) and not the other fundamental factors that explain the movements of both \( f_t - (p_t - p_t^*) \) and \( s_t - f_t \). That this single factor can explain so much of the movements in \( f_t - (p_t - p_t^*) \) and \( s_t - f_t \) also explains
why an idiosyncratic factor in the \( s_t - f_t \) equation is estimated to contribute so little to the fluctuations in \( s_t - f_t \).

The fluctuations in \( r_t^{md} \) we find are also linked to the instability of the money demand that has been documented in the literature. Figure 12 breaks up \( r_t^{md} \) into a money demand factor for the UK, \( r_t^{md,UK} = m_t - y_t - p_t + \lambda_i t \), and a money demand factor for the US, \( r_t^{md,US} = m_t^* - y_t^* - p_t^* + \lambda_i^* t \). From Figure 18, it is clear that most of fluctuations in \( r_t^{md} \) have their source in the UK rather than the US. Nielsen (2007) studies money demand in UK from 1873 to 2001 and documents a considerable amount of variation in the money velocity. His finding of a large decline in money velocity in the UK, starting from the early 1980s is consistent with large increases in \( r_t^{md,UK} \) (and in \( r_t^{md} \)) that we find in our analysis. Apparently, a large portion of money in UK starting from early 1980s cannot be accounted for by interest rates and nominal income levels. These unobserved monetary factors turns out to be important in explaining the movements of both \( f_t - ( p_t - p_t^* ) \) and \( s_t - f_t \).

7.2. The importance of the asset pricing approach

The fact that monetary fundamentals, here \( f_t - r_t^{md} \), can capture the long swings in exchange rates (\( s_t - ( f_t - r_t^{md} ) \) is substantially less volatile and persistent than \( s_t \)) might suggest that the asset pricing approach to exchange rate determination is not really needed to explain exchange rate movements. This would, however, be an incorrect interpretation. In fact, the asset pricing approach is crucial for getting the monetary (both observed and unobserved) fundamentals to matter so much. Recall that the asset pricing approach to exchange rates implies the following equation
\[ s_t = \sum_{i=0}^{\infty} \psi^i E_t \left[ (1 - \psi)(f_{t+i} - r_{t+i}^{md}) + \psi r_{t+i}^{up} + (1 - \psi)r_{t+i}^{ppp} \right] \] (7.2)

Note that when \( f_t \) and \( r_t^{md} \) are very persistent (i.e. have a near unit root), we can write (7.2) as

\[ s_t = f_t - r_t^{md} + \sum_{i=1}^{\infty} \psi^i E_t \left[ \Delta f_{t+i} - \Delta r_{t+i}^{md} \right] + \sum_{i=0}^{\infty} \psi^i E_t \left[ \psi r_{t+i}^{up} + (1 - \psi)r_{t+i}^{ppp} \right] \] (7.3)

The direct effect of monetary fundamentals is \( (1 - \psi)(f_t - r_t^{md}) \) while effect of expectations of future monetary fundamentals is \( \psi(f_t - r_t^{md}) + \sum_{i=1}^{\infty} \psi^i E_t \left[ \Delta f_{t+i} - \Delta r_{t+i}^{md} \right] \). Note that because we estimate \( \psi \) to be in the vicinity of .9, then nearly 90\% of the effect of \( f_t - r_t^{md} \) on \( s_t \) is due to expectations of future values of \( f_t - r_t^{md} \). Thus, the asset pricing approach allows \( f_t - r_t^{md} \) to have a large impact on \( s_t \) that would otherwise be missed if one only considers the direct effect of monetary fundamentals.

8. Conclusion

In this paper, we use the asset pricing approach proposed by Engel and West (2005, 2006) to quantify the contribution of monetary fundamentals to exchange rate movements within a monetary model of exchange rate determination. Using a state-space framework to model both the predictable and unpredictable components of fundamentals, we derive the restrictions implied by a rational expectations, present value model of the exchange rate for the observation equations in the state-space model. Employing annual data on the pound to the dollar exchange rate, money, output, prices, and interest rates for the UK and US from 1880 to 2010, we estimate various version of this state-space model by Bayesian methods.
We show that using information on just directly observed monetary fundamentals, 
\( m_t - m_t^* - (y_t - y_t^*) \), and exchange rates is plagued by weak identification of the expected future fundamentals. The posterior distribution of the contribution of observed fundamentals to the variance of exchange rates is bimodal, with roughly equal weight placed on close to a zero contribution and on close to a 100% contribution. We solve the identification problem by using the i) additional data on interest rate and price differentials, and ii) prior information about key parameters in the model. We find that directly observed monetary fundamentals and money demand shifters contribute most to movements in exchange rates and while deviations from uncovered interest parity and purchasing power parity contribute to a lesser extent. The results suggest that monetary fundamentals, as defined in this paper, appear to explain long-run movements in exchange rates (consistent with the monetary approach to exchange rate determination) while the risk premium associated with deviations from uncovered interest parity and deviations from purchasing power parity explain only a fraction of the short-run movements in exchange rates.
References


Figure 1. UK-US Exchange Rate and Observed Monetary Fundamentals

Log exchange rate ($s$) and observed level of monetary fundamentals ($f$)

- **log exchange rate ($s$)**
- **observed monetary fundamentals ($f = m - m^* - (y - y^*)$)**

$s(t) - f(t)$ and $f(t) - f(t-1)$

- **$s(t) - f(t)$**
- **$f(t) - f(t-1)$**
Figure 2. Histograms of posterior distribution of variance decomposition of $s_t - f_t$ for various models.
Figure 3. Prior and posterior distributions for the interest semi-elasticity of money demand, the discount factor, and the autoregressive parameters for the benchmark model.
Figure 4. Posterior distribution of the variance decomposition of $s_t - f_t$ for the benchmark model.
Figure 5. Historical decomposition of $s_t - f_t$ for the benchmark model.

Median, 5th, and 95th percentiles of the posterior distribution.
Figure 6. Posterior distribution of variance decompositions of $s_t - f_t$
for model that drops $f_t - (p_t - p_t^*)$ and includes PPP deviations and $s_t - f_t$ as observables.
Figure 7. Smoothed posterior distributions for key parameters for the model with diffuse priors for the discount factor.
Figure 8. Historical decomposition of $s_t - f_t$ for model with nonstationary $r^{md}$. Median, 5\textsuperscript{th}, and 95\textsuperscript{th} percentile of posterior distribution.
Figure 9. Historical decomposition of $s_t - f_t$ for model that allows for changes across fixed and floating exchange rate regimes. Median, 5th and 95th percentiles of the posterior distribution.
Figure 10. Historical decomposition of $s_t - f_t$ for model that includes a nonstationary idiosyncratic component in the $s_t - f_t$ observation equation. Median, 5th and 95th percentiles of the posterior distribution.
Figure 11. Historical decomposition of $f_t - (p_t - p_t^*)$ for model that allows for changes across fixed and floating exchange rate regimes. Median, 5th and 95th percentiles of the posterior distribution.
Figure 12. Decomposition of money demand shifter, $r_{md}$, into UK and US components for model that allows for changes across fixed and flexible exchange rate regimes.
Supplementary Appendix for

The Contribution of Economic Fundamentals to Movements in Exchange Rates

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Contents:

S-1: Details of Markov Chain Monte Carlo.

S-2: Details on data construction.

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S-4: State-space model for the benchmark model in section 5.

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S-7: Additional figures.
Appendix S-1. Details of Markov Chain Monte Carlo.

Each of the three models has an empirical state-space model of the form:

\[ Y_{t}^{\text{obs}} = H(\theta)S_{t} \]  \hspace{1cm} (S1.1)

\[ S_{t} = F(\theta)S_{t-1} + V_{t}, \quad V_{t} \sim \text{MVN}(0, Q(\theta)) \]  \hspace{1cm} (S1.2)

where \( Y_{t}^{\text{obs}} \) is the vector of observable time series, \( S_{t} \) is the vector of unobserved state variables, and \( \theta \) is the vector of structural parameters. The predictive log likelihood of the state-space model is given by:

\[
\log(L(Y_{T}, \theta)) = \sum_{t=1}^{T} \left\{ -\frac{1}{2} \log(\det(H(\theta)P_{t|t-1}H(\theta))) \\
- \frac{1}{2}(Y_{t} - H(\theta)S_{t|t-1})' (H(\theta)' P_{t|t-1}H(\theta))^{-1}(Y_{t} - H(\theta)S_{t|t-1}) \right\}
\]  \hspace{1cm} (S1.3)

where \( S_{t|t-1} \) and \( P_{t|t-1} \) are the conditional mean and variance of \( S_{t} \) from the Kalman filter.

Given a prior distribution over parameters, \( h(\theta) \), the posterior distribution, \( P(\theta|Y_{T}) \), is

\[ P(\theta|Y_{T}) \propto L(Y_{T}, \theta)h(\theta). \]  \hspace{1cm} (S1.4)

Because the log-likelihood is a nonlinear function of the structural parameter vector, it is not possible to write an analytical expression for the posterior distribution. As a result, we use Bayesian Markov Chain Monte Carlo methods to estimate the posterior distribution of the parameter vector, \( \theta \). In particular, we employ a Metropolis-Hasting sampler to generate draws from the posterior distributions. The algorithm is as follows:

(i) Given a previous draw of the parameter vector, \( \theta^{(i-1)} \), draw a candidate vector \( \theta^{i} \) from the distribution \( g(\theta|\theta^{(i-1)}) \).

(ii) Determine the acceptance probability for the candidate draw.
\[
\alpha(\theta^c, \theta^{(i-1)}) = \min \left[ \frac{L(Y_T, \theta^c) h(\theta^c)}{L(Y_T, \theta^{(i-1)}) h(\theta^{(i-1)})} \frac{g(\theta^{(i-1)} | \theta^c)}{g(\theta^c | \theta^{(i-1)})}, 1 \right].
\]

(iii) Determine a new draw from the posterior distribution, \( \theta^{(i)} \).

\[ \theta^{(i)} = \theta^c \quad \text{with probability} \quad \alpha(\theta^c, \theta^{(i-1)}) \]

\[ \theta^{(i)} = \theta^{(i-1)} \quad \text{with probability} \quad 1 - \alpha(\theta^c, \theta^{(i-1)}) . \]

(iv) Return to (i).

Starting from an initial parameter vector and repeating enough times, the distribution parameters draws, \( \theta^{(i)} \), will converge to the true posterior distribution.

In our application, \( \theta^c = \theta^{(i-1)} + \nu \), where \( \nu \) is drawn from a multivariate t-distribution with 50 degrees of freedom and a covariance matrix \( \Sigma \). We set \( \Sigma \) to be a scaled value of the Hessian matrix of \(-\log(L(Y_T, \theta))\) evaluated at the maximum likelihood estimates. We choose the scaling so that around 50 percent of the candidate draws are accepted. We set a burn-up period of 500,000 draws and then sampled the next 500,000 draws.

To calculate the historical decompositions, the parameters of the state-space model are drawn from their posterior distribution using Metropolis-Hasting MCMC. For each parameter draw \( \theta^{(i)} \), we draw \( S_T^{(i)} = [S_t^{(i)}], \ldots, S_t^{(i)}, \ldots, S_t^{(i)} ] \) from the conditional posterior distribution for the unobserved states, \( P(S_T | \theta^{(i)}, Y_T) \) using the “filter forward, sample backward” approach of Carter and Kohn (1994) (see also Kim and Nelson (1999)). The contribution of the states in time period t for a given parameter and state draw is \( H(\theta^{(i)}) S_t^{(i)} \).

References

Appendix S-2. Details on Data Construction

This appendix describes the sources of the data used in the text.

The US/UK nominal exchange rate comes from Taylor (2001): specifically the pre-1948 data are from the statistical volumes of Brian Mitchell; and the series after 1948 are period average observations taken from the IMF’s *International Financial Statistics (IFS)*.

UK real national income. Data series of 1880 – 1948 are real GDP taken from Bordo et al. (1998) that are originally from Mitchell (1988); and data series after 1948 are real GDP taken from the *IFS*. When combining two series we adjust those observations of the earlier period by using the formula: \( y_t \times \frac{y_{1948,IFS}}{y_{1948,Bordo}} \) for \( 1880 \leq t \leq 1948 \).

US real national income. Data series of 1880 – 1948 are real GNP taken from Bordo et al. (1998) that are originally from Balke and Gordon (1986); and data series after 1948 are real GDP taken from the *IFS*. When combining two series we adjust those observations of the earlier period by using the formula: \( y_t \times \frac{y_{1948,IFS}}{y_{1948,Bordo}} \) for \( 1880 \leq t \leq 1948 \).

UK money supply. Data series of 1880 – 1966 are net money supply (M2) taken from Bordo et al. (1998) and data series after 1966 are money plus quasi-money taken from the *IFS*. When combining two series we adjust those observations of the earlier period by using the formula: \( y_t \times \frac{y_{1966,IFS}}{y_{1966,Bordo}} \) for \( 1880 \leq t \leq 1966 \).

US money supply. Data series of 1880 – 1971 are money supply (M2) taken from Bordo et al. (1998) that are originally from Balke and Gordon (1986); and data series after 1971 are money plus quasi-money taken from the *IFS*. When combining two series we adjust those observations of the earlier period by using the formula: \( y_t \times \frac{y_{1971,IFS}}{y_{1971,Bordo}} \) for \( 1880 \leq t \leq 1971 \).

UK price level. Data series of 1880 – 1988 are from Rapach and Wohar (2002); and data series after 1988 are CPI taken from the *IFS*. When combining two series we adjust those observations of the earlier period by using the formula: \( y_t \times \frac{y_{1988,IFS}}{y_{1988,RW}} \) for \( 1880 \leq t \leq 1988 \).
US price level. Data series of 1880 – 1948 are from Rapach and Wohar (2002); and data series after 1948 are CPI taken from the *IFS*. When combining two series we adjust those observations of the earlier period by using the formula: \( y_t \times \frac{y_{1948,IFS}}{y_{1948,RW}} \) for \( 1880 \leq t \leq 1948 \).

UK interest rate. Prime bank bill rates from NBER macro history database.

US interest rate. Commercial paper rates from NBER macro history database.

**References**


Appendix S-3. Identification of model in section 3.

This appendix presents the mapping between the state-space model and its reduced-form VARMA representation, and discusses relevant identification issues. Here we assume AR(2) specifications for both $\phi_g(L)L$ and $\phi_g(L)L$ in equations (3.3) and (3.4) in the text.

Using equations (3.1)-(3.4) and assuming rational expectations, we can solve for the following expectations:

$$
E_t \left[ \sum_{j=1}^{\infty} \psi^j \cdot \Delta f_{t+j} \right] = [1 \ 0] \cdot \psi \cdot \left( I - \psi F_g \right)^{-1} \cdot \begin{bmatrix} g_t \\ g_{t-1} \end{bmatrix}
$$

(S3.1)

$$
E_t \left[ \sum_{j=0}^{\infty} \psi^{j+1} \cdot r_{t+j} \right] = \psi \cdot \mu_{t-1} + \psi \cdot \varepsilon_t' + [1 \ 0] \cdot \psi^2 \cdot \left( I - \psi F_\mu \right)^{-1} \cdot \begin{bmatrix} \mu_t \\ \mu_{t-1} \end{bmatrix}
$$

(S3.2)

where, $F_g = \begin{bmatrix} \phi_{g1} & \phi_{g2} \\ 1 & 0 \end{bmatrix}$, $F_\mu = \begin{bmatrix} \phi_{\mu1} & \phi_{\mu2} \\ 1 & 0 \end{bmatrix}$. Equation (S1.1) corresponds to the contribution of expected future observed monetary fundamentals to current deviation of exchange rate from $f_t$; Equation (S1.2) denotes the contribution of expected future remainder to current deviation of exchange rate from $f_t$.

Denote the 1 by 2 row vectors $[1 \ 0] \cdot \psi \cdot \left( I - \psi F_g \right)^{-1} = [B_{11} \ B_{12}]$, and $[1 \ 0] \cdot \psi^2 \cdot \left( I - \psi F_\mu \right)^{-1} = [B_{21} \ B_{22}]$. Then, we can set up the following state-space model for the exchange rate model:

Measurement Equations:
\[
\begin{bmatrix}
\Delta f_{t+1} \\
 s_{t+1} - f_{t+1}
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 \\
B_{11} & B_{12} & B_{21} & B_{22} & \psi & 0
\end{bmatrix}
\begin{bmatrix}
g_{t+1} \\
g_t \\
\mu_{t+1} \\
\mu_t \\
\varepsilon_t^f \\
\varepsilon_t^r
\end{bmatrix}
\]

(S3.3)

Transition Equations:

\[
\begin{bmatrix}
g_{t+1} \\
g_t \\
\mu_{t+1} \\
\mu_t \\
\varepsilon_t^f \\
\varepsilon_t^r
\end{bmatrix}
= \begin{bmatrix}
\phi_{g1} & \phi_{g2} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \phi_{\mu1} & \phi_{\mu2} & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
g_t \\
g_{t-1} \\
\mu_t \\
\mu_{t-1} \\
\varepsilon_t^f \\
\varepsilon_t^r
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(S3.4)

where, the variance-covariance matrix of the vector of four shocks \( V_{t+1} = \begin{bmatrix} \varepsilon_{t+1}^g & \varepsilon_{t+1}^\mu & \varepsilon_{t+1}^f & \varepsilon_{t+1}^r \end{bmatrix} \) is:

\[
\Omega = \text{Var}(V_{t+1}) = \begin{bmatrix}
\sigma_g^2 & - & - & - \\
\sigma_{g\mu} & \sigma_\mu^2 & - & - \\
\sigma_{gf} & \sigma_{gf}^2 & \sigma_f^2 & - \\
\sigma_{gr} & \sigma_{gr\mu} & \sigma_r & \sigma_r^2
\end{bmatrix}
\]

The above state-space model implies a VARMA reduced-form representation for \( \begin{bmatrix} \Delta f_{t+1} & s_{t+1} - f_{t+1} \end{bmatrix} \) and implies the specific mapping between the structural and reduced-form model. Following Morley, Nelson, and Zivot (2003), we show the above structural model is identified. Plug the transition equation into the measurement equation to obtain:
\[
\begin{pmatrix}
\Phi_g(L) & 0 \\
0 & \Phi_g(L)\Phi_\mu(L)
\end{pmatrix}
\begin{pmatrix}
\Delta f_{t+1} \\
s_{t+1} - f_{t+1}
\end{pmatrix}
= 
\begin{pmatrix}
L \\
(B_{11} + B_{12}L)\Phi_\mu(L) \\
(B_{21} + (B_{22} + \psi)L)\Phi_g(L) \\
0 & \Phi_g(L)\Phi_\mu(L)
\end{pmatrix}
\begin{pmatrix}
e_g^{t+1} \\
e_\mu^{t+1} \\
e_f^{t+1} \\
e_r^{t+1}
\end{pmatrix}
\] (S3.5)

Where, \( \Phi_g(L) = (1 - \phi_{g1}L - \phi_{g2}L^2) \) and \( \Phi_\mu(L) = (1 - \phi_{\mu1}L - \phi_{\mu2}L^2) \). Denote the LHS of (S1.5) by:

\[
\begin{pmatrix}
x_{1t+1} \\
x_{2t+1}
\end{pmatrix}
= 
\begin{pmatrix}
\Phi_g(L) & 0 \\
0 & \Phi_g(L)\Phi_\mu(L)
\end{pmatrix}
\begin{pmatrix}
\Delta f_{t+1} \\
s_{t+1} - f_{t+1}
\end{pmatrix}
\]

Then:

\[
\begin{pmatrix}
x_{1t+1} \\
x_{2t+1}
\end{pmatrix}
= (C + DL + EL^2 + FL^3 + GL^4) \cdot V_{t+1}
\] (S3.6)

Where, \( V_{t+1} = (e_g^{t+1} \quad e_\mu^{t+1} \quad e_f^{t+1} \quad e_r^{t+1}) \), and its variance is \( \Omega \).

\[C = \begin{pmatrix}
0 & 0 & 1 & 0 \\
B_{11} & B_{12} & 0 & 1
\end{pmatrix};
\]
\[D = \begin{pmatrix}
1 & 0 \\
-B_{11}\phi_{g1} + B_{12} & -B_{21}\phi_g + (B_{22} + \psi) \\
0 & -\phi_{g1} - \phi_{g2}
\end{pmatrix};
\]
\[E = \begin{pmatrix}
0 & 0 \\
-B_{11}\phi_{\mu2} - B_{12}\phi_{g1} & -B_{21}\phi_g - (B_{22} + \psi)\phi_g + 0 \\
0 & -\phi_{\mu1} - \phi_{\mu2}
\end{pmatrix};
\]
\[F = \begin{pmatrix}
0 & 0 \\
-B_{12}\phi_{g2} - (B_{22} + \psi)\phi_g & 0 \\
0 & \phi_{g2} + \phi_{\mu2}\phi_g
\end{pmatrix};
\]
\[G = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \phi_{\mu2}\phi_g
\end{pmatrix}.
\]

From the eq. (S.2), we can derive the second moments of its RHS:
\[
\text{Var}\left(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}\right) = C\Omega C' + D\Omega D' + E\Omega E' + F\Omega F' + G\Omega G'
\]  
(S3.7)

\[
\text{Cov}\left(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}, \begin{pmatrix} x_{t} \\
 x_{2t} \end{pmatrix}\right) = D\Omega C' + E\Omega D' + F\Omega E' + G\Omega F'
\]  
(S3.8)

\[
\text{Cov}\left(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}, \begin{pmatrix} x_{t-1} \\
 x_{2t-1} \end{pmatrix}\right) = E\Omega C' + F\Omega D' + G\Omega E'
\]  
(S3.9)

\[
\text{Cov}\left(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}, \begin{pmatrix} x_{t-2} \\
 x_{2t-2} \end{pmatrix}\right) = F\Omega C' + G\Omega D'
\]  
(S3.10)

\[
\text{Cov}\left(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}, \begin{pmatrix} x_{t-3} \\
 x_{2t-3} \end{pmatrix}\right) = G\Omega C
\]  
(S3.11)

Therefore, by Granger and Newbold’s Theorem (1986), the structure of the second moments implies that the \(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}\) has a reduced-form VMA(4) process. The AR parameters of

\[
\phi_g(L) = (1 - \phi_{g1}L - \phi_{g2}L^2)
\]

and

\[
\phi_\mu(L) = (1 - \phi_{\mu1}L - \phi_{\mu2}L^2)
\]

can be identified by the AR structure of

\[
\begin{pmatrix} \Delta f_{t+1} \\
 d_{t+1} \end{pmatrix}
\]

The parameters left in the state-space model as set up in (S1.3) and (S1.4) are 10 variance and covariance parameters and these parameters can be identified by the moving average terms of the \(\begin{pmatrix} x_{t+1} \\
 x_{2t+1} \end{pmatrix}\) as shown above. Therefore, the state-space model is identifiable.

**References**

Appendix S-4. State-space model for the benchmark model in section 5.

The state vector is:

\[
S_t = \begin{bmatrix}
g_t & g_{t-1} & \mu_{i}^{\text{np}} & \mu_{i}^{\text{up}} & \mu_{i}^{\text{ppp}} & \mu_{i}^{md} & \mu_{i-1}^{md} & \epsilon_{t}^{f} & \epsilon_{t}^{up} & \epsilon_{t}^{ppp} & \epsilon_{t}^{md}
\end{bmatrix}\]

and the transition equation is

\[
S_t = FS_{t-1} + V_t
\]

with

\[
F = \begin{bmatrix}
\phi_{\mu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \phi_{\text{up}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \phi_{\text{ppp}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \phi_{\text{md}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

and

\[
V_t = \begin{bmatrix}
\epsilon_{t}^{g} & 0 & \epsilon_{t}^{\mu,\text{np}} & 0 & \epsilon_{t}^{\mu,\text{ppp}} & 0 & \epsilon_{t}^{\mu,\text{md}} & 0 & \epsilon_{t}^{f} & \epsilon_{t}^{up} & \epsilon_{t}^{ppp} & \epsilon_{t}^{md}
\end{bmatrix}
\]

We denote the variance-covariance matrix of \( V_t \) by \( Q \).

Taking expectations and writing equations (5.6)-(5.9) in terms of the state variables, \( S_t \), yields the following measurement equation:

\[
\begin{bmatrix}
f_t - (p_t - p_t^*) \\
i_t - i_t^* \\
\Delta f_t \\
S_t - f_t
\end{bmatrix} = \begin{bmatrix}
\frac{\omega}{1-\omega} \left( H_{sf} + H_{sf} \right)F - H_{sf} + H_{up} \right) + H_{md} \\
( H_{sf} + H_{sf} )F - H_{sf} + H_{up} \\
H_{sf} \\
H_{sf}
\end{bmatrix} S_t,
\]

(S4.3)
where

\[
H_{nf} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad H_{uip} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}
\]

\[
H_{md} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
H_{sf} = \begin{bmatrix} B_{11} & B_{12} & B_{21} & B_{22} & B_{31} & B_{32} & B_{41} & B_{42} & 0 & \psi & 1-\psi & -(1-\psi) \end{bmatrix},
\]

\[
[B_{11} \quad B_{12}] = \left[ \psi \cdot \left(1 - \psi \phi_{\phi_x}\right)^{-1} \quad 0 \right],
\]

\[
[B_{21} \quad B_{22}] = \left[ \psi^2 \cdot \left(1 - \psi \phi_{\psi_{uip}}\right)^{-1} \quad \psi \right],
\]

\[
[B_{31} \quad B_{32}] = \left[ (1-\psi) \psi \cdot \left(1 - \psi \phi_{\psi_{ppp}}\right)^{-1} \quad (1-\psi) \right],
\]

\[
[B_{41} \quad B_{42}] = \left[ -(1-\psi) \psi \cdot \left(1 - \psi \phi_{md}\right)^{-1} \quad -(1-\psi) \right].
\]
Appendix S-5. State-space model for the case of nonstationary money demand shifter.

The state vector is given by:

\[
S_t = \begin{bmatrix}
    L_i^{md} & g_t & g_{t-1} & \mu_t^{uip} & \mu_{t-1}^{uip} & \mu_t^{ppp} & \mu_{t-1}^{ppp} & \mu_t^{md} & \mu_{t-1}^{md} & \varepsilon_t^f & \varepsilon_t^{uip} & \varepsilon_t^{ppp} & \varepsilon_t^{md}
\end{bmatrix}
\]

and the transition equation to be:

\[
S_t = FS_{t-1} + V_t
\]

with

\[
F = \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_{md} & 0 & 0 & 0 & 0 & 0
    0 & \phi_{g} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
    0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & \phi_{uip} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & \phi_{ppp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 0 & \phi_{md} & 0 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_{md} & 0
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
V_t = \begin{bmatrix}
    \varepsilon_t^{md} & \varepsilon_t^{uip} & 0 & \varepsilon_t^{muip} & 0 & \varepsilon_t^{mupp} & 0 & \varepsilon_t^{mu} & 0 & \varepsilon_t^{f} & \varepsilon_t^{uip} & \varepsilon_t^{ppp} & \varepsilon_t^{md}
\end{bmatrix}
\]

Taking expectations and writing equations (5.6)-(5.9) in terms of the state variables, \(S_t\), yields the following measurement equation:

\[
\begin{bmatrix}
    f_t - (p_t - p_t^*) \\
    i_t - i_t^* \\
    \Delta f_t \\
    s_t - f_t
\end{bmatrix} = \begin{bmatrix}
    \frac{-\psi}{1-\psi} \left( \left( H^{sf} + H^{sf} \right) F - H^{sf} + H^{uip} \right) + H^{md} \\
    H^{sf} \\
    H^{sf}
\end{bmatrix}
\]

where
\[ H_{sf} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \]

\[ H_{usp} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \]

\[ H_{mof} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ H_{sf} = \begin{bmatrix} -1 & B_{11} & B_{12} & B_{21} & B_{22} & B_{31} & B_{32} & B_{41} & B_{42} & 0 & \psi & 1 - \psi & 0 \end{bmatrix}, \]

\[ [B_{11} \quad B_{12}] = [\psi \cdot (1 - \psi \phi_e)^{-1} \quad 0], \]

\[ [B_{21} \quad B_{22}] = [\psi^2 \cdot (1 - \psi \phi_{usp})^{-1} \quad \psi], \]

\[ [B_{31} \quad B_{32}] = [(1 - \psi) \psi \cdot (1 - \psi \phi_{ppp})^{-1} \quad (1 - \psi)], \]

\[ [B_{41} \quad B_{42}] = [-\psi \cdot (1 - \psi \phi_{mof})^{-1} \quad 0] \]
Appendix S-6. State-space model for the case of idiosyncratic component in the exchange rate observation equation.

The state vector is given by:

\[
S_t = \begin{bmatrix}
g_t & g_{t-1} & \mu_{t}^{up} & \mu_{t-1}^{up} & \mu_{t}^{ppp} & \mu_{t-1}^{ppp} & \mu_{t}^{md} & \mu_{t-1}^{md} & \epsilon_t^f & \epsilon_t^{up} & \epsilon_t^{ppp} & \epsilon_t^{md} & x_t
\end{bmatrix}^T
\]

where \( x_t \) is the idiosyncratic component in the exchange rate observation equation. For the case where the idiosyncratic component is assumed to be stationary, it follows the following autoregressive model:

\[
x_t = \phi_x x_{t-1} + \epsilon_t^x \quad \text{(S6.1)}
\]

and for the case where it is nonstationary

\[
\Delta x_t = \phi_x \Delta x_{t-1} + \epsilon_t^x \quad \text{(S6.2)}
\]

The measurement equation is given by:

\[
\begin{bmatrix}
f_t - (p_t - p_t^* ) \\
i_t - i_t^*
\end{bmatrix} = \begin{bmatrix}
-\frac{\psi}{1-\psi} \left( (H_{sf} + H_{sf})F - H_{sf} + H_{up} \right) + H_{md} \\
( H_{sf} + H_{sf} )F - H_{sf} + H_{up}
\end{bmatrix} \begin{bmatrix}
\frac{\psi}{1-\psi} \left( (H_{sf} + H_{sf})F - H_{sf} + H_{up} \right) + H_{md} \\
H_{sf}
\end{bmatrix} S_t \quad \text{(S6.2)}
\]

where

\[
H_{sf} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H_{up} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H_{md} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
H_{sf} = \begin{bmatrix}
B_{11} & B_{12} & B_{21} & B_{22} & B_{31} & B_{32} & B_{41} & B_{42} & 0 & \psi & 1-\psi & -(1-\psi) & 1
\end{bmatrix},
\]

\[
\begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix} = \begin{bmatrix}
\psi \cdot (1-\psi \phi_x)^{-1} & 0 \\
\psi^2 \cdot (1-\psi \phi_{up})^{-1} & \psi
\end{bmatrix},
\]

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\[
[B_{31} \quad B_{32}] = \begin{bmatrix}
(1 - \psi) \cdot \phi_{ppp}^{-1} & (1 - \psi) \\

[B_{41} \quad B_{42}] = \begin{bmatrix}
-\psi \cdot \phi_{mat}^{-1} & 0
\end{bmatrix}
\]
Appendix S-7: Additional Figures

Figure S-1. Historical decomposition of UK-US exchange rate, benchmark model. Median, 5th, and 95th percentiles of the posterior distribution.
Figure S-2. Historical decomposition of $\Delta f_t$.
Median, 5th, and 95th percentiles of the posterior distribution.
Figure S-3. Prior and posterior distribution of key parameters for the model that drops $f_t - (p_t - p_t^*)$ and includes PPP deviations and $s_t - f_t$ as observables
Figure S-4. Posterior distribution of variance decompositions of $s_t - f_t$ for model that drops $s_t - f_t$ and includes PPP deviations and $f_t - (p_t - p_t^*)$ as observables.
Figure S-5. Posterior distribution of variance decomposition of $s_t - f_t$ for model with diffuse priors for the discount factor.
Figure S-6. Smoothed posterior distributions for key parameters for the model with diffuse priors for the semi-elasticity of money demand.
Figure S-7. Posterior distribution of variance decomposition of $s_t - f_t$ for model with diffuse priors for semi-elasticity of money demand.
Figure S-8. Smoothed posterior distributions for semi-elasticity of money demand and the discount factor for the model with nonstationary $r_{md}^n$. 

[Graphs showing prior and posterior distributions for different variables]
Figure S-9. Posterior distribution of key parameters across fixed and flexible exchange rate regimes.
Figure S-10. Posterior distributions of variances of components across fixed and flexible exchange rate regimes.
Figure S-11. Historical decomposition of $s_t - f_t$ for model that includes a stationary idiosyncratic component in the $s_t - f_t$ observation equation. Median, 5$^{th}$ and 95$^{th}$ percentiles of the posterior distribution.