Contributions

Nathan S. Balke and Zheng Zeng*
Credit demand, credit supply, and economic activity

Abstract: In this paper, we attempt to identify the separate contributions of credit demand, supply of financial intermediation, and supply of funds to fluctuations in indicators of credit conditions and to fluctuations in economic activity. We estimate a common factor model in which the six factors correspond to supply of funds, financial intermediation, credit demand, aggregate uncertainty, real economic activity, and inflation. We use a simple model of financial intermediation to motivate restrictions on the factor loadings designed to identify supply of funds, uncertainty, credit demand, and financial intermediation factors. We find that the supply of funds and financial intermediation factors explain most of the variation in interest rates spreads, while the financial intermediation and credit demand factors typically contribute to most of the fluctuations in credit quantity variables. For credit indicators, the 2008–2009 financial crisis appears to be largely due to a decline in the financial intermediation. However, this decline in financial intermediation seems to have originated from output and uncertainty shocks, rather than shocks to financial intermediation itself.

Keywords: aggregate uncertainty; credit demand; credit supply; financial conditions indices; financial intermediation.

JEL Classification: E44; E51; C32.

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1 Introduction

The modern approach to analyzing credit market frictions is to consider the role that asymmetric information between borrowers and lenders plays in the functioning of credit markets as a whole (not just bank lending). In particular, the
“broad credit view” highlights the role of balance sheet considerations of potential borrowers (Bernanke and Gertler 1989; Bernanke, Gertler, and Gilchrist 1996, 1999; Kiyotaki and Moore 1997). Asymmetric information between borrowers and lenders results in a wedge between the costs of borrowing (raising funds externally) and the costs of using internal funds. Deterioration in the balance sheets of potential borrowers can exacerbate this agency problem and increase the external finance premium. The “broad credit view” with its focus on the balance sheet conditions and the riskiness of the ultimate borrowers emphasizes the effect of shifts in credit demand.

On the other hand, Woodford (2010) suggests that the recent crisis, “at least in its initial phase, resulted more from obstacles to credit supply... than from a reduction in credit demand owing to the problems of ultimate borrowers.” Cúrdia and Woodford (2010), Adrian, Estrella, and Shin (2010), Adrian and Shin (2010) and Gertler and Kiyotaki (2010) stress the importance of a “risk-taking channel” in which the balance sheet conditions of financial intermediaries, rather than the ultimate borrowers, play a crucial role. For example, weakening of financial intermediaries’ balance sheets decreases the willingness of financial intermediaries to supply credit. In addition to financial intermediaries’ willingness to lend, the total amount of funds in the financial system can also affect the supply of credit.

In this paper, we seek to separately identify factors associated with demand for credit, the supply of financial intermediation, and the supply of funds. With this in mind, we estimate a common factor model that extracts co-movements from 39 monthly and quarterly indicators. We impose restrictions on the factor loadings in the common factor model to identify six aggregate factors: real output, inflation, credit demand, financial intermediation, supply of funds, and aggregate uncertainty. We let these six factors interact with one another through a vector autoregression (VAR), so that the factors can be correlated contemporaneously and over time. In order to identify the credit demand, the supply of financial intermediation, and the supply of funds factors, we apply restrictions on the factor loadings of credit quantities and interest rate spreads based on a simple model of financial intermediation adapted from Woodford (2010). We also include a factor that reflects aggregate uncertainty because of the potential effect that uncertainty may have on credit conditions as well as on economic activity in general [see Alexopoulos and Cohen (2009), Bloom (2009), Bloom, Floetotto and Jaimovich (2009), Butzen, Fuss and Vermeulen (2002)].

Using common factor models to capture the co-movements among various indicators is not new in the literature [e.g., Stock and Watson (1989, 2002, 2003, 2005)]. Examples that come closest to what we do in this paper include Hakkio and Keeton (2009), who extract the principal component of eleven financial
variables consisting of interest rate spreads, implied volatility indices, and volatility and dispersion of bank stock returns; and Hatzius et al. (2010), who construct a financial conditions index using measures of stock market volatility, indicators of prices, credit spreads, as well as monetary and loan quantities. Our model differs from these two papers in that we break up credit conditions into credit demand, financial intermediation, and source of funds by imposing restrictions on the factor loadings on these different credit factors. Unlike these other two papers, we also include indicators of economic activity, inflation, and uncertainty. This enables us to construct a multi-factor model that allows for interactions among factors.

Using Bayesian methods to estimate the common factor model and the contributions of these factors to observed variables, we find that the supply of funds and the financial intermediation factors explain most of the variation in short-term interest rates and in interest rate spreads. With respect to indicators of credit quantities, the financial intermediation factor and to a lesser extent credit demand factor typically contribute to the majority of the fluctuations in credit quantity indicators. The Great Recession of 2007–2009 shows up in our factor model first as a large decline in the output factor and then later as a large decline in the financial intermediation factor. We also find that innovations in financial intermediation and credit demand factors are strongly negatively correlated and may reflect shocks that affect both the demand for credit and the willingness of financial intermediaries to supply credit such as a balance sheet shock. When considering orthogonal shocks in the factors, credit factor shocks contribute little to output fluctuations over our sample while output and uncertainty shocks do contribute to fluctuations in the credit factors, particularly the financial intermediation factor.

The rest of the paper is organized as follows. In the next section, we present a simple model of financial intermediation that will be used to motivate the identification employed in our empirical analysis. In Section 3, we discuss the identification of our “structural” factor model. Section 4 reports the empirical results. We examine properties of the estimated factors and, in particular, the contributions of these factors to interest rates and credit quantity indicators. Also, we consider the interaction among the factors and examine impulse responses for the factors and a few key indicator variables. In Section 5, we offer the concluding remarks.

2 A simple model of financial intermediation

To motivate our identification of credit factors, we posit a simple, static, partial equilibrium model of financial intermediation similar to that in Woodford (2010).
Financial intermediaries borrow funds from savers and then lend to ultimate borrowers. The willingness of financial intermediaries to supply financial intermediation depends upon the spread between the rate at which financial intermediaries lend and the rate at which they borrow.

The demand for loans on the part of ultimate borrowers is given by

$$L(r_L, X_L),$$  \hspace{1cm} (1)

where $r_L$ is the interest rate on loans to ultimate borrowers with $\frac{\partial L(\cdot)}{\partial r_L} < 0$. $X_L$ represents variables that shift the demand for loans. The supply of loanable funds on the part of ultimate savers is given by

$$S(r_S, X_S),$$  \hspace{1cm} (2)

where $r_S$ is the interest rate on loans from ultimate lenders with $\frac{\partial S(\cdot)}{\partial r_S} > 0$. $X_S$ represents variables that shift the supply of funds in the financial system. Changes in $X_S$ could be due to things such as changes in net savings, the willingness of savers to put funds in the financial system, or central bank open market operations. In a world of perfect capital markets and no financial intermediaries, $L(r_L, X_L) = S(r_S, X_S)$ and $r_L = r_S$.

In an economy without perfect capital markets, there exists a need for financial intermediaries to link up ultimate borrowers and ultimate savers. Rather than a single market, there will exist two markets: one for final loans which we call the credit market and the other we call the funding market. Financial intermediaries participate in both markets; they borrow funds from ultimate savers in the funding market then turn around and lend to ultimate borrowers in the credit market. The supply of financial intermediation (and the demand for funds) by financial intermediaries is given by:

$$N(r_L - r_S, X_N),$$  \hspace{1cm} (3)

where $r_L - r_S$ is the lending-borrowing spread which determines the profitability of supplying credit. As the spread rises, financial intermediaries are more willing to borrow from ultimate lenders and lend to ultimate borrowers, so that $\frac{\partial N(\cdot)}{\partial (r_L - r_S)} > 0$.

The positive relation between the spread, $r_L - r_S$, and the quantity of financial intermediation reflects the frictions (and/or costs) present in financial intermediation. $X_N$ reflects the willingness of financial intermediaries to supply credit at a given interest rate spread. It could include administrative costs or production technology of issuing loans (Cúrdia and Woodford 2009; Christiano, Motto, and Rostagno 2010), the value of financial intermediaries’ capital or their capital requirements,
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or the willingness of financial intermediaries take risks (Adrian, Estrella, and Shin 2010; Adrian and Shin 2010; Gertler and Kiyotaki 2010).

An equilibrium requires

\[ L(r_L, X_L) = N(r_L - r_S, X_N) \]  (4)

and

\[ N(r_L - r_S, X_N) = S(r_S, X_S). \]  (5)

It is easy to show that changes in \( X_L \) which increase the demand for credit result in an increase in the spread and an increase in the quantity of loans \( \left( \frac{d(r_L - r_S)}{dX_L} > 0 \right. \) and \( \frac{dL(\cdot)}{dX_L} > 0 \). The increase in credit demand also raises interest rates generally,

\[ \frac{dr_L}{dX_L} > 0 \text{ and } \frac{dr_S}{dX_L} > 0. \]

An increase in supply of financial intermediation (due to change in \( X_N \)) reduces the rate paid by the ultimate borrowers \( \left( \frac{dr_L}{dX_N} < 0 \right) \) and raises the rate received by the ultimate savers \( \left( \frac{dr_S}{dX_N} > 0 \right) \), and hence, results in a decline in the spread but a rise in the quantity of loans \( \left( \frac{d(r_L - r_S)}{dX_N} < 0 \text{ and } \frac{dL(\cdot)}{dX_N} > 0 \right) \). For example, an increase in the “risk appetite” of financial intermediaries would correspond to an increase in financial intermediation and would result in a decrease in the spread and an increase in the quantity of credit.

Finally, an increase in the source of funds available from the ultimate savers will reduce the level of interest rates \( \left( \frac{dr_L}{dX_S} < 0 \text{ and } \frac{dr_S}{dX_S} < 0 \right) \), but the spread actually rises along with the quantity of credit \( \left( \frac{d(r_L - r_S)}{dX_S} > 0 \text{ and } \frac{dL(\cdot)}{dX_S} > 0 \right) \). An exogenous increase in the supply of funds lowers rates in the funding market, \( r_S \), and raises the quantity of financial intermediation. This in turn spills over into the credit market as the quantity of credit rises while interest rates to ultimate borrowers, \( r_L \), fall. Nonetheless, the spread, \( r_L - r_S \), rises as \( r_S \) falls more than \( r_L \). Note that in a general equilibrium framework, \( X_L, X_S, \) and \( X_N \) would depend on underlying preferences, policy, and technology shocks. For example, Cúrdia and Woodford (2010) in their dynamic general equilibrium model with credit frictions show that an increase in consumption demand on the part of savers (similar to an exogenous decline in \( X_L \)) decreases spreads while increases in consumption demand on the part of borrowers (similar to an exogenous rise in \( X_L \)) increases spreads.
In our empirical work below, we will use various spreads ($r_L - r_S$), short-term interest rates ($r_S$), and credit quantities ($L$) as indicators of credit market conditions. The above model’s predictions for the response of spreads, short-term rates, and credit quantities to shifts in $X_t$, $X_{nt}$, and $X_S$ will be used to help identify changes in credit demand, the quantity of financial intermediation, or the supply of funds in the data.

3 The common factor model

Rather than relying on a single indicator of interest rates, interest rate spreads, and credit quantities, we use multiple indicators and extract common factors corresponding to credit demand, financial intermediation, and the supply of funds from these indicators. In addition to the credit factors, we also construct factors for real economic activity (hereafter “output factor”), inflation, and uncertainty from numerous indicators of these factors.

The observation equation(s) for the indicators are given by:

$$Y_t = A(L)Y_{t-1} + H(L)S_t + \omega_t, \omega_t \sim N(0, R); \quad (6)$$

where $Y_t$ is the vector of all observable indicators and $S_t$ is the vector of unobserved factors including real output ($s_{yt}$), inflation ($s_{\pi t}$), uncertainty ($s_{u,t}$), credit demand ($s_{cd,t}$), financial intermediation ($s_{fi,t}$), and supply of funds ($s_{sf,t}$). $R$ is a diagonal variance-covariance matrix of the innovations $\omega_t$. For the $j$th individual indicator, $y_{jt}$, the observation equation is given by:

$$y_{jt} = A_j(L)y_{jt-1} + H_j(L)S_t + \varpi_{jt},$$

where $\varpi_{jt}$ represents the $j$th idiosyncratic innovation at time period $t$.

$A_j(L) = \sum_{k=0}^{K} A_{jk}L^k$ and $H_j(L) = \sum_{n=0}^{N} H_{jn}L^n$ are lag polynomials. In our specification, the number of idiosyncratic lags in the $A_j(L)$ matrix is set equal to 2. $H_j(L)$ is the factor loading matrix on current and two lagged values of the common factors.1 In order to provide economic interpretations of the common factors, we place sign restrictions and zero restrictions on $H_j(L)$ to help in identifying different factors. In particular, for the credit factors, we identify unobserved factors of credit demand ($s_{cd}$), financial intermediation ($s_{fi}$), and supply of funds ($s_{sf}$) by imposing

1 Including two lags of the idiosyncratic factors and two lags on the common factors provides a relatively parsimonious yet flexible model to capture the dynamics between the observables and the unobserved factors. We experimented with other lag lengths and the results were fairly similar.
restrictions implied by the model in Section 2 on the factor loadings of a common factor model.

The evolution of the observed factors is assumed to be a stationary VAR(k):\[ S_t = FS_{t-1} + \nu_t, \quad \nu_t \sim N(0, Q); \] (7)
where \(S_t = [s_t, s_{t-1}, \ldots, s_{t-k+1}]^\prime,\)
and \(\nu_t\) is a vector of shocks to the common factors:
\[ \nu_t = [\nu_{y,t}, \nu_{\pi,t}, \nu_{cd,t}, \nu_{fi,t}, \nu_{sf,t}, 0, \ldots, 0]^\prime.\]
Matrix \(F\) captures the unrestricted interactions across the six states:
\[ F = \begin{pmatrix} \rho \\ I \\ O \end{pmatrix} \]
where \(I\) denotes \(6(k-1)\) identity matrix, \(O\) denotes a \(6(k-1)\times6\) zero matrix, \(\rho = [\rho^{(1)}, \rho^{(2)}, \ldots, \rho^{(k)}]\) and each of the \(\rho^{(l)}\) is a \(6 \times 6\) matrix given \(l=1, 2, \ldots, k.\)

The state equation describes the interactions among various factors in the model. This is where the “general equilibrium” nature of the macroeconomy is captured by our empirical model. We allow innovations in the state equation, \(\nu_t\), to be correlated with one another. Later in our analysis, when we consider orthogonal shocks to the common factors, we impose a recursive structure so that \(S_t = FS_{t-1} + P \mu_t;\)
where \(\mu_t \sim N(0, I)\) and \(PIP' = Q.\)

### 3.1 Selection of indicators and restrictions on factor loadings

Table 1 contains the 39 indicators included in the model, their frequency of observation, any data transformation employed, and the range of observations. Also included in the table are the restrictions placed on the factor loadings in the observation equations in the table. These restrictions take the form of sign

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2 We estimated the state equation with VARs of three and six lags respectively, and the results of these two specifications are similar. In this paper we only report the results of the model with three VAR lags as this VAR yielded slightly more precise estimates of the factors.
### Table 1  Indicators and data specification.

<table>
<thead>
<tr>
<th></th>
<th>$s_y$</th>
<th>$s_n$</th>
<th>$s_{cd}$</th>
<th>$s_p$</th>
<th>$s_{df}$</th>
<th>$s_u$</th>
<th>Freq.</th>
<th>Specification</th>
<th>Start</th>
<th>End</th>
<th>Data source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Real GDP</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Q</td>
<td>% growth</td>
<td>1985Q1</td>
<td>2011Q4</td>
<td>BEA</td>
</tr>
<tr>
<td>2. IP – Final goods</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>% growth</td>
<td>1985M1</td>
<td>2011M12</td>
<td>FRB</td>
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<tr>
<td>3. Nonfarm bus empl</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>% growth</td>
<td>1985M1</td>
<td>2011M12</td>
<td>BLS</td>
</tr>
<tr>
<td>4. NAPM composite index</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>Level</td>
<td>1985M1</td>
<td>2011M12</td>
<td>ISM</td>
</tr>
<tr>
<td>5. CPI-U</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>% growth</td>
<td>1985M1</td>
<td>2011M12</td>
<td>BLS</td>
</tr>
<tr>
<td>6. CPI-core</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>% growth</td>
<td>1985M1</td>
<td>2011M12</td>
<td>BLS</td>
</tr>
<tr>
<td>7. PCE</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>% growth</td>
<td>1985M1</td>
<td>2011M12</td>
<td>BEA</td>
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<tr>
<td>8. GDP deflator</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Q</td>
<td>% growth</td>
<td>1985Q1</td>
<td>2011Q4</td>
<td>BEA</td>
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<tr>
<td>14. 3 m LIBOR rate</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>?</td>
<td>?</td>
<td></td>
<td>M</td>
<td>Level</td>
<td>1985M1</td>
<td>2011M12</td>
<td>FRB</td>
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<tr>
<td>15. SP 500 volatility</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>std. dev.</td>
<td>1985M1</td>
<td>2011M12</td>
<td>CRSP</td>
</tr>
<tr>
<td>16. Broad volatility</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>std. dev.</td>
<td>1985M1</td>
<td>2011M12</td>
<td>CRSP</td>
</tr>
<tr>
<td>17. VIX index</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>Level</td>
<td>1990M1</td>
<td>2011M12</td>
<td>CBOE</td>
</tr>
<tr>
<td>18. VXO index</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>Level</td>
<td>1986M1</td>
<td>2011M12</td>
<td>CBOE</td>
</tr>
<tr>
<td>22. Dist. to default (75th pct.)</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>Level</td>
<td>1985M1</td>
<td>2010M12</td>
<td>Gilchrist and Zakrajske (2010)</td>
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<tr>
<td>36. Broker-Dealer Assets</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>Q</td>
<td>% growth</td>
<td>1985Q1</td>
<td>2011Q4</td>
<td>FRB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>37. Shadow bank assets</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>Q</td>
<td>% growth</td>
<td>1985Q1</td>
<td>2011Q4</td>
<td>FRB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38. Comm Bank total loans</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>Q</td>
<td>% growth</td>
<td>1985Q1</td>
<td>2011Q4</td>
<td>FRB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39. Total credit liabilities</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>?</td>
<td>Q</td>
<td>% growth</td>
<td>1985Q1</td>
<td>2011Q4</td>
<td>FRB</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $s_y$, Real Economic factor; $s_x$, Inflation factor; $s_{sc}$, Supply of funds factor; $s_u$, Uncertainty factor; $s_{sf}$, Financial Intermediation factor; $s_{sd}$, Credit demand factor; +, positive sign restriction; –, negative sign restriction; ?, no sign restriction.
restrictions on the current values (but not on the lags) of the unobserved common factors and zero restrictions on current and lagged values of the factors. This combination of sign and zero restrictions allows us to identify multiple unobserved factors. We also use a normalization restriction which sets the coefficient on an observation equation for each factor equal to 1. This essentially sets the scale of the unobserved factor which is otherwise not identified.

Most of the sign and zero restrictions given in Table 1 are straightforward. We select real GDP, industrial production, total non-farm payrolls, and the National Association of Purchasing Managers composite index as indicators. All of these variables are assumed to have a positive factor loading on the output factor and zero factor loadings on the other factors. We view these variables as indicators of the output factor solely. Any correlation between these output indicators and other observed indicator variables in the model is due solely to the correlation of the output factor with the other common factors. Similarly, the percentage changes in GDP deflator (implicit price deflator), aggregate Personal Consumption Expenditure (PCE) index, aggregate Consumer Price Index (CPI-U), and CPI-core are assumed to have a positive factor loading on the inflation factor and zero factor loadings on the others.

To extract credit demand, financial intermediation, and supply of funds factors, we employ various short-term interest rates, interest rate spreads, and credit quantities as indicators of credit conditions. These indicators roughly correspond to $r_s$, $r_L-r_s$, and $L$ from the model in Section 2. The short-term rates we include as indicators are the 3-month commercial paper rate and the 3-month Libor rate as well as the Fed Funds rate and Treasury Bill rates for 3-month, 6-month, and 1-year maturities. The simple model of financial intermediation presented in Section 2 suggests that short rates ($r_s$) respond positively to an increase in the credit demand factor or in the supply of financial intermediation factor while responding negatively to an increase in the supply of funds factor. Thus, we impose those restrictions on the factor loadings of the 3-month commercial paper and Libor rates. For the government rates, we leave their factor loadings unrestricted as while one might expect these variables to be closely related to the borrowing rates of financial intermediaries, they are not rates at which financial intermediaries actually borrow.\(^3\) We include these government rates as they are often included in empirical work examining credit conditions.\(^4\)

\(^3\) We obtain very similar results when we impose the same sign restrictions on government securities' factor loadings as on the commercial paper and Libor rates.  
\(^4\) The commercial paper/T-Bill spread has often been used as an indicator of credit conditions [see for example, Friedman and Kuttner (1992)]. Similarly, the financial conditions indices of Hakkio and Keeton (2009) and Hatzius et al. (2010) include yields on government securities among their indicators.
We also include several lending-borrowing spreads as indicators. Since most financial intermediaries tend to borrow short and lend long [see Adrian, Estrella, and Shin (2010), Adrian and Shin (2010)], we select the spreads between AA, A, BBB, BB (S&P rated), as well as Aaa, Baa and High Yield (Moody’s rated) corporate bond rates and the 3-month Commercial paper rate, respectively, as indicators for the lending-borrowing spread.\(^5\) As described in Section 2, the lending-borrowing spreads are assumed to have positive factor loadings for the credit demand and supply of funds factors, and negative factor loadings for the financial intermediation factor. The selected credit quantity variables include Commercial and Industrial (C&I) loans, commercial paper outstanding, consumer installment credit outstanding, the ratio of total net borrowings of household and nonfinancial business sectors to GDP,\(^6\) total assets of security brokers and dealers, total loans of commercial banks, shadow bank assets [as in Adrian and Shin (2010)], and total credit liabilities of nonfarm nonfinancial businesses from the Flow of Funds data. The factor loadings of the credit quantity variables are positive for credit demand, supply of financial intermediation, and supply of funds factors. We also include the net percent of loan officers reporting tightening commercial credit standards reported by Senior Loan Officer Opinion Survey on bank lending practices, and NFIB survey of small business loan availability compared to 3 months ago (Dunkelberg and Scott 2009). These variables have been used as indicators of credit market tightness elsewhere in the literature.\(^7\) However, as it is not clear whether “tightness” is due to high credit demand or low credit supply, we leave the factor loadings of these indicators unrestricted.

Among the indicators of uncertainty, we include a few stock market volatility variables: the standard deviations of the returns in the Standard and Poor’s 500 composite index, standard deviation of the NYSE-AMEX-NASDAQ value weighted accumulated return index, the VIX and VXO indices, as well as cross-sectional dispersion of firms’ stock returns.\(^8\) These stock market volatility variables are the exclusive indicators of our uncertainty factor and have positive factor loadings. In addition to these volatility variables, we also include as indicators of aggregate uncertainty the Consumer Confidence Index (survey of present situation) and the

\(^{5}\) The results are insensitive to the choice of alternative short-rates with which to calculate the spreads.

\(^{6}\) Total net borrowings of household sectors and nonfinancial business are obtained from the Federal Reserve Statistical Release, Z.1, Flow of Funds Accounts, Table F.1 “Total Net Borrowing and Lending in Credit Markets” and are the sum of lines 3–6.

\(^{7}\) Lown and Morgan (2004) and Asea and Blomberg (1996) found that the lending standard survey is informative in explaining variation in business loans and real economic activity.

\(^{8}\) The series is calculated following Gilchrist, Sim and Zakrajsek (2009a) and Bloom, Floetotto and Jaimovich (2009).
“Distance to Default” for corporate bonds (25th and 75th percentiles) from Gilchrist and Zakrajsek (2010). For these two indicators, we allow them to respond to the output factor in addition to the uncertainty factor. Finally, we allow short-term interest rates, interest rate spreads, and credit quantities to respond to the uncertainty factor (in addition to the three credit factors) but impose no restrictions on those factor loadings. As suggested by Chari, Christiano and Kehoe (2008) and Hall (2010), asset reallocation as a result of changes in uncertainty can result in interest rate changes independently of credit market conditions.

4 Empirical results

Our data sample runs from January 1985 to December 2011. We estimate the model using Bayesian Markov Chain Monte Carlo (MCMC) methods, here the Gibbs sampler. The Markov chain ran for a 100,000 iteration burn-in period after which the empirical posterior distribution is obtained by sampling every tenth draw from the Gibbs sampler for a total of 10,000 draws from the posterior distribution. By taking the unobservable states (common factors) as parameters, we can obtain the joint posterior distributions of the parameters and the states. Appendix A provides a detailed description on applying the Bayesian estimation approach.

4.1 Behavior of six common factors

Figure 1 displays the median of the posterior distribution along with the 10th and 90th percentiles for the six common factors. The output factor captures the co-movements of several indicators of output growth and mostly coincides with business cycles. The largest decline in the output factor occurs during the “Great Recession” in 2007–2009. In fact, this factor displays negative values starting in October of 2007 all the way through June 2009. The inflation factor reflects the fact that since the mid-1980s there has been very little persistence in the growth rate of aggregate prices. Interestingly, it does show a steep (but temporary) decline in late 2008.

The aggregate uncertainty factor primarily mirrors stock market volatility. There are large increases in the uncertainty factor during the stock market crash in October 1987 and the 2007–2008 subprime mortgage crisis. Also the 1997 Asian financial crisis, September 2001, the Flash Crash in May 2010, and the volatility associated with European Debt crisis in August 2011 are all reflected as large increases in the estimated uncertainty factor.
Figure 1: Mean, 10th and 90th percentiles of the posterior distributions of common factors.
The supply of funds factor displays two large spikes upward in 2008. The first
is in January through March of 2008 and corresponds with the round of interest
rates cuts initiated by the Federal Reserve in early 2008. The second spike
occurs in October and November of 2008 and reflects the dramatic expansion of
the Federal Reserve Balance sheet that occurred during this period.9 The credit
demand factor displays declines around recessions, and in 1990 and 2001 the
decline in credit demand actually preceded the recession. During the 2007–2009
recession, the credit demand factor fell precipitously during the September
through December 2008 period before recovering relatively quickly. The financial
intermediation factor displays small declines in the early 1990s and in 2001–2002,
around these recessions, but the financial intermediary factor typically lagged
behind economic activity. Like the credit demand factor, it also displays a large
decline starting in late 2008. However, the decline in the financial intermediation
factor, which reaches a trough in May–June 2009, is much more persistent
than the decline in the credit demand factor. All these periods have been sug-
gested as periods of deteriorating credit conditions: the credit crunch following
the S&L crisis of the early 1990s, the credit bear market following the dot-com
bubble burst of early 2000s, and the most recent subprime mortgage crisis
(Bernanke and Lown 1991; Clair and Tucker 1993; Gilchrist, Yankov, and Zakrajsek
2009b; Hatzius et al. 2010). The credit conditions index in Hatzius et al. (2010)
also shows a substantial deterioration in 2008 and a rebound towards the end of
2009. Also, our results are consistent with the claim of Woodford (2010), Adrian
and Shin (2010), and others who argue that financial intermediation played an
important role in the 2007–2009 financial crisis. That credit conditions have been
slow to recover seems in our analysis to be due more to the supply of financial
intermediation rather than the supply of funds or the demand for credit.

4.2 Contributions of factors to fluctuations in indicators

To assess how well our factors capture the fluctuations in the indicators, we
calculate the contribution of each factor to fluctuations in selected indicators.
Figures 2–6 present the median of the posterior distribution for historical decom-
positions of the selected indicators.10 The indicators are displayed using dotted
lines and the solid lines plot the contributions of the common factors. Note that

9 We do not include Federal Reserve Balance sheet variables in our set of indicators precisely
because these variables move so dramatically over this period.
10 To save space we present only a selected number of indicators. The historical decompositions
for all of the other indicators are available upon request.
Figure 2  Contributions of output, inflation and uncertainty factors to their indicators.
Figure 3 Contributions of uncertainty and credit factors to short rates.
Figure 4 Contributions of uncertainty and credit factors to low risk corporate – 3-month commercial paper spreads.
Figure 5 Contributions of uncertainty and credit factors to high risk corporate – 3-month commercial paper spreads.
Figure 6  Contributions of uncertainty and credit factors to credit quantity variables.
these are not orthogonal decompositions, as the factors are generally correlated with one another.

Figure 2 displays the contribution of the output factor, inflation factor, and uncertainty factor to four of each factor’s indicators. The output and inflation factors track fluctuations in output and inflation indicators well. For uncertainty indicators, it is clear that the uncertainty factor captures movements not only in realized stock market volatility but also implied volatility as measured by the VIX and VXO indices. The uncertainty factor also captures a large portion of the fluctuations in the distance-to-default measure (DD75) with distance-to-default rising when the uncertainty factor is low and vice versa.

Figure 3 displays the contributions to short-term interest rates of the supply of funds, demand for credit, and financial intermediation factors along with the contribution of the uncertainty factor. One observes that the supply of funds factor explains much of the movements in the level of short-term interest rates over the sample. Interestingly, the intermediation factor also contributes substantially to the contribution of short-term interest rates. On the other hand, the contributions of the credit demand factor and the uncertainty factor are relatively small.

Figures 4 and 5 present the decompositions of interest rate spreads. Figure 4 displays the contributions of supply of funds, intermediation, credit demand, and uncertainty factors to the spreads between, respectively, Aaa, AA, A, Baa rated corporate bonds and the 3-month commercial paper rate. Figure 5 displays the decompositions of the spreads between, respectively, BBB, BB, HY rated bond yields and the 3-month commercial paper rate. These two figures suggest that the contributions of supply of funds factor are important to interest rate movements. The channel here is primarily through the effect of the supply of funds on short-term rates – the steepening of the yield curve when the supply of funds factor increases is largely due to short rates falling. The financial intermediation factor also contributes to fluctuations in interest spreads. This effect tends to lag behind the effect of the supply of funds factor. Interestingly, as the bond rating gets lower the effect of the uncertainty factor gets more important for yield spreads. For High Yield bonds, the uncertainty factor tends to be the biggest contributor to the yield spread.

Turning to Figure 6, which displays the nonstructural decompositions of a few credit quantity variables, one observes that much of the movements of these variables are driven by the financial intermediation and credit demand factors with the intermediation factor being the principle source of fluctuations

11 We only report the C&I loans, net credit tightening standard survey, loan availability survey, commercial bank total loans, and total credit liabilities. The decompositions of other series are consistent with the results of the selected indicators, and are available upon request.
Credit demand, credit supply, and economic activity

in commercial and industrial loans, commercial bank loans, and total credit liabilities. Focusing on the financial crisis of 2007–2008, the credit demand factor contributes to the decline in credit quantities (and the rise in loan officer survey) earlier in the downturn while the financial intermediation factor contributes later in the downturn. The supply of funds and uncertainty factors contribute little to movements in credit quantities. Taking Figures 3–6 as a whole, changes in the supply of funds results in interest rate changes but very little change in credit quantities.

4.3 Orthogonal shocks in credit, output, inflation, and uncertainty factors

Recall that the model allows the factors to be correlated with one another through the state equation, which is a VAR, and also through correlation in the innovations in the factors. Table 2 displays the median of the posterior distribution (along with the 10th and 90th percentiles) of correlations between innovations in the factors. The estimated correlations are generally small with the interior 80% of the posterior distribution including the value zero. The exception is the correlation between innovations in the credit demand factor and the financial intermediation factor, which is negative and relatively large (in absolute value). This suggests that shocks tend to cause credit demand and financial intermediation factors to move in opposite directions. In models of credit frictions based on asymmetric information, a negative balance sheet shock – a shock that results in a deterioration of the balance sheets of borrowers and lenders – forces borrowers to turn to external financing for investment projects and, hence, raises credit demand, while that same balance sheet shock reduces the risk appetite of financial intermediaries which results in a decrease in the supply of financial intermediation.

To understand better the dynamic interaction among factors, we examine the response of the underlying factors and the indicators with respect to orthogonal shocks in the estimated factors. The orthogonal shocks are identified by assuming recursive structure using the following Cholesky ordering: real output, inflation, credit demand, financial intermediation, supply of funds, and lastly uncertainty. The ordering of real output and inflation before the credit factors is conventional and in line with much of the monetary VAR literature. Based on the “broad credit view,” where the demand for credit directly also reflects the creditworthiness of borrowers and, hence, affects the financial intermediation decision, we assume a shock to the credit demand factor to have contemporaneous effect on the supply of financial intermediation, and, therefore, order credit...
Table 2  Correlations between innovations in factors.

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Inflation</th>
<th>Credit demand</th>
<th>Financial intermediation</th>
<th>Supply of funds</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>–0.038 (–0.155, 0.080)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit demand</td>
<td>0.065 (–0.106, 0.233)</td>
<td>0.026 (–0.144, 0.197)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial intermediation</td>
<td>–0.042 (–0.210, 0.121)</td>
<td>–0.003 (–0.174, 0.164)</td>
<td>–0.696 (–0.827, –0.513)</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supply of funds</td>
<td>–0.084 (–0.232, 0.066)</td>
<td>–0.029 (–0.169, 0.111)</td>
<td>–0.278 (–0.557, 0.030)</td>
<td>0.222 (–0.064, 0.497)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Uncertainty</td>
<td>0.016 (–0.116, 0.147)</td>
<td>–0.076 (–0.179, 0.026)</td>
<td>–0.153 (–0.434, 0.214)</td>
<td>0.101 (–0.213, 0.363)</td>
<td>0.055 (–0.280, 0.345)</td>
<td>1</td>
</tr>
</tbody>
</table>

Median, 10th, and 90th percentiles of the posterior distribution.
demand before financial intermediation. As the supply of funds factor would, in part, reflect monetary policy responses to the current state of the economy, we order the supply of funds factor after the other credit factors. Finally, we order aggregate uncertainty last because our uncertainty factor largely reflects volatility in financial markets, which is sensitive to almost all economic and credit conditions. Most of the results are robust to alternative orderings.12

Figure 7 displays the median (solid line), 10th percentile (dashed line), and 90th percentile (dashed line) from the posterior distribution of the responses of the factors to output, inflation, and uncertainty shocks. The rows represent the type of shock while the columns represent the response of the factor. An output shock results in increases in the output factor and a delayed increase in the inflation factor. The financial intermediation factor tends to increase, with some delay, after an output shock as does the credit demand factor while the supply of funds factor and the uncertainty factor tend to exhibit small declines. These responses appear to be similar to those one might expect from a positive aggregate demand shock. Inflation shocks largely have insignificant effects on the other factors, although the median response suggests responses reminiscent of a negative aggregate supply shock. Uncertainty shocks result in declines in the output and inflation factors, a contraction in the financial intermediation and credit demand factors, but an increase in the supply of funds factor. The uncertainty shock seems to have effects similar to what one might expect if an increase in uncertainty has a negative effect on aggregate demand.

Figure 8 displays the responses of all the factors to orthogonal shocks in credit demand, financial intermediation, and supply of funds. A credit demand shock results in increases in the output and inflation factors as well as a temporary decrease in the uncertainty factor. Such a shock also has a negative effect on financial intermediation and the supply of funds factors. Financial intermediation shocks tend to result in increases in the output factor and a decline in the credit demand factor, although the interior 80 percentiles of responses include zero. The response of inflation, uncertainty factors, and supply of funds factors are, on the other hand, close to zero. Finally, a supply of funds shock tends not to have significant effects on the output and inflation factors. While the median responses of the uncertainty and financial intermediation factors are negative and the response of the credit demand factor is positive, the interior 80% intervals for these responses include zero.

12 Perhaps, this is not too surprising given the correlations between innovations in the factors were relatively small, with the exception of credit demand and supply of financial intermediation. Changing the order of credit demand and financial intermediation affects the impulse response analysis somewhat but had little effect on the historical decompositions.
Figure 7: Impulse responses of factors to output, inflation, and uncertainty shocks. Median, 10th, and 90th percentile of the posterior distribution.
Figure 8. Impulse responses of factors to supply of funds, financial intermediation, and credit demand shocks.

Median, 10th, and 90th percentile of the posterior distribution.
As most of the output, inflation, and uncertainty indicators respond to those factors only, the impulse responses of those indicators will look similar to the responses of their underlying factors displayed in Figure 7. However, since interest rates, interest rate spreads, and credit quantity indicators are affected by the three credit factors as well as uncertainty, it is useful to examine the response of these indicators to various orthogonal shocks. Figure 9 displays the median, 10th, and 90th percentiles of the posterior distribution of responses of the Fed Funds rate, the Baa-Commercial paper spread, and total credit liabilities to orthogonal shocks. Moving across the first row of Figure 9, we find that the Fed Funds rate responds positively to an orthogonal output shock but ambiguously to an inflation shock. As the output factor does not have a direct effect on interest rates, the impact of an output shock on interest rates must be through its effect on the credit factors. From Figure 7, output shocks tend to cause credit demand and financial intermediation to rise and the supply of funds to fall. All of these effects tend to result in an increase in the Fed Funds rate. Not surprisingly, both orthogonal credit demand and financial intermediary shocks result in an increase in the Fed Funds rate while a supply of funds shocks results in a decline in the Fed Funds rate. Interestingly, an uncertainty shock results in a decline in the Fed Funds rate.

The response of the Baa-Commercial paper spread to an output shock is negative while an inflation shock has an ambiguous effect on the spread. Orthogonal shocks in the financial intermediation factor lower the Baa-Commercial paper spread while shocks to supply of funds raise the spread, much like one might expect given the simple model of Section 2. On the other hand, orthogonal shocks in credit demand raise the spread initially but have an ambiguous effect thereafter. Uncertainty shocks raise the Baa-Commercial paper spread. Finally, the last row of Figure 9 presents the response of total credit liabilities. Shocks to the output and financial intermediation factor increase total credit liabilities as one might expect, while shocks to other factors have only ambiguous effects. Note that as the output factor has no direct impact on credit quantity indicators, its effect on total credit liabilities is through the credit factors.

In summary, orthogonal shocks to credit factors have the anticipated effects on the various credit indicators. Of the credit factors, orthogonal financial intermediation shocks tend to have the largest impacts on interest rates, spreads, and credit quantities. Positive orthogonal shocks to both financial intermediation and credit demand result in an increase in output. Our model

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13 The responses of the rest of the indicators are all consistent with the ones of the representative indicators. They are not reported here but are available upon request.
14 Note that we do not impose any sign restriction on the factor loadings of short-term interest rates or interest rate spreads with respect to the uncertainty factor.
Figure 9: Impulse responses of funds rates, Baa-CP spread and total credit liabilities. Median, 10th, and 90th percentile of the posterior distribution.
also exhibits evidence of feedback from output and uncertainty shocks to the credit factors, with positive orthogonal output shocks resulting in an increase in the financial intermediation factor (to a lesser extent the credit demand factor as well) and a decline in the supply of funds factor. Uncertainty shocks result in an increase in the supply of funds and a contraction in the supply of financial intermediation.

4.3.1 Historical decompositions with orthogonal shocks

To assess how important the various shocks were in actual fluctuations, we examine historical decompositions based on orthogonalized shocks. Figures 10 and 11 decompose fluctuations in the underlying factors while Figures 12 and 13 display historical decomposition of selected indicators. The decomposition presented are the median contribution from the posterior distribution.

From Figure 10, we observe that most of the fluctuations in the output factor are due to output shocks; the recessions of 1990–1991, 2001–2002, and 2007–2009 appear to be driven largely by output shocks. Of the other factors, uncertainty shocks also seem to have a role in contributing to output fluctuations, albeit a relatively small one. Inflation fluctuations are almost entirely due to inflation shocks. Finally, while fluctuations in the uncertainty factor are largely driven by uncertainty shocks, output shocks do seem to contribute to an increase in uncertainty during the 2008–2009 period.

Figure 11 displays the historical decomposition for the credit factors. For the credit demand factor, most of the fluctuations are due to shocks in the credit demand factor itself – the shocks to the other factors have only minor contributions. Fluctuations in the financial intermediation factor, however, have several contributors. While financial intermediation shocks are the largest contributor, output shocks also contribute to declines in the financial intermediation factor during recessions, particularly the recession of 2007–2009. Uncertainty shocks and credit demand shocks also are contributors to the decline in the financial intermediation factor in 2008. The supply of funds factor is largely driven by its own shocks, but output shocks do contribute to the increase in the supply of funds factor in late 2008 perhaps reflecting the actions taken by the Fed in response to the financial crisis.

Figure 12 reports the historical decompositions of a select number of indicators, including real GDP, CPI inflation, and the VXO. Unlike the individual factors, the shocks in the common factor need not explain all the fluctuations in an indicator variable as each indicator variable has its own idiosyncratic component. The decompositions of real GDP growth show that output shocks are responsible
Figure 10  Historical decompositions of output, inflation and uncertainty factors.
Figure 11: Historical decompositions of credit demand, financial intermediation and supply of funds.
Figure 12: Historical decompositions of real GDP, CPI and VXO index.
Figure 13 
Historical decompositions of fed funds rate, Baa-CP spread and total credit liabilities.
for the vast majority of its fluctuations, although the uncertainty shock contributes partially to the dip during the “Great Recession.” Not surprisingly, given that the inflation factor is largely unaffected by shocks in the other factors, CPI inflation is driven almost entirely by inflation shocks. Fluctuations in the VXO on the other hand appear to be due to both uncertainty shocks and to output shocks, particularly in 2008–2009.

Turning to indicators of credit conditions, in Figure 13 shocks to the output factor play a large role in the fluctuations in these variables. The large decline in the Fed Funds rates in 2008 is largely due to the response to output shocks over this period. Shocks to the financial intermediation, the supply of funds, and the uncertainty factors also contribute to Fed Funds rate fluctuations over the sample. The output and uncertainty factors also are important contributors to fluctuations in the Baa-Commercial paper spread as are shocks to the financial intermediation and the supply of funds factors. The dramatic increase in Baa-Commercial paper spreads in 2008–2009 is largely attributed to output shocks, although supply of funds shocks also contribute early and uncertainty shocks contribute later in this period to the increase in spreads. Finally, turning to an indicator of credit quantity, total credit liabilities from the Flow of Funds, we observe that output and financial intermediation shocks contribute most to fluctuations in this variable. The decline in credit liabilities in the early 1990s was due primarily to financial intermediation shocks. During the 2008–2009 period, output shocks contribute early while financial intermediation shocks contribute later to the large and persistent decline in total credit liabilities.15

Overall, these results suggest that shocks originating in the financial sector were not a particularly important source of fluctuations in economic activity in our sample. Given the financial dislocations during the “Great Recession,” these results would on the surface appear to be surprising. However, given the narrative of the 2008–2009 Financial Crisis, the precipitating “shock” was the stalling of house price appreciation, the consequent stagnation of residential construction, and general economic activity. All these occurred before there were significant changes in aggregate financial variables. This is not to say that financial

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15 On the advice of an anonymous referee, we checked to see whether the results were sensitive to the inclusion of the 2008–2009 financial crisis in the sample. We estimated the model up the 2006 and then used the Kalman filter to decompose the observed time series into contribution of the underlying factors and structural shocks for the entire sample. Most of the results using the short sample to estimate the model are similar to those of the model estimated to the full sample. Where the results differ are in decomposition of short-term interest rates. In the short sample model, the credit demand factor plays a larger role in the decomposition of short-term interest rates than in the full sample model.
frictions were not important in propagating shocks originating elsewhere, only that changes in these other variables temporally preceded changes in the credit indicators we employ.\footnote{Balke (2000) examines a threshold VAR where the threshold variable depends on credit conditions. In this case, credit conditions can have an effect independent of credit shocks by triggering a change in regime.}

5 Concluding remarks

In this paper, we attempt to separate credit conditions into factors representing credit demand, supply of financial intermediation, and supply of funds. We estimate a dynamic factor model whose factors correspond to measures of aggregate real economic activity, overall inflation, aggregate uncertainty, credit demand, supply of financial intermediation, and the supply of funds. When estimating the credit factors, the supply of funds and financial intermediation factors tend to be associated with fluctuations in interest rates and interest rate spreads while the demand for credit and financial intermediation factors tend to be associated with credit quantities. Imposing a recursive structure on the innovations in the factors, we find that shocks to the credit factors have contributed only a little to output fluctuations while output and uncertainty shocks have contributed to fluctuations in the credit factors especially the financial intermediation factor. This suggests that credit’s role is more of a propagator of shocks rather than a source of shocks.

Acknowledgments: Many thanks to Nick Bloom and William Dunkelberg for supplying their data. We also thank Tim Fuerst, Kundan Kishore, and Peter VanderHart as well as seminar participants at the June 2010 Western Economic Association Meetings and the February 2011 Eastern Economic Association Meetings for helpful comments and suggestions. The views expressed in this paper are solely those of the authors and not those of the Federal Reserve Bank of Dallas or the Federal Reserve System.
Appendix A

A. Bayesian estimation

Kim and Nelson (1998) and Johannes and Polson (2003) provide detailed introductions to Bayesian Markov Chain Monte Carlo methods. The objective of the estimation is to find the posterior distribution of both the unknown parameters, \( \Theta \), and the unobservable state vector, \( S \), given the observable indicators, \( Y \). By sequentially sampling \( S^{(i)} \) from posterior distribution \( P(S|\Theta, Y) \) and \( \Theta^{(i)} \) from \( P(\Theta|S, Y) \), the resulting sample distribution of \( (S^{(i)}, \Theta^{(i)}) \) converges to \( P(S, \Theta|Y) \).

Define \( \tilde{S}_t = [S_1, S_2, \ldots, S_T] \)' and \( \tilde{Y}_t = [Y_1, Y_2, \ldots, Y_T] \)' . The steps taken in the Gibbs Sampler are as follows:

1. Taking the parameters of the state space model as given, draw a realization of the state vector \( S_t \), conditional on the model’s parameters and the observed data. Because the state space model presented by equation (1) and (2) is linear and Gaussian, the distribution of \( S_t \) given \( \tilde{Y}_t \), and that of \( S_t \) given \( S_{t+1} \) and \( \tilde{Y}_t \) for \( t=T-1, T-2, \ldots, 1 \) are also Gaussian:

\[
S_t \sim N(S_{t|\tilde{Y}_t}, P_{t|\tilde{Y}_t}),
\]

\[
S_t | \tilde{Y}_t, S_{t+1} \sim N(S_{t|\tilde{Y}_t, S_{t+1}}, P_{t|S_{t+1}});
\]

where \( S_{t|\tilde{Y}_t} = E(S_t | \tilde{Y}_t) \), \( P_{t|\tilde{Y}_t} = \text{Cov}(S_t | \tilde{Y}_t) \); \( S_{t|S_{t+1}} = E(S_t | \tilde{Y}_t, S_{t+1}) = E(S_t | S_{t+1}, \tilde{Y}_t) \), \( P_{t|S_{t+1}} = \text{Cov}(S_t | \tilde{Y}_t, S_{t+1}) = \text{Cov}(S_t | S_{t+1}, \tilde{Y}_t) \).

We can take Kalman filter approach to obtain \( S_{t|\tilde{Y}_t} \) and \( P_{t|\tilde{Y}_t} \) by filtering forward:

\[
S_{t|t-1} = S_{t-1} + P_{t|t-1} \cdot H'(H'P_{t-1} + H'+R)^{-1}\eta_{t-1},
\]

\[
P_{t|t-1} = P_{t-1} - P_{t|t-1} \cdot H'(H'P_{t-1} + H'+R)^{-1}H'P_{t|t-1},
\]

where \( \eta_{t-1} = (Y_t - AY_{t-1} - HS_{t-1}) \) in our model is the new information that \( Y_t \) can bring to forecasting \( S_t \), and is weighted by the variance of the conditional forecast error \( (H'P_{t-1} + H'+R) \).

We draw \( S_{t(i)} \) from the conditional distribution, and obtain \( S_{t|S_{t+1}} \) and \( P_{t|S_{t+1}} \) by sampling backwards:

\[
S_{t|S_{t+1}} = S_{t|t} + P_{t|t} \cdot F'(FP_{t|t}F'+Q)^{-1}\eta_{t+1},
\]

\[
P_{t|S_{t+1}} = P_{t|t} - P_{t|t} \cdot F'(FP_{t|t}F'+Q)^{-1}FP_{t|t}.
\]

17 In our model, the elements of matrices \( A, H, F, Q \) and \( R \).
where $\eta^*_{t+1}$ is the new information updated by the state equation and equals to $(S_t^{t+1} - F S_t^t)$ in our model, and is weighted by the variance $(FP_t F^t + Q)$.

2. Taking state vector $\tilde{S}_t$ and variance-covariance matrix $Q$ as given, draw parameters of the state equation, i.e., the elements in $F$ matrix. We can rewrite the state equation in matrix form as below:

$$y = \rho X + \nu, \quad \nu \sim \mathcal{N}(0, Q);$$

where $X = (S_0, S_1, \ldots, S_{T-1})'$ and $y = (S_1, S_2, \ldots, S_T)'$.

We employ a multivariate normal prior distribution for $\rho$ given by

$$\rho \sim \mathcal{N}(\alpha, \Sigma) I[\{s(\rho)\}]$$

where $\alpha$ and $\Sigma$ are known and $I[\{s(\rho)\}]$ is an indicator function used to denote that roots of $F(L)$ lie outside the unit circle. The posterior distribution for $\rho$ is given by

$$\rho | \tilde{S}_t \sim \mathcal{N}(\tilde{\alpha}, \tilde{\Sigma}) I[\{s(\rho)\}]$$

where

$$\tilde{\alpha} = (\Sigma^{-1} + X'X)^{-1}(\Sigma^{-1} \hat{\alpha} + X' \tilde{S}_t),$$

$$\tilde{\Sigma} = (\Sigma^{-1} + X'X)^{-1}.$$

We draw $\rho$ from the posterior and discard the draws if the roots lie outside the stationary region.

3. Taking the state vector $\tilde{S}_t$ and $F$ matrix as given, draw elements in the variance-covariance matrix $Q$. We employ an Inverse-Wishart prior distribution $W^{-1}(\Psi_0, \kappa_0)$, where $\Psi_0$ and $\kappa_0$ are known. The posterior distribution is given by $W^{-1}(\Psi_1, \kappa_1)$ where

$$\Psi_1 = \Psi_0 + \sum_{t=1}^{T} v_t v_t',$$

$$\kappa_1 = \kappa_0 + T.$$

4. Taking the state vector $\tilde{S}_t$ and the $R$ matrix as given, draw parameters in the measurement equation, i.e., the elements in $A$ and $H$ matrices. The posterior distributions have similar forms as those in step 2 with the linear regressions given by the measurement equations. Draws of parameters which resulted in roots lying outside the stationary region or violated sign restrictions were discarded and another new draw was made.

In order to help setting the scale of the factors, we set tight priors (around one) on the coefficients of the Fed Funds rate, Commercial and Industrial loan growth, and S&P 500 composite volatility to set the scales of our credit, and
uncertainty factors, respectively. Also, we normalize the factor loadings of real GDP and GDP deflator to fix the scale of our output and inflation factors.

5. Taking the state vector $S_t$ and the parameter vector $A(L)$ and $H(L)$ as given, draw variances in $R$ matrix from the Inverse-Wishart posterior distribution similar as those in step 3 with linear regressions given by the measurement equations.

6. Repeat steps 1–5. This is done for a burn-in buffer of 100,000 draws after which every tenth draw is used to form the posterior distribution consisting of total 10,000 draws.

References


