The Empirical Content of Nash-Bargained Household Behavior

Marjorie B. McElroy

ABSTRACT

In households whose allocation decisions can be represented as Nash-bargained household decisions, extrahousehold environmental parameters (EEPs) serve as pure shifters of the threat points. The comparative statics of changes in demands due to changes in these EEPs are given. These are incorporated into a comprehensive statement of the empirical content of Nash-bargained household behavior, including a Nash generalization of Barten’s (1966) fundamental matrix equation of the theory of consumer demand. Estimation and data requirements are discussed along with nested testing of the following structure: the neoclassical model is nested in the Nash-bargained model which, in turn, is nested in an unrestricted model of household demands. Emphasized throughout is the enriched menu of explanatory variables for demand analysis provided by the Nash model, as well as the model’s ability to jointly analyze (i) household formation and (ii) intrahousehold allocation decisions.

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I. Introduction

McElroy and Horney (1981) (hereafter MH) proposed a co-operative Nash-bargained model of household behavior. Nash models permit and resolve conflicts among family members. Each household member has a utility function and a threat point (maximal level of utility outside of the household). The greater a member's threat point, the more strongly that member's relative valuation of goods is reflected in the household demands. Prices, individual-specific nonwage incomes, and parameters controlling how well an individual can do outside the household all affect threat points. For the Nash model, MH derived the associated system of household demand (and labor supply) functions, showed that the comparative statics of the Nash model generalize those of the neoclassical model (including generalizations of the income and substitution effects, the Slutsky equation, Engel aggregation, and the symmetry and negative semidefiniteness of the substitution matrix), and suggested certain tests.

In contrast to the Nash-bargained models of household behavior, neoclassical models essentially lead to the conclusion that households behave as if they have one neoclassical utility function. There are several ways to rationalize this approach. One is Becker's (1981) celebrated concept of altruism. Another is Samuelson's (1963) social welfare function coupled with an intrafamily income distribution determined by its members' marginal "ethical worths." A key issue that separates bargaining from neoclassical models is the treatment of income: in neoclassical models, only pooled family income matters; in the bargaining approach, who has control over the various income sources matters. In an old (1979) but recently published working paper, Horney and McElroy (1988) found weak evidence against the neoclassical model. More recently, Altonji, Hayashi, and Kotlikoff (1989), as well as Cai (1989), rejected the pooling hypothesis of the neoclassical model. In this issue Schultz (1990) and Thomas (1990) also find evidence against the income-pooling restrictions imposed by the neoclassical model. All of these empirical findings are readily explained by the Nash-bargained model of household behavior.

A second key issue that separates neoclassical from Nash-bargained household decisions is highlighted in this paper. It is whether the opportunity cost of family membership matters for the intrafamily distribution of income and therefore for the household demands: in the Nash model the opportunity cost of family membership matters; in the neoclassical model it does not. The person-specific nonwage incomes discussed above are

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1. Manser and Brown (1980) independently proposed a different Nash-bargained model. The relationship between the models in MH and Manser and Brown is discussed in MH.
only one of the potentially numerous determinants of the opportunity cost of family membership. In this paper the variables that determine the opportunity cost of family membership are taken as parametric and called extrahousehold environmental parameters. Although not specifically based on a Nash model, several studies have found that the intrafamily distribution of resources favors family members with better (current or future) outside opportunities; see, for example, Rosenzweig and Schultz (1982), Senauer, Garcia, and Jacinto (1988), and Grossbard-Schechtman and Neuman (1988). Again, such results are difficult to explain with a neoclassical model, but are readily explained by the Nash-bargained model of household behavior.

Nevertheless, none of the above papers directly pose the empirical implications of the Nash model as either a null or specific alternative hypothesis. This leaves undone nested testing of the following structure: the neoclassical model is a special case of the Nash model, which in turn is a special case of an unrestricted demand system. Chiappori (1988b) appropriately drew attention to the lack of testing of the null hypothesis that the neoclassical restrictions hold versus the specific alternative hypothesis that the more general Nash restrictions hold. But, his mistaken claim that the Nash model has "virtually" no empirical content highlights the need for a clear statement of the empirical implications of the Nash model; see McElroy and Horney's (1990) reply. This paper gives a comprehensive statement of the empirical implications of the Nash model, paying attention to the data requirements. It also lays out the comparative statics of the Nash demand system associated with changes in parameters that shift the threat points. The paper is organized as follows: Section II reviews salient elements of the Nash model; Section III derives the comparative statics of changes in household demands due to pure shifts in the threat points. Section IV explicitly states the empirical content of the Nash-bargaining hypothesis and the fundamental matrix equation of the Nash-bargained theory of household behavior—this is shown to be a generalization of Barten's (1966) fundamental matrix equation of the neoclassical theory of consumer demand. Section V augments the neoclassical null hypothesis and then nests this "augmented" neoclassical hypothesis within the Nash hypothesis; while Section VI discusses the data requirements for testing. Section VII eliminates redundant restrictions in both the Nash and neoclassical models; Section VIII discusses the invariance of utility functions across marital statuses; and Section IX sketches one way to estimate the Nash model and test the Nash versus the neoclassical model. Section X concludes.

In what follows each individual can be in one of two states, "married" or "unmarried." Most narrowly, one may interpret the model as describing the intrafamily allocation of resources for a legally married couple.
But a much broader range of interpretations is available. The model is
certainly not restricted to legal marriage. Nor is the unmarried state
restricted to being single forever; one might, for example, be searching for
another mate. Further, the players need not be of the opposite sex. They
may, for example, be of different generations: McElroy (1985) interpreted
the model as describing the joint decisions of young men and their parents
concerning both where the son lived and whether and how much he
worked. Most generally, being "married" can be interpreted as being
one of two players sharing resources in a cooperative game, played volun-
tarily in the expectation of gains from cooperation with the other player.
The "unmarried" state, then, defines the opportunity cost of playing the
game. For economy of language, I denote the states as "married" and
"unmarried" and the players as $m$ and $f$ (male and female, or mother and
father), but the reader is asked to bear in mind that a much broader range
of interpretations is available.

II. Review of the Nash Model

This section draws freely on MH. The model postulates
two individuals, $m$ and $f$, who, if married to each other, jointly allocate
resources according to the solution to a two-person, Nash, cooperative
game. A key feature of the game is the threat point of each player. Person
$m$'s threat point is $m$'s opportunity cost of being married—the best that
$m$ can do on his own. Person $f$'s threat point is defined analogously. This
section, after first establishing the threat points, states the Nash game
and its solution and discusses properties of the solution. It highlights the
special assumptions that reduce the comparative statics of the Nash
model to those of the neoclassical model, thereby laying the basis for the
remainder of the paper.

First, to establish the threat points, assume that if $m$ and $f$ are unmar-
rried, then $m$'s utility function is given by $U^m_0(x_0, x_1, x_3)$ and $f$'s by $U^f_0(x_0, x_2, x_4)$. Here $x_1$ is a good consumed by $m$, $x_2$ is a good consumed by $f$, $x_3$ is the leisure time of $m$, and $x_4$ is the leisure time of $f$. Lastly, $x_0$ is an
additional private good that, if $f$ and $m$ marry, will be a household good
(a Samuelsonian pure public good within that household). Assume that $x = (x_0, x_1, x_2, x_3, x_4)'$ can be bought at prices $p = (p_0, p_1, p_2, p_3, p_4)'$. Let
$T$ be the time endowment for both $m$ and $f$, and $I_m$ and $I_f$ their respective nonwage incomes. If not married, $m$ and $f$ each maximize their separate
utilities subject to their individual full income constraints [$p_1x_0 + p_1x_1 + p_3x_3 = I_m + p_3T$ for $m$, and similarly for $f$], giving rise to their respective indirect utility functions,
(1) \( V_0^m(p_0, p_1, p_3, I_m; \alpha_m) \) and \( V_0^f(p_0, p_2, p_4, I_f; \alpha_f) \).

In (1) note the appearance of the vectors \( \alpha_m \) and \( \alpha_f \), or the extrahousehold environmental parameters (or EEPs, for short). Formally, EEPs are any variables that shift the maximum value of utility attainable by the individual outside of the marriage. Thus, in turn, changes in EEPs shift the threat points in the Nash bargain and are, therefore, parametric to the bargaining outcome. For example, if \( V_0^m \) is thought of as the value of the game of being single, then one element of \( \alpha_m \) would be the ratio of the number of potential spouses for \( m \) to the number of \( m \)'s competitors in the relevant marriage market, and similarly for \( f \). Hence, men born at the beginning of the baby boom who desired spouses younger than themselves had bigger \( \alpha_m \)'s than did men born after the peak of the boom, and conversely for women. Several additional examples of EEPs are given in Section III.

If \( m \) and \( f \) are married, \( V_0^m \) and \( V_0^f \) serve as their respective threat points. They also are assumed to have utility functions defined over their own and their spouses' consumptions, \( U^m(x) \) and \( U^f(x) \), with \( 5 \times 1 \) gradients, \( U_x^m \) and \( U_x^f \), assumed to be nonnegative. As discussed in Section VII, \( m \) and \( f \) may each be assumed to have an invariant meta utility function. For example, for \( m \) this meta utility function might embody state dependent utilities \( U^m(\cdot) \) and \( U_0^m(\cdot) \) for the unmarried and married states, respectively. This, as well as other interpretations of the relationship between married and unmarried utility functions, is discussed in Section VIII. Parallel interpretations are available for \( f \). In this paper, however, no particular relationship between the married and unmarried utility functions is assumed, although such restrictions would presumably lead to additional exploitable empirical restrictions.

The Nash-bargained solution to the allocation problem of \( m \) and \( f \) dictates that \( m \) and \( f \) jointly choose \( x \) to maximize the product of their gains from marriage, namely,

(2) \( N = [U^m(x) - V_0^m(p_0, p_1, p_3, I_m; \alpha_m)][U^f(x) - V_0^f(p_0, p_2, p_4, I_f; \alpha_f)], \)

subject to full household expenditures equalling full household income,

(3) \( p'x = (p_3 + p_4)T + I_m + I_f, \)

or, \( p'q = I_m + I_f \) where \( q \) is the vector of excess demands,

\[
q = (x_0, x_1, x_2, x_3 - T, x_4 - T).
\]

The last two elements of \( q \) are the negatives of the labor supplies of \( m \) and \( f \), respectively. It follows that \( m \) and \( f \) are married only if their marriage is efficient and that gains from marriage are positive for both, or \( g^m = U_0^m \)
The solution to maximization of (2) subject to (3) is a system of demand equations

\[ x_i = h_i(p, I_m, I_f, \alpha_m, \alpha_f), \quad i = 0, 1, 2, 3, 4, \]

a key result. Note that the arguments of the Nash demand system include all prices, separate measures of nonwage income for \( m \) and \( f \), and the extrahousehold environmental parameters, \( \alpha_m \) and \( \alpha_f \).

It is immediately apparent that this Nash demand system (4) is more general than its neoclassical counterpart. First, the arguments of a neoclassical demand system would not include person-specific nonwage incomes for \( m \) and \( f \), but would include only pooled or total family nonwage income, \( y = I_m + I_f \), which would be a restriction on (4). Second, in a neoclassical demand system, the opportunity cost of being married would be irrelevant and consequently the extrahousehold environmental parameters (\( \alpha \)'s) would not be included as arguments—another restriction on (4).

The demand system (4) exhibits well defined (i) Nash uncompensated and (ii) Nash compensated price effects, as well as (iii) separate income effects for \( I_m \) and \( I_f \). These three effects are related by (iv) the Nash generalization of the Slutsky equation. Of special importance is the Nash generalization of the substitution matrix,

\[ SG^{-1}, \]

and (v) its properties of symmetry and negative semidefiniteness. Here \( S \) and \( G \) are defined as

\[ S \equiv (X_p + \frac{1}{2} X_f q'), \]

\[ G \equiv \lambda I_3 + D(V_p' + \frac{1}{2} V_f q'). \]

The members of \( S \) are \( X_p \), the matrix of uncompensated price effects,

\[ X_p = \frac{\partial x_i}{\partial p}; \]

\( X_f \), the matrix of the nonwage income effects \( I_m \) and \( I_f \) (hereafter simply income effects),

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2. This model is consistent with Becker's model of marriage (1981 and elsewhere). In particular, it dictates that so long as it is possible to allocate expenditures on \( x \) so that both parties have positive gains from marriage, this will be done. It is not possible to have one party's gain be positive while the other's is zero or negative. Hence, the implicit model of divorce is one of efficient divorce.
\[ X_t = [X_{I_m} : X_{I_f}] = \left[ \frac{\partial x}{\partial I_m} : \frac{\partial x}{\partial I_f} \right]; \]

and \( u \), a \( 2 \times 1 \) vector of ones. The new members of \( G \) are \( \lambda \), the Lagrange multiplier associated with the full income constraint; \( I_5 \), the identify matrix of rank five; \( D \), which arrays the gradients of the utility functions,

\[ D = [U_{I_m}^t : U_{I_f}^t]; \]

and \( V_p \) and \( V_I \), which record the threat point shifts due to price and income changes,

\[ V_p = \begin{bmatrix} \frac{\partial V_{I_m}^m}{\partial p_1} & \frac{\partial V_{I_m}^m}{\partial p_2} \\ \frac{\partial V_{I_f}^m}{\partial p_1} & \frac{\partial V_{I_f}^m}{\partial p_2} \end{bmatrix} \text{ with } \frac{\partial V_{I_m}^m}{\partial p_2} = \frac{\partial V_{I_f}^m}{\partial p_2} = \frac{\partial V_{I_m}^f}{\partial p_3} = \frac{\partial V_{I_f}^f}{\partial p_3} = 0; \]

\[ V_I = \begin{bmatrix} \frac{\partial V_{I_m}^m}{\partial I_m} & 0 \\ 0 & \frac{\partial V_{I_f}^f}{\partial I_f} \end{bmatrix}. \]

Recall from the introduction that a key distinction between the Nash and neoclassical models is whether or not \( I_m \) and \( I_f \) have separate effects on demands. Also note that the \( i, j \)th element of the second term in \( S \) is just the average of \( m \)'s and \( f \)'s income effects weighted by the (excess) demand for the commodity whose price has changed, or \( 1/2[(\partial x_i/\partial I_m) + (\partial x_i/\partial I_f)]q_i \). Hence, in the special case that the effects on demands of changes in \( m \)'s and \( f \)'s incomes are equal, \( S \) reduces to the neoclassical substitution matrix,

\[ S_c = X_p + x_yq', \]

where \( y \) is pooled family income, \( y = I_m + I_f \), and \( x_y = \partial x/\partial y \).

Note also that \( G \) captures that part of the Nash generalization of the substitution matrix stemming from the impact of income and of price changes (\( V_I \) and \( V_p \)) on the threat points. Hence, if these are null, \( G = \lambda I \) and the symmetry and negativity of the Nash generalized substitution matrix, \( SG^{-1} \), reduces to the symmetry and negativity of the neoclassical substitution matrix, \( S_c \).

For future reference, result (iii), the separate income effects of \( I_m \) and \( I_f \), is recorded as

\[ (6) \quad X_t = [X_{I_m} : X_{I_f}] = SG^{-1}DV_I + bu', \]

where \( b \) is a \( 5 \times 1 \) vector defined (for the purpose of saving space) in (8) below.
Finally, differentiation of the budget restraint (3) with respect to prices and both nonwages incomes yields, respectively, (vi) Cournot Aggregation \( X'_p p = -q \) and (vii) the Nash generalization of Engel Aggregation,

\[ p'X_{I_m} = 1 \text{ and } p'X_{I_f} = 1. \]

The above italicized results (i) through (vii) summarize the comparative static results of MH. The next section extends these results to include the comparative statics associated with extrahousehold environmental parameters.

III. Extrahousehold Environmental Changes: Uses and Comparative Statics

Recall that extrahousehold environmental parameters (EEPfs) are parameters that shift the threat points but do not affect the prices and nonwage incomes faced by married individuals. Since they play a key role in the identification of the Nash model and since they are a rich source of additional variables for explaining household behavior, a number of examples follow.

In Section II, a measure of competitiveness in the marriage market was given as an example of an EEP. But what if \( m \) and \( f \) are already married and there is no divorce? Take, for example, certain areas of rural India where \( m \) and \( f \) are likely to have an arranged marriage and no divorce option. Women may, nonetheless, choose to return to their parents' household rather than continue in an unhappy marriage. Hence, \( \alpha_f \) would include \( f \)'s parents' wealth (or unexpected increments in their wealth).\(^3\)

In addition, an entire class of examples of \( \alpha_m \) and \( \alpha_f \) can be generated from systematic differences in prices and nonwage incomes between married and unmarried states.\(^4\) For example, a woman may receive welfare payments if she is not married but lose them if she marries (or gain them if she divorces). In this case, one element of \( \alpha_f \) would be the additional nonwage income received in the form of welfare when unmarried. More generally, any type of government or private transfer that is not portable

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3. This example is from Mark Rosenzweig.
4. The Nash model could have an equivalent and alternative set up in which there is one set of prices and nonwage incomes for the married state and a different set of prices and nonwage incomes for the unmarried state (as determined, for example, by tax rates and transfer payments that are conditioned on marital status). The \( \alpha \)'s would then only contain variables that shift the threat points that are independent of prices and nonwage incomes. The way the model is set up in the text, the price vector, \( p \), and the nonwage incomes, \( I_m \) and \( I_f \), are those that pertain to the married state and deviations therefrom are captured in the \( \alpha \)'s. It is clear that one parameterization is equivalent to the other.
into and out of the married state should be included in the $\alpha$'s. Likewise, taxes change systematically with marital status. For example, if marriage raises the marginal income tax rate of a woman, then $\alpha_f$ would include the decrease in her marginal tax rate due to leaving marriage. Similarly, if, as predicted by marriage models (e.g., Peters 1986), no fault divorce leads to lower support payments to divorced women, then, other things equal, women living in a state with no fault divorce laws have lower $\alpha_f$'s. In general, the EEPs cover a wide range of parameters, including, but not limited to, parameters that describe marriage markets, parameters that characterize the legal structure within which marriage and divorce occur, and parameters that characterize government taxes and government or private transfers that are conditioned on marital or family status.

The first order conditions of the Nash model obtained by partial differentiation of the appropriate Lagrangian function with respect to each $x_i$ are

$$ N_i = U_i^m(U^f - V_i^0) + U_i^f(U^m - V_i^m) = \lambda p_i, \quad i = 0, 1, 2, 3, 4, $$

or in matrix notation,

$$ Dg = \lambda p, $$

where $g$ is the vector of gains from marriage, $g = (g^m, g^f)^T$.\(^5\) Partial differentiation of these first order conditions and of the budget restriction with respect to $\alpha_m$ and $\alpha_f$ yields

$$ (7) \begin{bmatrix} J & p \\ p' & 0 \end{bmatrix} \begin{bmatrix} X_{\alpha} \\ \lambda'_{\alpha} \end{bmatrix} = \begin{bmatrix} DV_{\alpha} \\ 0 \end{bmatrix}, $$

where

$$ X_{\alpha} \equiv \begin{bmatrix} \frac{\partial x}{\partial \alpha_m} : \frac{\partial x}{\partial \alpha_f} \end{bmatrix}, $$

$$ \lambda'_{\alpha} \equiv \begin{bmatrix} \frac{\partial \lambda}{\partial \alpha_m} : \frac{\partial \lambda}{\partial \alpha_f} \end{bmatrix}, $$

$$ V_{\alpha} \equiv \begin{bmatrix} \frac{\partial V_0^m}{\partial \alpha_m} & 0 \\ 0 & \frac{\partial V_0^f}{\partial \alpha_f} \end{bmatrix}. $$

\(^5\) The matrix form of the first-order conditions did not appear in MH. For this paper, I redefined $g$ to have its elements in reverse order from the $g$ defined in MH, p. 340.
and $J$ is the matrix of second partial derivatives of $N$,

$$ J_{5 \times 5} = \left[ \frac{\partial N}{\partial x_i \partial x_j} \right]. $$

Solving (7) for the effects of changes in $\alpha_m$ and $\alpha_f$ on $X$ and $\lambda$ yields

$$ \begin{bmatrix} X_{\alpha} \\ \lambda'_{\alpha} \end{bmatrix} = \begin{bmatrix} B \\ b' \end{bmatrix} \begin{bmatrix} 5 \times 5 \\ 1 \times 5 \end{bmatrix} \begin{bmatrix} DV_a \\ 0' \end{bmatrix} = \begin{bmatrix} BDV_a \\ b'DV_a \end{bmatrix}, $$

where $B$, $b$, and $c$ are the submatrices of the partitioned inverse of $[I_B \ 0]$ that correspond to $J$, $p$, and $0$, respectively.

It is not hard to show that $B = SG^{-1}$. Hence, from (8), changes in the $\alpha$'s result in changes in household demands according to

$$ X_{\alpha} = SG^{-1}DV_a. $$

This can be expanded as

$$ \frac{\partial x}{\partial \alpha_m} = SG^{-1}U_{0}^{m} \frac{\partial V_{0}^{m}}{\partial \alpha_m} \text{ and } \frac{\partial x}{\partial \alpha_f} = SG^{-1}U_{0}^{m} \frac{\partial V_{0}^{f}}{\partial \alpha_f}. $$

In other words, the bigger the effect of $\alpha_m$ on $V_{0}^{m}$, the bigger the effect of a change in $\alpha_m$ on the demand for each $x_i$; parallel results hold for $\alpha_f$.

 Likewise, extract from (7) what I will call the Nash aggregation condition,

$$ p'X_{\alpha} = 0'. $$

This aggregation condition confines changes in demand due to changes in the extramarital environmental parameters, $\alpha_m$ and $\alpha_f$, to movements along the budget line.

**IV. Empirical Content of the Nash Model**

Barten (1966) codified the fundamental matrix equation of the theory of consumer demand in terms of partial derivatives. In this section I present the Nash generalization thereof, and then show how Barten's equation is a special case of the Nash generalization. The empirical implications of the Nash model are then summarized.

Collecting the partial derivatives of the first order conditions with respect to $\alpha$, $p$, and $I$ yields

$$ \begin{bmatrix} J \\ p' \end{bmatrix} \begin{bmatrix} X_{\rho} \\ X_{I} \\ X_{\alpha} \end{bmatrix} = \begin{bmatrix} \lambda I_{5} + DV_{p} \\ -\lambda'_{\rho} \\ -\lambda'_{I} \end{bmatrix} \begin{bmatrix} 5 \times 5 \\ 1 \times 2 \\ 1 \times 2 \end{bmatrix}. $$

6. This equation is formed from (7) above and from (A-1) and (A-7) from MH.
which I call the fundamental matrix equation of the Nash bargained theory of household behavior in terms of partial derivatives. To paraphrase and generalize Goldberger (1967), it is (11) along with the full income constraint (3) that permits us to deduce the general features which a complete system of Nash-bargained demand functions will possess, regardless of the underlying utility functions.

Barten (1966) codified the "fundamental matrix equation of the theory of consumer demand in terms of partial derivatives." Generalizing his equation to accommodate both the time endowments associated with labor supply and (in the last two columns) the zero derivatives of the neoclassical first order conditions with respect to \( \alpha \)

\[
\begin{bmatrix}
U & p \\
p' & 0
\end{bmatrix}
\begin{bmatrix}
X_p \\
-\lambda'_p
\end{bmatrix}
\begin{bmatrix}
x_p \\
-\lambda'_x
\end{bmatrix}
\begin{bmatrix}
X_a \\
\lambda'_a
\end{bmatrix}
= 
\begin{bmatrix}
\lambda I_5 & 0 \\
-q' & 1
\end{bmatrix}
\begin{bmatrix}
0_s \\
0_a
\end{bmatrix}
\begin{bmatrix}
0_f \\
0_t
\end{bmatrix},
\]

where \( U \) is the Hessian of the relevant utility function. I will call this the augmented neoclassical fundamental matrix equation. It is readily shown that restricting the Nash fundamental equation (11) so that \( V_p = 0 \), \( V_f = 0 \), and \( V_a = 0 \) yields the neoclassical fundamental equation (12). Hence the comparative statics of the neoclassical system are formally nested within those of the Nash. That is, the Nash demand system is a generalization of the neoclassical one.

The empirical content of the Nash model can be derived from (11) and (3) and can be summarized in four propositions. These are given as (N.1) through (N.4) in the upper left panel of Table 1. The upper right panel of Table 1 gives the corresponding "augmented" list of neoclassical propositions, (C.1) through (C.4). Propositions (N.1) and (N.2) (from \( \nu \) above) are readily recognizable as generalizations of their neoclassical counterparts, (C.1) and (C.2), the symmetry and negativity of the neoclassical substitution matrix, \( S_e = X_p + x_q' \). Propositions (N.1) and (N.2) were essentially derived in MH. The only part not proved there is that for all

*7. First, on the RHS of (11), set \( V_p, V_f, \) and \( V_a \) equal to zero matrices. Then, note that in the left-most matrix in (11), the Nash Hessian \( J \) collapses to a neoclassical Hessian, denoted here as \( U \). (This follows from the fact that once \( V_p, V_f, \) and \( V_a \) vanish, the neoclassical and Nash Hessians are empirically indistinguishable.) Recall that pooled (or family) income is \( y = f_m + f_f \). Note that when \( V_f = 0 \), then \( \partial x / \partial f_m = \partial x / \partial f_f = \partial x / \partial y = x_y \), so that we can set \( X_f = [x_0 : x_y] \), and similarly set \( \lambda f = (\lambda_0, \lambda_y) \). Having executed all of the above substitutions, postmultiplication of (11) by the operator that averages columns six and seven,

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

delivers (12).
Table 1
The Empirical Content of the Nash and the Neoclassical Models of Household Demand

<table>
<thead>
<tr>
<th>Nash Hypothesis</th>
<th>Augmented Neoclassical Hypothesis*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Version</strong></td>
<td></td>
</tr>
<tr>
<td>(N.1) $SG^{-1}$ is symmetric</td>
<td>(C.1) $S_c$ is symmetric</td>
</tr>
<tr>
<td>(N.2) $SG^{-1}$ is negative semidefinite (with $z'SG^{-1}z = 0$ iff $z \propto p$)</td>
<td>(C.2) $S_c$ is negative semidefinite (with $z'S_cz = 0$ iff $z \propto p$)</td>
</tr>
<tr>
<td>(N.3) $X_m - X_p = SG^{-1}DV_l \begin{pmatrix} 1 \ -1 \end{pmatrix}$</td>
<td>(C.3) $X_m - X_p = 0_5$</td>
</tr>
<tr>
<td>(N.4) $X_a = SG^{-1}DV_a$</td>
<td>(C.4) $X_a = 0_{5 \times 2}$</td>
</tr>
<tr>
<td><strong>Deleted Version</strong></td>
<td></td>
</tr>
<tr>
<td>(N.1) $\tilde{S}\tilde{H}$ is symmetric</td>
<td>(C.1) $\tilde{S}_c$ is symmetric</td>
</tr>
<tr>
<td>(N.2) $\tilde{S}\tilde{H}$ is negative definite</td>
<td>(C.2) $\tilde{S}_c$ is negative definite</td>
</tr>
<tr>
<td>(N.3) $\tilde{X}_m - \tilde{X}_p = \tilde{S}G^{-1}DV_l \begin{pmatrix} 1 \ -1 \end{pmatrix}$</td>
<td>(C.3) $\tilde{X}_m - \tilde{X}_p = 0_4$</td>
</tr>
<tr>
<td>(N.4) $\tilde{X}_a = \tilde{S}G^{-1}DV_a$</td>
<td>(C.4) $\tilde{X}<em>a = 0</em>{4 \times 2}$</td>
</tr>
</tbody>
</table>

a. Nash Hypothesis with $V_p = 0$, $V_I = 0$, and $V_a = 0$.

$5 \times 1$ vectors $z$, $z'SG^{-1}z = 0$ iff $z \propto p$. The proof of this is a straightforward analogue of the corresponding proof in the neoclassical case.⁸

Proposition (N.3) is obtained by postmultiplying (6) by the vector $(1, -1)'$ to obtain (N.3). Finally, (N.4) is Equation (9) above.

V. The Augmented Neoclassical Null Hypothesis

As shown in MH, if the threat points are invariant with respect to changes in prices and nonwage income (i.e., if $V_p = 0$ and $V_I = 0$), then $G = \lambda I_I$ and $S = S_c$. It follows that (N.1), (N.2), and (N.3) collapse, respectively, to their neoclassical counterparts, (C.1), (C.2), and (C.3). Likewise, if the threat points are insensitive to the EEPs ($V_a = 0$), then (N.4) collapses to (C.4). The proofs of these nestings are trivial.

Taken together, I will call (C.1), (C.2), (C.3), and (C.4) the "augmented" neoclassical hypothesis. The standard neoclassical hypothesis as given, for example, in Samuelson's *Foundations* (1947, p. 113) consists

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⁸ For a succinct statement see Theil (1971, pp. 677–78).
of (C.1) and (C.2) only. (C.3) and (C.4) comprise the augmentation. Proposition (C.3) says that household demand responses to changes in $I_m$ are identical to responses to changes in $I$. Consequently, his and her non-wage incomes may be aggregated to total family nonwage income without loss. The second part of the augmentation, (C.4), says that extrahousehold environmental variables (the $\alpha$'s) do not affect the demands of married couples. Certainly the classic tests of the neoclassical hypothesis (including Barten 1969, Byron 1970a, 1970b, Christensen, Jorgenson, and Lau 1975, Deaton 1974, Deaton and Muellbauer 1980a, 1980b, pp. 67–78) do not even mention, much less test, (C.3) or (C.4). Outside of the context of a more general model, such as the Nash model, there is simply no reason to consider the augmentation (C.3) and (C.4).

A final point is that propositions (N.1), (N.2), and (N.3) provide overidentifying restrictions on the first partials of the Nash demand system. In contrast, (N.4) does not in general provide testable restrictions, although it may in certain cases; see Section IX and Appendix A.

VI. Data Requirements

Having laid out the empirical implications of the Nash and neoclassical models, some discussion of data requirements is in order.

Estimation of the Nash demand system requires more data than would estimation of the corresponding neoclassical equations. There are two extra data requirements, data on separate nonwage incomes for each family member and data on the EEPs. With regard to person-specific nonwage incomes, many extant microdata from the U.S. contain little or poor information on the individual control of nonwage income of family members. Nonetheless, the studies by Altonji, Huyashi, and Kotlikoff (1989), Horney and McElroy (1988), and Cai (1989) all used U.S. data sets. In addition, as the studies of Schultz (1990) and Thomas (1990) in this issue illustrate, more data where such distinctions are possible are coming online, especially in the form of microdata from developing countries. With regard to the EEPs, data are readily available. For U.S. data sets, for example, it is often possible to use information on the location of respondents to merge in data on the interstate variations on divorce laws, taxes, welfare payments, sex ratios, and the like. Of course, whether or not the biases resulting from ignoring individual nonwage incomes and EEPs are important is an empirical question.

Turning from estimation to empirical implications, the empirical implications of any model must always be stated relative to the relevant information set. For example, in the neoclassical model the least data required to test an empirical implication of the model is sufficient data to estimate
one demand equation. This enables one to construct the own substitution effect, \( \partial x_i / \partial p_i + x_i (\partial x_i / \partial y) \), and test its negativity, a necessary condition for (C.2). Thus one needs data on the commodity in question and on all prices and (pooled family) income. To exhaust the full implications of symmetry and negativity of the substitution matrix, \( S_e \) (that is, to test C.1 and C.2), one needs to estimate the entire demand system, and hence requires data on all of the commodities as well as all prices and income. To test the augmentation of neoclassical hypotheses, (C.3) and (C.4), however, one requires, in addition, data on the nonwage income of each family member and on the EEPs as well—as much data as is required to estimate the Nash demand system.

Suppose one rejects the augmented neoclassical hypothesis, (C.1) through (C.4). That would be consistent with the Nash model but does not single out the Nash model from other members of the (possibly large) class of models that would lead to this rejection, including no restrictions at all. To test the Nash model versus no restrictions at all, or to test the neoclassical model versus the specific alternative that the Nash model holds, the data requirements are stringent. First, one needs data to estimate the Nash system of demand equations (quantities, prices, individual nonwage incomes, and the EEPs). Second, one needs to be able to estimate the \( G \) matrix, that is, one needs to estimate \( V_p \) and \( V_i \), the responsiveness of the threat points to changes in the prices and person-specific nonwage incomes. Section IX of this paper sketches one way to do this using extant data sets. In the final section of the paper, I argue that rejection of, or lack of rejection of, (N.1), (N.2), (N.3), and (N.4) is not nearly as interesting as the ability of separate nonwage incomes and of the EEPs to explain important empirical phenomena. For the latter, of course, we need only data on prices, person-specific nonwage incomes and the EEPs.

VII. Eliminating Redundant Restrictions to Obtain the “Deleted” Nash Hypothesis

Although for empirical testing it is crucial to delete an equation from the Nash generalized substitution matrix, the disinterested reader may skip this section without loss of continuity.

In the Nash model, as in the neoclassical model, the budget restraint (assumed to be satisfied identically by the data) implies a set of linear restrictions on the price and income derivatives of the demand system. These are (i) Nash aggregation, given in (10) above, (ii) Cournot aggregation,

\[
p'X_p = -q'.
\]
and (iii) the *Nash generalization of Engel aggregation*,

\[(14) \quad p' X_f = \nu', \]

respectively. Premultiplying \(S\) by \(p'\) and substituting (13) and (14) yields

\[(15) \quad S' p = 0, \]

a straightforward generalization of the neoclassical result that \(S_c p = 0\).

Since the data satisfy the budget restriction (3) identically, these three aggregation conditions are automatically satisfied as well. As a consequence each restriction (N.1), (N.2), (N.3), and (N.4) contains redundancies. Without loss of generality, these redundancies are eliminated by deleting the equations corresponding to the last commodity.

Let \(H = G^{-1}\), and define \(\hat{r} = -(p_1, p_2, p_3, p_4)' / p_3\) and partition\(^9\)

\[S = \begin{bmatrix} \hat{S} \\ \hat{S}' \end{bmatrix}, \quad H = \begin{bmatrix} H_0 \\ h^{(5)} \end{bmatrix}.\]

Then, as shown in Appendix B, (15) and (N.1) yield

\[(16) \quad SH = \begin{bmatrix} \hat{S}H & \hat{S}H\hat{r} \\ \hat{r}'\hat{S}H & \hat{r}'\hat{S}H\hat{r} \end{bmatrix}.\]

This partitioning of \(SH\), along with the following lemma, forms the basis for deleting a redundant equation from (N.1) and from (N.2). The lemma is a straightforward result of the linear dependence of the rows and columns of \(SH\) stemming from (15).

**Lemma:** (i) \(SH\) is symmetric iff \(\hat{S}H\) is symmetric. (Proof by inspection of (16).)

(ii) \(SH\) is negative semidefinite (with \(z' SH z = 0\) iff \(z \propto p\)) iff \(\hat{S}H\) is negative definite. (Proof in Appendix B.)

The lemma allows us to replace (N.1) and (N.2) with their respective "deleted" versions,

(N.1) \(\hat{S}H\) is symmetric

and

(N.2) \(\hat{S}H\) is negative definite.

These properties are recorded in the lower left panel of Table 1.

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9. Note that a single tilde denotes deletion of the fifth row or column, double tilde denotes deletion of both the fifth row and column, subscripts are used to denote row numbers, and superscripts in parentheses are used to denote column numbers.
With regard to (N.3), premultiplication by \( p' \) yields (via equations 14 and 15) zeros on both sides of the equation. Hence, the last equation in (N.3) is a linear combination of the first four. Deleting it leaves

\[
(\bar{N}.3) \quad \bar{X}_t m - \bar{X}_f = \bar{S}G^{-1}DV_f \begin{pmatrix} 1 \\ -1 \end{pmatrix}.
\]

Here \( \bar{X}_t m \) is the first four elements of \( X_t m \), and so forth.

Finally, note that via Nash aggregation (10), (N.4) reduces to

\[
(\bar{N}.4) \quad \bar{X}_a = \bar{S}G^{-1}DV_a,
\]

where \( \bar{X}_a \) is defined as the \( 4 \times 2 \) matrix formed by deleting the last row from \( X_a \). The "deleted" version of the Nash hypothesis, (N.1) through (N.4), appears in the lower left panel of Table 1.

It is trivial to show that if \( V_p = 0 \), \( V_f = 0 \), and \( V_a = 0 \), this "deleted" Nash hypothesis collapses to the "deleted" version of the augmented neoclassical hypothesis. The lower right panel of Table 1 records this "deleted," augmented neoclassical hypothesis. Here \( \bar{S}_c \) is defined as \( S_c \) with its last row and column deleted, a single tilde denotes the last element has been deleted from \( X_t m \), and so forth.

**VIII. On the Invariance of Utility Across Marital Statuses**

What is the relationship between an individual's utility function as a married person and that same individual's utility function as an unmarried person? At an abstract level each individual has an invariant meta utility function that applies in all states of the world. For current purposes, a state corresponds to marital status. A simple example that is used in Section IX is \( m \)'s meta utility function,

\[
U^m = \begin{cases} 
U^m(x_0, x_1, x_3, x_4), & \text{if } m \text{ is married} \\
U^0_m(x_0, x_1, x_3), & \text{if } m \text{ is unmarried.}
\end{cases}
\]

This paper uses the most general assumption that \( U^m(\cdot) \) and \( U^0_m(\cdot) \) are unrelated, state-dependent utility functions.

A more restrictive alternative approach would be to assume that the unmarried utility function is the same as the married utility function but subject to the preallocation of zero spousal consumption. For \( m \), for example, this is expressed as

\[
U^m_0(x_0, x_1, x_3) = U^m(x_0, x_1, 0, x_3, 0).
\]

Here preallocation is used in the sense of Pollak (1969), guaranteeing \( m \)
a well-defined neoclassical demand system when unmarried. One way to justify this assumption is to think of $U^m(\cdot)$ as the reduced-form utility function resulting from household production where the inputs to household production are goods and time. Obviously, spouse-specific inputs ($x_2$ and $x_4$) are zero if one is "unmarried." The preallocation of zero spousal consumption when unmarried would rule out some functional forms for $U^m(\cdot)$ such as Cobb–Douglas. It would also result in empirical restrictions. For once $U^m(\cdot)$ is specified, $U^m_0(\cdot)$ can be derived, and $V^m_0(\cdot)$ in turn. Once this is done for both $m$ and $f$, then $V_p$ and $V_f$ can be derived as well.

IX. Nesting and Testing the Nash and Neoclassical Models

This section sketches an approach to estimation and nested testing of the Nash and neoclassical models. Estimation of a neoclassical demand system requires data on quantities demanded, pooled family income and prices. Estimation of the Nash demand system requires these data plus the resolution of family income into person-specific incomes ($I_m$ and $I_f$) and also data on the EEPs ($\alpha$'s). But testing the Nash Hypotheses (N.1, N.2, and N.3) or using it as a specific alternative to the Neoclassical Hypothesis requires an estimate of the matrix $G$. To obtain this, one needs, in addition to the above, information on the responsiveness of the threat points to changes in prices and in person-specific nonwage incomes. With due allowance for selectivity into marriage, this additional information can be obtained from samples of unmarried individuals.

To be concrete, suppose one had a cross-section of data consisting of married couples and unmarried men and women. Obtain some homogeneity by limiting the age range (e.g., 25 to 35 years). In order for the sample of unmarried individuals to contain information on the opportunity cost of marriage, limit the sample to couples in their first marriage and previously married individuals who have left a first marriage.

The steps for estimation and testing are as follows:

Step 1. Estimate a married versus divorced probit regression for men (using the divorced men and the married men). For each individual, save the predicted probability of marriage to use in subsequent selectivity corrections.

10. The reader is reminded that "unmarried" here is the complement to the relevant definition of "married," which was defined very broadly in the Introduction.
11. If $U^m(\cdot)$ is Cobb–Douglas, then $U^m_0(\cdot) = 0$, which is undesirable.
Step 2. Specify an indirect utility function for the divorced men. Use Roy's identity to derive the associated demand system. Estimate it, using selectivity corrections based on step 1.

Steps 3 and 4. Repeat steps 1 and 2 for the divorced women.12

Step 5. Specify the married utility functions for $m$ and $f$ and combine them with the threat points (indirect utilities) from steps 2 and 4 to form the Nash criterion function. Derive and estimate the associated Nash demand system using the subsample of couples, using selectivity corrections based on step 2.

Step 6. Estimate a neoclassical demand system for the married couples.

Step 7. From the estimates in steps 2, 4, and 5, retrieve estimates of $V_p$, $V_I$, and $\lambda$ and hence of $G$.

Step 8. Do nested testing of the following hierarchy of restrictions on $X_i$ and $X_p$. The hypotheses are listed in increasing order of restrictiveness.

- $H_0$: No restrictions.
- $H_N$: $\tilde{N}.1$ and $\tilde{N}.2$ and $\tilde{N}.3$ hold.
- $H_C$: $\check{C}.1$ and $\check{C}.2$ and $\check{C}.3$ and $\check{C}.4$ hold.

Step 9. Use $\tilde{N}.4$ to estimate $V_a$.

A few observations are in order. First, sample selectivity corrections are well-trod territory that will not be reviewed here (see Heckman 1974 and elsewhere). Second, there are various estimators that one could use in the above steps, but maximum likelihood estimators in steps 2, 4, 5, and 6 would produce a maximum likelihood estimator of $G$ in step 7 and feed naturally into nested testing based on likelihood ratios in step 8.

Third, with regard to the parameters that characterize $V_a$, the above steps apply to the most general case in which $V_a$ is not identified in the estimation of the divorced men's and divorced women's demand systems. When it is not, then ($\tilde{N}.4$) is used to identify $V_a$ as in step 9. However, there are many specifications of $V_0^m$ and $V_0^f$ under which $V_a$ is identified in steps 2 and 4. In these cases ($\tilde{N}.4$) provides overidentifying restrictions. Hence, ($\tilde{N}.4$) is added to the joint hypothesis, $H_N$, in step 8. An example that makes clear when $V_a$ is and is not identified by steps 2 and 4 is given in Appendix A.

Finally, note that the above framework lends itself naturally to studying the relationship between the joint decisions of divorce and labor supply

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12. This assumption does not imply that the metautilities of men and women differ, but only that the constraints on their behaviors differ systematically. For example, due to past specialization and investments in household production and child care vis-à-vis her former spouse a woman is likely to have less market capital, more home-production capital, larger marriage-specific home-production capital losses, and poorer prospects in the remarriage market.
(T – x₁, T – x₂). Each male and each female is postulated to have a labor supply equation while married and another if divorced. These are related by the appearance of the indirect utility function for the divorced state as the threat point for the married state. Hence, for both men and women, one can readily analyze the effect of divorce on labor supply and vice versa. This is just one example of a class of examples where the Nash model provides a natural framework for analyzing the joint decisions of household formation and intrahousehold allocations.

X. A Perspective on Nash-Bargained Household Behavior

One obtains economic perspective by reflecting on the role of neoclassical consumer theory in general, and then drawing an analogy between the Nash model and standard neoclassical consumer theory. At its most practical and fundamental level, the “empirical implications” of neoclassical consumer theory (i) specify that prices and nonwage income constitute the appropriate arguments of demand functions, and (ii) guide our interpretation of the effects of these variables as, for example, income and substitution effects. These “empirical implications” of the model have been, and no doubt will continue to be, used on a daily basis by economists of every stripe.

At a second more sophisticated but less fundamental and less practical level, neoclassical consumer theory specifies a set of restrictions on the demand system that exhaust the implication of the theory: the symmetry and negativity of the neoclassical substitution matrix (C.1 and C.2 above). These conditions exhaust the implications of neoclassical theory in that (under appropriate regularity conditions) the satisfaction of these conditions guarantees that the underlying utility function can be recovered and hence that all other empirical implications (such as homogeneity) can be derived from them. These symmetry and negativity conditions have been the appropriate objects of standard tests of the neoclassical theory. They have not, however, borne up well under testing; see Deaton and Muellbauer (1980a) for a survey combined with a catalogue of deficiencies of these tests, including the prevalent use of macro time series data. Micro-data based studies of the joint labor supply of husbands and wives have only added to the cumulating rejections of the empirical content of the neoclassical model; see McElroy (1981) for a survey and Lundberg (1988) for recent evidence in a different context. In 1943 Samuelson (1947) sagely speculated, “I wonder how much economic theory would be changed if either of these two conditions [symmetry and negativity] . . .
were found to be empirically untrue. I suspect very little." Forty-seven years of theoretical and empirical economics have certainly borne him out.

So what explains our collective unbudging faith in neoclassical consumer theory? On the one hand, we know that the data we use arise from much more complicated realities than can be captured by this beautiful, spare, all-encompassing abstraction. On the other hand, our intuition suggests that any useful abstraction whatever will give a similar key role to prices and income and embody interpretations of the type dictated by neoclassical consumer theory. This source of this intuition is, no doubt, the long history of fruitful and incredibly diverse uses of these empirical implications in both theoretical and empirical applications of neoclassical theory (from the demand for apples to patterns of church attendance).

By analogy to the neoclassical model, the Nash model's fundamental and practical role is to specify and interpret the role of explanatory variables in a demand system. The list of explanatory variables for the Nash model is fundamentally richer than that for the neoclassical model. At the intensive margin the Nash model extends the neoclassical list of explanatory variables by requiring the disaggregation of family nonwage income into separate components for each family member—\( I_m \) and \( I_f \) replace \( I_m + I_f \). At the extensive margin the Nash model adds an entirely new set of explanatory variables, the extrahousehold environmental parameters (\( \alpha \)'s), in principle including every variable that affects how well each family member could do in the next best alternative outside of the family. Depending on the governmental and social setting these latter extrahousehold environmental variables might include, but would not be limited to, indices of each member’s control over resources outside the family (including indices of wages, nonwage income, and employability of each family member), measures of how well each family member could do in the marriage or remarriage market (e.g., appropriate age-specific sex ratios) or returning to their family of origin (e.g., parental wealth), measures of supporting social networks and social restrictions (such as religion and caste), parameters that describe the legal structure (such as the rules for the determination of property settlements, alimony, and child support), parameters that describe government and private taxes and transfers that are conditioned on marital and family status (such as AFDC payments and other payments to custodial parents), and so forth. There is mounting evidence that this enrichment of the range of explanatory variables provided by the Nash model is useful.\(^{13}\)

Also at this fundamental and practical level, the Nash model provides a natural vehicle for analyzing problems that involve joint decisions to

\(^{13}\) See the evidence cited in the Introduction.
form (or dissolve) households and to allocate resources within households. The Nash model automatically links these decisions by using the indirect utility functions from the ‘‘unmarried’’ states in describing decisions in the ‘‘married’’ state. Three examples make the point. As shown in Section IX, the Nash model provides a natural framework for the joint analysis of labor supply and divorce. A second example is provided by McElroy’s (1985) analysis of sons and their parents jointly determining the son’s labor force participation when he leaves his parental home; for rural areas of developing countries the analogue would be parents and youths jointly deciding on labor supply and migration of youths to the city. A third example of application would be an analysis of the marriage tax. Here, with a progressive income tax and a husband earning income, the marginal wage rates of many women decrease upon marriage and increase upon divorce, systematically deterring labor force participation while married as well as deterring marriage itself. As these examples demonstrate, the analytical link that the Nash model provides between household formation (dissolution) and intrahousehold allocation is a potentially fruitful area for future applications.

Proceeding with the analogy of the Nash to the neoclassical model, I turn to the more sophisticated but less fundamental and less practical implications of the Nash model. These are the negativity and symmetry of the Nash generalization of the substitution matrix—(N.1) and (N.2)—and the Nash restrictions relating person-specific income effects (N.3). These are straightforward generalizations of their neoclassical counterparts—(C.1), (C.2), and (C.3). Direct tests of these Nash propositions and of the nesting of the neoclassical model within the Nash model remain to be done. Results of these tests will be of interest in their own right. Yet, just as for the neoclassical model, the ultimate utility of the Nash model will depend not on the outcomes of these tests, but on the judgment of the profession as to the usefulness of the enriched list of explanatory variables provided the Nash model, as well as the expanded class of problems amenable to analysis with it.

As always the appropriate model depends on the problem at hand. There is increasing evidence that a wide class of problems can be fruitfully attacked with Nash or other bargaining models of family behavior, models that reflect the opportunity cost of family membership.

Appendix A

In general, $V_{\alpha}$ is not overidentified and (N.4) provides no basis for testing the Nash model. There are, however, many specifications of $V_{\alpha}(\cdot)$ and
$V_0'()$ under which $V_0$ is overidentified. This is easily shown by example.

In the following example, $V_0''()$ is specified in the spirit of the indirect translog utility function, the associated demand system is derived, and then the identification of the parameters of $\partial V_0'' / \partial \alpha_m$ are discussed. Of course, parallel results would obtain for $\partial V_0'/\partial \alpha_f$.

Suppose that $V_0''()$ takes the form

$$V_0''(p_0, p_1, p_3, I_m) + \left( \gamma_{0m} + \gamma_{1m} I_m + \sum_{j=0,1,3} \gamma_j p_j \right) \alpha_m,$$

where $v()$ is a standard neoclassical indirect utility function (translog, for example). Note that the effect of $\alpha_m$ on $V_0''$ is assumed to be 1

$$\frac{\partial V_0''}{\partial \alpha_m} = \left( \gamma_{0m} + \gamma_{1m} I_m + \sum_{j=0,1,3} \gamma_j p_j \right).$$

Via Roy's identity, $m$'s demand system is

$$x_i = \frac{\frac{\partial v}{\partial p_i} + \gamma_i \alpha_m}{\frac{\partial v}{\partial I_m} + \gamma_{1m} \alpha_m}, \quad i = 0, 1, 3.$$

Estimation of this demand system will yield estimates of $\gamma_0$, $\gamma_1$, $\gamma_3$, and $\gamma_{1m}$, but not of $\gamma_{0m}$.

An estimator for this last parameter follows. Extract the first columns from both sides of (N.4)

$$\begin{pmatrix} \frac{\partial V_0''}{\partial \alpha_m} \end{pmatrix} = \hat{S} G^{-1} D \begin{pmatrix} \frac{\partial V_0''}{\partial \alpha_m} \\ 0 \end{pmatrix} = \hat{S} G^{-1} U' \frac{\partial V_0''}{\partial \alpha_m}.$$

Let $d = \hat{S} G^{-1} U'$. Solving yields

$$\frac{\partial V_0''}{\partial \alpha_m} = (d'd)^{-1} d' \hat{X}_{\alpha m},$$

or, upon substitution of (A.1) into (A.4),

$$\gamma_{0m} = (d'd)^{-1} d' \hat{X}_{\alpha m} - \sum_{j=0,1,3} \gamma_j p_j - \gamma_{1m} I_m.$$

Estimators of $\hat{X}_{\alpha}$ and $d$ can be retrieved from steps 5 and 7 in Section IX; estimates of the $\gamma_j$'s and $\gamma_{1m}$ come from the men's demand system. Hence, (A.5) can be used as the basis for estimating $\gamma_{0m}$ and (A.1) can be used to recover $\partial V_0'' / \partial \alpha_m$. Similar analysis for $f$ yields $\partial V_0' / \partial \alpha_f$ and hence...
\(V_n\). Of course, if \(\gamma_{0m}\) is specified as zero, then \((A.4)\) does provide overidentifying restrictions.

More generally, if \(V''_0(\cdot)\) and \(V'_0(\cdot)\) are specified such that the \(\alpha\)'s only affect utilities via interactions with prices and nonwage incomes, then \((N.4)\) provides overidentifying restrictions on the Nash model.

**Appendix B**

To prove the lemma in the text write the partitioned version of \((15)\) as

\[
S'p = [\bar{S}' \quad s_5'] \begin{pmatrix} \bar{p} \\ p_5 \end{pmatrix} = \bar{S}'\bar{p} + s_5p_5 = 0_5,
\]

and solve for the last row of \(S\) as a linear combination of the first four,

\[
(B.1) \quad s_5' = \bar{p}'\bar{S} \begin{pmatrix} -1 \\ p_5 \end{pmatrix} = \bar{r}'\bar{S}.
\]

Second, note that via \((N.1)\) and \((15)\),

\[
(B.2) \quad SHp = \begin{bmatrix} \bar{S} \\ s_5' \end{bmatrix} [\bar{H} \quad h^{(5)}] \begin{pmatrix} \bar{p} \\ p_5 \end{pmatrix} = H'S'p = 0_5.
\]

Now from \((B.1)\), the last equation in \((B.2)\) is a linear combination of the first four and can be dropped, leaving

\[
(B.3) \quad \bar{S}[\bar{H} \quad h^{(5)}] \begin{pmatrix} \bar{p} \\ p_5 \end{pmatrix} = 0_4.
\]

Solving \((B.3)\), the last column of \(\bar{S}H\) can be written as a linear combination of the first four columns

\[
(B.4) \quad \bar{S}h^{(5)} = \bar{S}\bar{H}\bar{r}.
\]

Now via partitioning

\[
(B.5) \quad SH = \begin{bmatrix} \bar{S}\bar{H} & \bar{S}\bar{h}^{(5)} \\ s_5'\bar{H} & s_5'h^{(5)} \end{bmatrix}.
\]

Applying the linear dependence of the rows of \(S\) \((B.1)\) and columns of \(\bar{S}H\) \((B.5)\) to the border in \((B.4)\) yields

\[
(B.6) \quad SH = \begin{bmatrix} \bar{S}\bar{H} & \bar{S}\bar{H}\bar{r} \\ \bar{r}'\bar{S}\bar{H} & \bar{r}'\bar{S}\bar{H}\bar{r} \end{bmatrix}.
\]
From (B.6) we can see that part (i) of the Lemma holds. Further, partition an arbitrary $5 \times 1$ vector $z$ as $(\tilde{z}', z_5)$. Then via (B.6)

\[(B.7) \quad z' \tilde{S} H z = \tilde{z}' \tilde{S} \tilde{H} \tilde{z} + 2z_5 \tilde{z}' \tilde{S} \tilde{H} \tilde{r} + z_5' \tilde{S} \tilde{H} \tilde{r} = (\tilde{z} + z_5 \tilde{r})' \tilde{S} \tilde{H} (\tilde{z} + z_5 \tilde{r}) = z_*' \tilde{S} \tilde{H} z_*,\]

where $z_* = \tilde{z} + z_5 \tilde{r}$. Note that $z_* = 0$ iff $z \propto p$. Consequently (B.7) implies that (N.2) holds iff for all $4 \times 1$ vectors $z_* \neq 0_4$,

\[(B.7) \quad z_*' \tilde{S} H z_* < 0 \quad \text{and} \quad z_*' \tilde{S} H z_* = 0 \iff z_* = 0_4.\]

Part (ii) of the Lemma follows and the Lemma is proven.

References


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