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Source: *The American Economic Review*, Vol. 88, No. 1, (Mar., 1998), pp. 33-62

Published by: American Economic Association

Stable URL: <http://www.jstor.org/stable/116817>

Accessed: 11/08/2008 17:18

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# Competition Between Private and Public Schools, Vouchers, and Peer-Group Effects

By DENNIS EPPLE AND RICHARD E. ROMANO\*

*A theoretical and computational model with tax-financed, tuition-free public schools and competitive, tuition-financed private schools is developed. Students differ by ability and income. Achievement depends on own ability and on peers' abilities. Equilibrium has a strict hierarchy of school qualities and two-dimensional student sorting with stratification by ability and income. In private schools, high-ability, low-income students receive tuition discounts, while low-ability, high-income students pay tuition premia. Tuition vouchers increase the relative size of the private sector and the extent of student sorting, and benefit high-ability students relative to low-ability students. (JEL H42, I28)*

Discontent in the United States with the primary and secondary educational system has become the norm. The decline in SAT scores in the 1970's, embarrassing international comparisons of student achievement, slow growth in productivity measures, and increasing disparity in earnings all call into question the quality of the educational system.<sup>1</sup> Education

policy figured prominently in recent presidential elections. The debate has centered on issues of school choice, including voucher systems (Karen De Witt, 1992). Typical voucher proposals provide students attending private schools a tax-financed, school-redeemable voucher of fixed amount toward (or possibly covering) tuition. Although a 1993 California referendum for vouchers was defeated, policy change at state and local levels abounds, as does change in the private educational sector. The state of Minnesota and school districts in 30 states allow residents to choose the public school their children attend.<sup>2</sup> The city of Milwaukee introduced a voucher system in the 1989–1990 school year. A number of private school and private–public school initiatives are developing (see e.g., John F. Witte et al., 1993; Steve Forbes, 1994; Steven Glazerman and Robert H. Meyer, 1994; Joe Nathan, 1994; *Newsweek*, 1994; *Wall Street Journal*, 1994; Steven Baker, 1995; Jay P. Green et al., 1996). Educational reform emphasizing increased school competition with an increased

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<sup>1</sup> The provocatively titled report of the National Commission on Excellence in Education (1983), *A Nation at Risk*, details the decline of performance of U.S. students in the 1970's. More recent data can be found in Daniel M. Koretz (1987). Modest gains in performance on standardized achievement tests, followed by a leveling off, well below peak scores of the early 1960's, characterizes the late 1980's and 1990's.

<sup>2</sup> Public funding of nonsecular schools and considerable freedom of school choice has been practiced for years in England (Daphne Johnson, 1990) and much of Canada (Nick Kach and Kas Mazurek, 1986). These choice systems support horizontal differentiation in schooling and safeguards exist to limit vertical (quality) differentiation. Our analysis is concerned primarily with the effects of a voucher system on vertical differentiation.

role of the private sector is at the forefront of the policy debate and recent policy initiatives.

The modern economic case for vouchers and increased educational choice was made by Milton Friedman (1962). The academic educational and political-science professions have since considered the pros and cons of voucher systems and educational choice (John E. Coons and Stephen D. Sugarman, 1978; Myron Liberman, 1989; John Chubb and Terry Moe, 1990). Economic analysis of the interaction between public and private schools, and of related policy instruments like vouchers, is only beginning to emerge. This paper continues the study of the "market" for education by developing a model that focuses on the interaction between the public and private educational sectors and also examines the consequences of vouchers. We describe the equilibrium characteristics of the market for education with an open-enrollment public sector and a competitive private sector.

Our model embodies two key elements of the educational process. First, students differ in their abilities. Higher ability is assumed to increase a student's educational achievement *and* that of peers in the school attended. Second, households differ in their incomes, with higher income increasing the demand for educational achievement. A student in our model is then characterized by an ability and a household income, a draw from a continuous bivariate distribution. A school's quality is determined by the mean ability of the student body, reflecting the model's peer-group effect. We characterize the equilibrium distribution of student types across public and private schools and examine the tuition structure of private schools, assuming that student types are verifiable. We develop a theoretical and computational model in parallel, with the latter calibrated to existing estimates of parameter values. Equilibria are simulated for a range of voucher values.

Key characteristics of an equilibrium are the following. A hierarchy of school qualities will be present, with the set of (homogeneous) public schools having the lowest-ability peer group and a strict ability-group ranking of private schools. The equilibrium student bodies of schools correspond to a partition of the ability-income-type space of students with

stratification by income and, in many cases, stratification by ability. As Figure 1 from our computational model illustrates, type space is then carved into diagonal slices with each higher slice making up a private school's student body and with the bottom slice comprising the public sector.

The normality of demand for a good peer group leads relatively high-income students to cross subsidize the schooling of relatively high-ability students, producing the latter partition. Private schools attract high-ability, low-income students by offering them tuition discounts, sometimes fellowships. Even with free entry, schools price discriminate by income against students who are not on the margin between switching schools. The equilibrium differentiation of schools and economies of scale in education preclude perfect competition for every type of student. Nevertheless, this price discrimination does not disrupt the internalization of the peer-group externality by private schools. An equilibrium without a public sector is Pareto efficient given the equilibrium number of schools. Because free public schools do not price the peer-group externality, an equilibrium with public schools is Pareto inefficient.

In the computational model, we employ a Cobb-Douglas specification of utility and educational achievement which incorporates the peer-group effect. The parameters are calibrated to U.S. data from various sources. We compute approximate equilibria for voucher values ranging from \$0 to \$4,200 per student (\$4,222 equals the expenditure per student in public schools in 1988).<sup>3</sup> With no vouchers, the predicted percentage of students in the public sector is 90 percent (the actual value for the United States is 88 percent). As the voucher is increased, the size of and mean ability in the public sector decrease. With a \$2,000 voucher, for example, the percentage of students remaining in the public sector equals 70 percent, and the mean ability declines by 15.8 percent.

<sup>3</sup> The integer number of private schools in our model precludes existence of competitive equilibrium except in special cases. This integer problem and our approximation approach are discussed later in the paper.

The entry of private schools and consequent more efficient sorting of students across schools caused by vouchers increases average welfare (and achievement) only a little in our computational model, while having larger distributional effects. As we discuss in detail later, the magnitude of the aggregate effect depends on the extent of complementarity of peer ability and own ability in the educational production function. There is little empirical evidence to guide assessment of the extent of such complementarity. The voucher increases the premium to ability in private schools. The largest proportionate gains from the voucher then accrue to low-income, high-ability students. For example, a household with income of \$10,000 and student with ability at the 95th percentile has a welfare gain of about 7.5 percent of income from a \$2,000 voucher. Students of low income and low ability who remain in the public sector when a \$2,000 dollar voucher is available experience small welfare losses but make up a majority. It bears emphasizing that our model takes public and private schools to be equally effective providers of education, however. Some argue that private schools are more productive and that the competitive effect of a voucher program will increase public-school effectiveness.<sup>4</sup> For example, Hoxby (1996) concludes from her empirical investigation that competition-induced performance improvement would increase public-school achievement by more than enough to offset

<sup>4</sup> Caroline M. Hoxby (1994, 1996) provides evidence that private-school competition increases public-school effectiveness. William N. Evans and Robert M. Schwab (1995) find that Catholic private schools are more effective in inducing students to complete high school and also to attend college. These studies take on the challenge of finding instruments that predict well private-school attendance while being independent of unobserved determinants of educational achievement. Controversy exists concerning the quality of the instruments used. See Thomas J. Kane (1996) for a discussion of Hoxby's methodology. David N. Figlio and Joe A. Stone (1997) employ a different set of instruments than Evans and Schwab and find that, at current input levels, religious private schools are less effective than public schools in producing achievement on standardized exams in math and science (but nonreligious private schools are more effective). See Witte (1996) and Figlio and Stone (1997) for references to other studies.

losses of the magnitude that emerge due to reduced peer quality in our computational model. Our analysis delineates the allocative effects of vouchers and demonstrates a potential for significant redistribution.

A theoretical-economics literature on education is beginning to emerge. Charles A. M. de Bartolomé (1990) develops a two-neighborhood model of the provision of public educational inputs (quality) with two ability types and peer-group externalities. He shows that the voting/locational equilibrium is inefficient because the median voter does not internalize the consequences of migration on peer groups in choosing the input level. No independent income variability characterizes students in his model. Raquel Fernandez and Richard Rogerson (1996) introduce income differences in a two-neighborhood model of the provision of inputs but abstract from peer-group effects. They examine the effects of redistributive policies and direct controls on inputs. Neither model has a private sector. Our analysis is differentiated by its consideration of a private sector and its two-dimensional, continuous type space. In a normative analysis of student groupings in the presence of peer-group effects, Richard Arnott and John Rowse (1987) show how a social planner would maximize the sum of achievements in allocating students of various abilities across classrooms. We analyze equilibrium outcomes, and most of our analysis is positive.

Joseph E. Stiglitz (1974), Norman J. Ireland (1990), Ben Eden (1992), Charles F. Manski (1992), Michael Rothschild and Lawrence J. White (1995), Epple and Romano (1996), and Gerhard Glomm and B. Ravikumar (1998) consider the consequences of a private sector for education. Stiglitz, Glomm and Ravikumar, and Epple and Romano are concerned with the existence and properties of voting equilibria over tax-financed, public-school expenditure in the presence of a private alternative. Ireland analyzes the effects of vouchers on utilities and the quality of the public alternative, taking the tax rate as exogenous. Individuals differ only by income, and the private alternative can be purchased continuously in all these analyses. Hence, the private sector is relatively passive, and issues of financial aid and differences in

student ability across schools do not arise. Our model is distinguished by having differences in ability and related peer-group effects, and by providing an active role for private-sector schools.

Eden (1992) analyzes vouchers in a purely private market system of provision of education having two ability types and peer-group effects. A voucher equal to the difference between the social and private benefit of education to each ability type is shown to induce socially optimal provision of education. Key differences in our analysis include our consideration of the interaction between the public and private sectors, our exploration of the implications of continuous differences in ability and income, and our attention to positive issues. Manski (1992) pursues a computational analysis of vouchers that also considers peer-group effects among other aspects of education (especially various objectives of public-school decision makers). Our models differ in a number of ways. Most importantly, we permit private schools to discriminate in their tuition policies, with many consequences. Rothschild and White (1995) analyze a competitive model with consumers also inputs to production (a peer-group effect), using higher education as their primary example. We share a concern for market pricing in the presence of an externality. Differences in our model, among others, are the presence of a public sector, a more detailed specification of peer effects and demand for education, and student variation in both ability and household income. Our attention to the implications for pricing, profitability, and school qualities of a peer-group effect deriving from student abilities, the allocation of students according to ability and household income and the related distribution of educational benefits, and the effects of vouchers are not concerns in Rothschild and White.

Private schools are cases of clubs with non-anonymous crowding due to the ability-dependent externality and schools' power to price it. Suzanne Scotchmer (1994) provides an excellent synthesis of this literature. We follow this literature in our competitive specification of private schools as further discussed below.

The next section presents the model. Section II develops the theoretical results. The computational results comprise Section III. Concluding remarks follow. An Appendix contains some of the detail.

### I. The Model

Household income is denoted  $y$ , and each household has a student of ability  $b$ . The joint marginal distribution of ability and income in the population is denoted  $f(b, y)$  and is assumed to be continuous and positive on its support,  $S \equiv (0, b_{\max}] \times (0, y_{\max}]$ . All students attend a school since we assume that free public schooling is preferred to no schooling. The household decision maker's utility function,  $U(\cdot)$ , is increasing in numeraire consumption and the educational achievement of the household's student, and it is continuous and twice differentiable in both arguments. Achievement,  $a = a(\theta, b)$ , is a continuous and increasing function of the student's ability and the mean ability of the student body in the school attended,  $\theta$ .<sup>5</sup> Let  $y$  denote after-tax income and

<sup>5</sup> The influence of ability on own educational achievement is well documented and not controversial. Eric Hanushek (1986) provides an excellent survey. In the economics literature, Anita A. Summers and Barbara L. Wolfe (1977) and Vernon Henderson et al. (1978) find significant peer-group effects. Evans et al. (1992) adjust for selection bias in the formation of peer groups and show that it eliminates the significance of the peer group in explaining teenage pregnancy and dropping out of school. They are careful to point out that their results should not be interpreted as suggesting that peer-group effects do not exist, but as demonstrating that scientific proof of those effects is inadequate. Note, too, that their work supports the notion that peer-group variables enter the utility function since a selection process does take place. The psychology literature on peer-group effects in education also contains some controversy. In their survey paper, Richard L. Moreland and John M. Levine (1992) conclude:

The fact that good students benefit from ability grouping, whereas poor students are harmed by it, suggests that the mean level of ability among classmates, as well as variability in their ability levels, could be an important factor. The results from several recent studies . . . support this notion.

This squares with our reading of the literature (Summers and Wolfe, 1977; Henderson et al., 1978; Chen-Lin Kulik and James A. Kulik, 1982, 1984; Aage B. Sorensen, 1984;

$p$  tuition expenditure, the latter equal to zero if a public school is attended. Thus,  $U = U(y_i - p, a(\theta, b))$ , with  $U_1, U_2, a_1$ , and  $a_2$  all positive. The achievement function captures the peer-group effect in our model, discussed further below. To maintain simplicity and highlight the role of peer groups, a school's quality is determined exclusively by the mean ability of its peer group.<sup>6</sup> In ongoing work we are extending the model to include variation in educational inputs.

$U$  is also assumed to satisfy everywhere the "single-crossing" condition (SCI):

$$(1) \quad \partial \left( \frac{\partial U / \partial \theta}{\partial U / \partial y_i} \right) / \partial y_i > 0.$$

Hence, for students of the same ability, any indifference curve in the  $(\theta, p)$ -plane of a higher-income household cuts any indifference curve of a lower-income household from below. This condition corresponds to an income elasticity of demand for educational quality that is positive at all qualities for all types.<sup>7</sup> One set of sufficient conditions on  $U$  for SCI is  $U_{11} \leq 0$  and  $U_{12} \geq 0$ , with at least one having strict inequality.<sup>8</sup>

Sorensen and Maureen T. Hallinan, 1986; Adam Gamoran and Mark Berends, 1987; Jennie Oakes, 1987; Gamoran, 1992). However, there are alternative interpretations (Robert E. Slavin, 1987, 1990).

<sup>6</sup> For simplicity, the possibility that dispersion in peer ability also affects achievement is not built into our model. Roland Benabou (1996b) explores the consequences for economic growth of dispersion in human capital.

<sup>7</sup> We believe this to be uncontroversial. While we know of no empirical studies that use direct measures of educational quality, a substantial empirical literature on the demand for educational expenditure exists. Although considerable diversity in magnitudes of estimates of the income elasticity of demand for educational spending are present, estimates using a variety of approaches find the income elasticity to be positive (Daniel Rubinfeld and Perry Shapiro, 1989).

<sup>8</sup> Households may consider education a consumption good, an investment good, or a combination of the two. Our formulation can be interpreted to accommodate any of these motives. However, for households not subject to borrowing constraints, a pure investment motive would imply a zero income elasticity of demand. For such households, this in turn would imply that the SCI condition in (1) would be only weakly satisfied. In light of the empir-

Preferences for school quality might also depend on ability. We say preferences satisfy *weak single crossing in ability* if

$$\partial \left( \frac{\partial U / \partial \theta}{\partial U / \partial y_i} \right) / \partial b \geq 0$$

which implies a weakly positive ability elasticity of demand for quality. However, because the pertinent empirical evidence is mixed and scarce, we postpone restricting preferences in this regard until necessary.<sup>9</sup> In our computational model and to illustrate our more general theoretical results, we adopt a Cobb-Douglas specification of the utility function:

$$(2) \quad U = (y_i - p)a(\theta, b)$$

$$a(\theta, b) = \theta^\gamma b^\beta$$

$$\beta > 0 \quad \gamma > 0.$$

While (2) satisfies SCI, it embodies the "neutral" assumption of zero ability elasticity of demand:

$$\partial \left( \frac{\partial U / \partial \theta}{\partial U / \partial y_i} \right) / \partial b = 0.$$

Our computational results are not driven by own-ability effects on the demand for education. Keep in mind, too, that the theoretical results do not assume specification (2).

A school's costs depend only on the number of students it enrolls, since inputs vary only with size. All schools, public and private, have the simple cost function:

$$(3) \quad C(k) = V(k) + F$$

$$V' > 0 \quad V'' > 0$$

ical evidence suggesting the income elasticity to be positive, we conserve space in the development that follows by assuming that SCI is strict for all households.

<sup>9</sup> Henderson et al. (1978) find no interaction between own ability and the benefits to an improved peer group, corresponding to  $\partial^2 U / \partial \theta \partial b = 0$  in our model. Summers and Wolfe (1977) find some support for higher peer-group benefits to lower-ability students, that is,  $\partial^2 U / \partial \theta \partial b < 0$ . Thus the literature provides limited evidence from which to draw conclusions.

where  $k$  denotes the number of attending students. Technical differences among schools are not an element of our model (for simplicity). Hence, vouchers cannot drive technically inefficient schools from the market, an effect predicted by some proponents of vouchers (see footnote 4). Let  $k^*$  denote the “efficient scale,”

$$(4) \quad k^* \equiv \text{ARGMIN}[C(k)/k].$$

The presumption of some economies of scale in education is realistic (Lawrence Kenny, 1982) and important. Otherwise, the private market would produce an infinite number of schools containing infinitely refined peer groups. Our model’s equilibrium will be consistent with the fact that the number of types of students greatly exceeds the number of schools.

Public-sector schools offer free admission to all students. This open-enrollment policy leads to homogeneous public schools in equilibrium because we assume no frictions in public-school choice are present. Without equalization of  $\theta$ ’s in public-sector schools, students would migrate to higher- $\theta$  schools to reap the benefits of a better peer group. With equalized  $\theta$ ’s, no incentives for switching schools within the public sector remain. We study the alternative of neighborhood school systems that impose residence requirements in Epple and Romano (1995).

Since all public schools will have the same  $\theta$ , one can think of the public sector as consisting of one (possibly large) school. Public-sector schooling is financed by a proportional income tax,  $t$ , paid by all households, whether or not the household’s child attends school in the public sector. Thus,  $y_i = (1 - t)y$ . The public sector chooses the (integer) number of schools and their sizes to minimize the total cost of providing schooling subject to (3). The tax rate adjusts to balance the budget. Because households are atomistic, there is no tax consequence to a household’s decision about school attendance. The public finance of schooling can then be largely suppressed in the analysis until the consideration of vouchers. The public sector is passive in this model for simplicity. Public-sector schools do not segment students by ability (track), increase educational inputs to compete more effectively with the private sector, or behave strategically

in any way. More realistic alternatives are important topics for research, some of which are discussed in the final section.

Private-sector schools maximize profits, and there is free entry and exit.<sup>10</sup> Modeling private schools as choosing an admission policy and tuition policy is convenient and involves no loss of generality. Student types are observable, implying that tuition and admission can be conditioned on ability and income as competition permits.<sup>11</sup> Private schools are an example of clubs with “non-anonymous crowding” (Scotchmer and Myrna H. Wooders, 1987; Scotchmer, 1997) because of the peer-group effect, and we model private-school behavior following the literature on *competitive* club economies. In particular, private schools maximize profits as *utility takers* (see Scotchmer, 1994), a generalization of price-taking when consumers (types) and products differ. Private schools believe they can attract any student-type by offering admission at a tuition yielding at least the maximum utility the student could obtain elsewhere.

Let an  $i$  subscript,  $i = 1, 2, \dots, n$ , indicate a value for the  $i$ th private school. A zero subscript does the same for “the” public school. Let  $p_i(b, y)$  denote the tuition necessary to enter school  $i$ , with  $p_0(b, y) = 0 \forall (b, y)$ . Let  $\alpha_i(b, y) \in [0, 1]$  denote the proportion of type  $(b, y)$  in the population that school  $i$  admits,

<sup>10</sup> Consideration of alternative objective functions to profit maximization is reasonable, especially given the significant proportion of nonprofit schools. Some private schools might, for example, pursue the objective of quality maximization. Quality maximization, like profit maximization, is a member of a set of objective functions that are utility independent in the sense that they place no weight on offering any student types higher utility than the student’s (equilibrium) reservation utility. Our preliminary analysis of this issue suggests that equilibria where some private schools pursue objectives from this set other than profit maximization must also be competitive equilibria. Roughly, the failure of any school to maximize profits would permit entry by a profit-maximizing school.

<sup>11</sup> The notion is that abilities can be determined through testing, and required financial disclosures permit determination of household income. At least in the case of Cobb-Douglas utility, equation (2), students will have no incentive to underperform on exams, since tuition will be nonincreasing in ability in equilibrium (proved in Epple and Romano [1993]). Incentive compatibility in the reporting of income is more complex.

with any  $\alpha_0(b, y) \in [0, 1]$  “optimal” for the public school as determined by the residual demand for public education. A private school’s profit-maximization problem can be written as

$$(5) \quad \begin{aligned} & \text{MAX}_{\theta_i, k_i, p_i(b, y), \alpha_i(b, y)} \pi_i \\ & \equiv \iint_S [p_i(b, y) \alpha_i(b, y) \\ & \quad \times f(b, y) db dy] - V(k_i) - F \end{aligned}$$

subject to

$$(5a) \quad \alpha_i(b, y) \in [0, 1] \forall (b, y);$$

$$(5b) \quad U(y_i - p_i(b, y), a(\theta_i, b))$$

$$\geq \text{MAX}_{j \in \{0, 1, \dots, n\} | j \neq i; \alpha_j(b, y) > 0 \text{ is in the optimal set of } j}$$

$$U(y, b);$$

$$(5c) \quad k_i = \iint_S \alpha_i(b, y) f(b, y) db dy;$$

$$(5d) \quad \theta_i = \frac{1}{k_i} \iint_S b \alpha_i(b, y) f(b, y) db dy.$$

Constraints (5c) and (5d) define, respectively, the size of the school’s student body and the mean ability. Constraint (5a) precludes a school from admitting a negative number of a type or more of a type than exists in the population.<sup>12</sup> Constraint (5b) imposes the utility-taking assumption. Students’ alternatives are limited to schools where they are admitted. Students always have the option of attending the public school. It is innocuous to require

that (5b) hold for all  $(b, y)$  as we have specified (i.e., including for nonadmitted students). Tuition charged to students for whom  $\alpha_i(b, y) = 0$  is school  $i$ ’s only optimal choice (i.e., nonadmitted students) is irrelevant. Note, too, that tuition such that (5b) holds with strict equality will be optimal.

Private schools enter so long as they expect to make positive profits as utility takers. Because incumbent private schools maximize profits as utility takers, entry results if and only if  $\pi_i > 0$  for some incumbent school. The public-sector/private-sector equilibrium is described by the following five conditions in addition to the government balanced-budget condition presented below in Section II, subsection C, for the more general case with vouchers.

Condition UM:

$$\begin{aligned} & U^*(b, y) \\ & = \text{MAX}_{i \in \{0, 1, \dots, n\} | \alpha_i(b, y) > 0 \text{ is in the optimal set of } i} U(y_i - p_i(b, y), a(\theta_i, b)) \end{aligned}$$

$$\forall (b, y).$$

Condition ΠM:

$$[\theta_i, k_i, p_i(b, y), \alpha_i(b, y)] \text{ satisfy (5),}$$

$$i = 1, 2, \dots, n.$$

Condition ZΠ:

$$\pi_i = 0 \quad i = 1, 2, \dots, n.$$

Conditions PSP:

$$p_0(b, y) = 0 \forall (b, y)$$

$$\alpha_0(b, y) \in [0, 1] \forall (b, y)$$

$$k_0 = \iint_S \alpha_0(b, y) f(b, y) db dy$$

$$\theta_0 = \frac{1}{k_0} \iint_S b \alpha_0(b, y) f(b, y) db dy.$$

<sup>12</sup> One might object to the presumption that “competitive schools” recognize the limit to demand. The presumption is analogous to a monopolistically competitive firm’s recognition of a limit on its demand curve. Dropping the presumption would lead to schools admitting infinite densities of some types. See Scotchmer (1994) for the analogue in the literature on club goods.

Condition MC:

$$\sum_{i=0}^n \alpha_i(b, y) = 1 \quad \forall (b, y).$$

Condition UM summarizes household utility maximization. Households choose a most-preferred private or public school, taking admission/tuition policies, school qualities, and taxes as given. Profit maximization of private schools (ΠM) and the public-sector policies (PSP) have been discussed. While the entry assumption above is formally part of the definition of equilibrium, it is convenient to substitute the implication that private schools must earn zero profits (ZΠ). The last condition is market clearance, which uses the simplifying assumption above that free public schooling is preferred to no schooling.

## II. Theoretical Results

### A. Solution to the Private School's Problem

Using UM, the first-order conditions for problem (5) can be written as follows:

$$(6a) \quad U(y_i - p_i^*, a(\theta_i, b)) \\ = U^*(b, y) \quad \forall (b, y);$$

$$(6b) \quad \alpha_i(b, y) \begin{cases} = 0 \\ \in [0, 1] \\ = 1 \end{cases}$$

$$\text{as } p_i^*(b, y, \theta_i) \begin{cases} < \\ = \\ > \end{cases} V'(k_i)$$

$$+ \eta_i(\theta_i - b) \quad \forall (b, y);$$

$$(6c) \quad \eta_i = \frac{1}{k_i} \iint_S \left[ \frac{\partial p_i^*(b, y, \theta_i)}{\partial \theta_i} \alpha_i(b, y) \right. \\ \left. \times f(b, y) db dy \right].$$

Condition (6a) describes school  $i$ 's optimal tuition function,  $p_i^*(b, y, \theta_i)$  and is just (5b)

with equality combined with the equilibrium condition UM;  $p_i^*(\cdot)$  is student-type  $(b, y)$ 's reservation price for attending school of quality  $\theta_i$ . Condition (6b) characterizes optimal admission policies.<sup>13</sup> The term  $\eta_i(\theta_i - b)$  may be thought of as the marginal cost of admission operating via the peer-group externality in school  $i$ . From (6c),  $\eta_i$  [the Lagrangian multiplier on (5d)] equals the per-student revenue change in school  $i$  deriving from a change in  $\theta_i$ . The appropriately scaled change in  $\theta_i$  due to admitting student of ability  $b$  equals  $(b - \theta_i)$ ; its negative is then multiplied by  $\eta_i$  to obtain the peer-externality cost. The peer cost of admitting students with ability below the school's mean is positive because the resulting quality decline dictates reduced tuition to all students, while the peer "cost" of admitting above-mean-ability students is negative. Let  $MC_i(b) \equiv V'(k_i) + \eta_i(\theta_i - b)$ , which we term *effective marginal cost*. Types with reservation prices below  $MC_i(b)$  are not willing to pay enough to cover their effective marginal cost and are not admitted. The school admits all of a type that has a reservation price above effective marginal cost, and any  $\alpha_i \in [0, 1]$  is optimal if  $p_i^* = MC_i$ .<sup>14</sup>

### B. Properties of Equilibrium

We now turn to the properties of equilibrium, assuming one exists. Existence issues are discussed below. Heuristic arguments have been substituted for formal proofs when reasonable.

The first result concerns the qualities of schools.

**PROPOSITION 1:** *A strict hierarchy of school qualities results, with the public sector*

<sup>13</sup> Results (6b) and (6c) are found by substituting  $p_i^*$  from (6a) into (5), and then forming a Lagrangian function to take account of (5c) and (5d). Result (6b) is then derived by pointwise optimization over  $\alpha_i$  while taking account of the constraint (5a).

<sup>14</sup> In the upper and lower lines of (6b), the solution for  $\alpha_i$  is at a corner, and the first-order conditions are also sufficient for a local maximum. In the middle line of (6b), where  $p_i^* = MC_i$  and any  $\alpha_i(b, y) \in [0, 1]$  satisfies the first-order conditions,  $V''$  sufficiently large implies local maximization.

having the lowest-ability peer group:  $\theta_n > \theta_{n-1} > \dots > \theta_1 > \theta_0$ .

Formal proof is in the Appendix. Here an economic interpretation is provided. All private schools must be of higher peer quality than schools in the public sector. Otherwise, no students would be willing to pay to attend any private school.

Why must a strict hierarchy of private schools characterize equilibrium? If two private schools were of the same quality, then they would compete perfectly for students. Consequently, they would have the same effective marginal costs of admitting all types, and their tuitions (to all admitted students) would equal effective marginal costs. An opportunity to increase profits would exist by varying admissions/tuitions in such a way to either: (a) increase quality and admit a student body that values quality by more, or (b) decrease quality and admit a student body that values quality by less. In either case, the school differentiates itself in quality, at the same time attracting a student body that permits profitable price discrimination over the quality change.

We sketch the example of a profitable quality improvement, beginning with schools having identical student bodies (the proof shows that this is without loss of generality). Let one school admit the same number of  $(b_2, y_2)$  types as it expels of  $(b_1, y_1)$  types, where  $b_2 > b_1$  and  $y_2 > y_1$ , implying an increase in  $\theta$  but no change in production costs,  $V(k) + F$ . Further, choose the types (which is always feasible) such that  $y_2 - y_1 > b_2 - b_1$  by enough that, using SCI, the  $(b_2, y_2)$ -types value increased quality by more than the  $(b_1, y_1)$ -types, even though their abilities differ. This permits the school to charge the newly admitted students tuitions higher than their effective marginal costs because they are selected to value quality increases by more than the expelled students. The profit increase occurs because the new student body values the quality increase by more than would the original student body;  $\theta$  and  $\eta$  rise in the school. It would not increase profits to substitute students in such a way that  $\theta$  rises without also changing the student body's average value of quality improvements, because tuitions equal

effective marginal costs in the initially non-differentiated schools.<sup>15</sup> This example assumes that a school substitutes students to increase quality, but alternative profitable substitutions exist that decrease quality, roughly, by also creating a lower-income student body.

In either case, the argument depends on SCI. It also identifies the model's force for "diagonal stratification" (see the examples in Figure 1). As developed more fully below, this stratification results because students having relatively high income and low ability within a school cross subsidize relatively low-income, high-ability students.

The strict hierarchy of Proposition 1 supports the equity-related concerns of some that private schools operate to the detriment of public schools by siphoning off higher-ability students. Whether a strict hierarchy is efficient is analyzed below. First we develop further the positive properties of equilibrium.

Proposition 2 describes equilibrium pricing, and Proposition 3 describes the resulting partition of types. Some definitions are useful. Let  $\mathcal{A}_i \equiv \{(b, y) \in S | \alpha_i(b, y) > 0 \text{ is optimal}\}$  denote the *admission space* of school  $i$ ,  $i = 0, 1, \dots, n$  (see Figure 1, for example). A locus of points  $(b, y) \in \mathcal{A}_i \cap \mathcal{A}_j$ ,  $i \neq j$ , assuming it exists, is referred to as a *boundary locus* between  $i$  and  $j$ . (Boundary loci have zero measure in  $S$ , as proved in Epple and Romano [1993].) Since any household prefers free public schooling to no schooling, the entire type space  $S$  is partitioned into admission spaces. Last, to avoid tedious qualification of statements for public-sector schools, we specify that  $MC_0 \equiv 0$  for all  $(b, y)$ . This notation is convenient since students see a zero cost of public education.

**PROPOSITION 2:** (i) *On a boundary locus between school  $i$  and  $j$ ,  $p_i = MC_i(b)$  and  $p_j = MC_j(b)$ ; pricing on boundary loci is strictly according to ability in private schools.* (ii)  *$p_i(b, y) > MC_i(b)$  for off-boundary students who attend private school  $i$ ; pricing off-*

<sup>15</sup> Mathematically, beginning with equal  $\theta$ 's, first-order effects on profits of varying admissions vanish, but the profit function is convex in some directions in  $[\alpha(b, y), p(b, y)]$ -space, allowing a profit increase.

boundary loci depends on income in private schools. (iii) Every student attends a school that would maximize utility if all schools instead set  $p_i$  equal to equilibrium  $MC_i$  for all students. The allocation is as though effective marginal cost pricing prevails in private schools.<sup>16</sup>

See Epple and Romano (1993) for proof. Competition between private schools that share a boundary locus forces prices to effective marginal costs for student-types on the locus. These students are indifferent to attending the schools sharing the locus. Private schools then have no power to price discriminate with respect to income on boundary loci. Prices are, however, adjusted to differing abilities because private schools internalize the peer-group effect. Tuition to private school  $i$  decreases with ability at rate  $\eta_i$  along its boundary loci, reflecting the value of peer-group improvements of the school's student body.

Moving inside a boundary locus in a private school's admission space, students' preferences change in such a way that they would strictly prefer the school attended if it practiced effective marginal-cost pricing. Part (ii) of Proposition 2 establishes that private schools exploit this by increasing price. These students are also indifferent between the private school attended or their best alternative by (6a), but this is a *result* of discriminatory pricing. Generally, then, price depends both on ability and income within admission spaces.<sup>17</sup>

Part (iii) of Proposition 2 follows because it is profitable for a private school to be sure to attract any student whose reservation price

exceeds the school's effective marginal cost. The student allocation's link to effective marginal costs, and hence abilities, will be shown to be efficient (except for the public sector). The income-related price discrimination that occurs does not disrupt the allocation consistent with effective marginal-cost pricing; rather, it is purely redistributive.

While this income-related price discrimination is of the first degree (à la Pigou), its magnitude is limited by competition for students among the differentiated schools. Near a boundary in a school's admission space, a student's preference for the school attended would be slight under effective marginal-cost pricing, so that the admitting school can capture little rent. The number and sizes of private schools then determine their power to price discriminate over income. All private schools have student bodies less than  $k^*$  by a similar argument to that in more standard monopolistically competitive equilibria.<sup>18</sup> Here school  $i$ 's marginal-revenue curve can be constructed by ordering from highest to lowest students' reservation prices minus peer costs [i.e.,  $p_i^* + \eta_i(b - \theta_i)$ ], and thus the associated *downward-sloping* average revenue curve may be derived. Zero profits then implies a scale below  $k^*$ . If we let  $k^*$  decline, then private schools become more numerous and less differentiated (have closer  $\theta$ 's), and income-related price discrimination declines.

Now consider the partition of types into schools. We say *stratification by income* (SBI) holds if, for any two households having students of the same ability, one household's choice of a higher- $\theta$  school implies it has a weakly higher income than the other household. Analogously, *stratification by ability* (SBA) is present if, holding income fixed, the household that chooses a higher- $\theta$  school must have a student of weakly higher ability. The combination of SBI and SBA implies a diagonalized partition as, for example, in Figure 1.

**PROPOSITION 3:** (i) SBI characterizes equilibrium. (ii) If preferences satisfy weak single crossing in ability (W-SCB) and  $\eta_1 \leq$

<sup>16</sup> The statements regard the *equilibrium* effective marginal cost. Income effects would cause these costs to change if tuition equaled effective marginal cost for all students. This has distributional (but not efficiency) implications.

<sup>17</sup> While there are no published studies of the allocation of financial aid by income and ability among private elementary and secondary schools, there is evidence on the allocation of financial aid by colleges and universities. There the evidence is that both ability and family income are significant determinants of whether and how much financial aid is received (J. Brad Schwartz, 1986; Sandra R. Baum and Saul Schwartz, 1988; Charles T. Clotfelter, 1991).

<sup>18</sup> The points made here are proved in Epple and Romano (1993).

$\eta_2 \leq \dots \leq \eta_n$ , then SBA also characterizes equilibrium.<sup>19</sup>

To confirm part (i), consider two households with students of the same ability but differing incomes  $y_i^2 > y_i^1$ . In the  $(\theta, p)$ -plane, indifference curves of a household are upward sloping. For the same ability, SCI implies that any indifference curve of  $y_i^2$  cuts any indifference curve of  $y_i^1$  from below. Allocations are as if tuitions equal effective marginal costs [part (iii) of Proposition 2]. Thus, the choice between schools  $i$  and  $j$  may be represented in the  $(\theta, p)$ -plane as a choice between  $(\theta_i, MC_i(b))$  and  $(\theta_j, MC_j(b))$ . If  $\theta_j > \theta_i$ , it must be that  $MC_j(b) > MC_i(b)$  if either type chooses  $i$ . A standard single-crossing argument then applies to complete the proof.

Part (ii) is proved in the Appendix; here we provide some intuition. Assume first that the demand for quality is independent of ability (e.g., as in the Cobb-Douglas specification) and that all private schools give the same discount to ability along their boundary loci (i.e., schools'  $\eta$ 's are the same). Holding nominal household income fixed, real income would rise with student ability due to tuition discounts at all private schools. SBA would then result by the same logic explaining SBI. Hence, the combination of a positive income elasticity (SCI) and discounts to ability alone would cause *both* SBI and SBA, the diagonalized partition as in Figure 1. Relatively high-income and low-ability students cross subsidize relatively low-income and high-ability students in private schools. The argument holds more strongly if the  $\eta$ 's strictly ascend or if W-SCB holds strictly. However, neither condition is necessary for SBA, nor do any of our other results require these conditions or SBA. It may be possible absent these conditions to get cases having nonmonotonic boundary loci in the  $(b, y)$ -plane.<sup>20</sup>

<sup>19</sup> We thank an anonymous referee for encouraging us to investigate bid-rent functions (see e.g., Masahisa Fujita, 1989), which ultimately led to part (ii) of Proposition 3.

<sup>20</sup> The alternative to W-SCB implies that lower-ability types are willing to pay more for a better peer group, and the alternative to weakly ascending  $\eta$ 's implies that lower-

We now turn to normative results which are quite intuitive. Again, see Epple and Romano (1993) for the formal analysis. Pareto efficiency requires: (i) a student allocation that internalizes the peer-group externality given the number of schools, and (ii) entry as long as aggregate household net willingness to pay for an allocation with one more school exceeds the change in all schools' costs. An equilibrium without a public sector would satisfy condition (i) but not condition (ii). Effective marginal cost includes the marginal value of the peer group externality, implying that  $MC_i(b)$  equals the social marginal cost of attendance at school  $i$  by a student of ability  $b$ . A purely private-school equilibrium then satisfies efficiency condition (i) by part (iii) of Proposition 2.

However, entry to the point of zero profits entails externalities so that efficient entry [condition (ii)] fails to hold in a fully private equilibrium. An entrant captures the full value of its product to the student body it admits but ignores utility changes of nonadmitted students and profit changes of other schools resulting from the reallocation.<sup>21</sup> Fixed costs,

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quality schools give bigger discounts to ability. Either would tend to work against pure ability stratification, though Proposition 1 implies that some degree of ability stratification would be present. It is desirable to demonstrate SBA without assuming ascending  $\eta$ 's, since these values are endogenous. However, providing general primitive conditions for SBA independent of assumptions concerning the equilibrium  $\eta$ 's is difficult, because their equilibrium values depend on the entire distribution of types in the population. For the Cobb-Douglas case and assuming independence of income and ability in the population, we (Epple and Romano, 1993) have shown SBA without assuming weakly ascending  $\eta$ 's.

<sup>21</sup> The comparison of the equilibrium number of schools in a fully private equilibrium to the Pareto-efficient number entails a trade-off. The entrant ignores the lost revenues and cost savings to other schools from the students that it admits. Since almost every student attracted away from incumbent schools is inframarginal (i.e., tuition exceeds effective marginal cost), the net effect here of entry is negative, tending to cause too much entry. Opposing this is the entrant's failure to capture the full returns from increased variety of school qualities that results. Although the entrant fully price discriminates to the students it admits, it cannot tax other students for the adjustments in the incumbent schools' qualities. A net benefit to other students is likely to result because the incumbent schools will better accommodate preferences.

hence the finite size of an entrant, underlie the entry externalities as in many models of monopolistic competition.

Introduction of the free public sector implies deviations from both efficiency conditions. In general, the public sector displaces multiple differentiated private schools, substituting the equivalent of one “large” homogeneous school. This effective reduction in the number of schools is without attention to costs and benefits, generally implying a deviation from efficiency condition (ii).

Holding fixed the number of schools in the public–private equilibrium (and counting the public sector as one school), zero pricing of public schooling violates condition (i). By just reallocating students between the public sector and private school 1 near their shared boundary locus, Paretian gains are feasible. Reference to the upper panel of Figure 1 from our computational equilibrium may help clarify the argument. Gains would result from shifting into private school 1 relatively lower-ability students below but near the boundary locus, students for whom the marginal social cost in the public sector is positive. These students are nearly indifferent between the two schools when facing the social cost of attending the private school but a tuition (zero) below the social cost of attending the public school. Students near the boundary locus and attending the private school may also be of sufficiently high ability that the social “cost” of attending the public school is negative. Gains from shifting such students into the public sector are then also feasible. Such students exist in our computational model, the rough prescription being to rotate the boundary locus counterclockwise at the point of ability having zero social marginal cost in the public school. Collecting these results, we have the following proposition.

**PROPOSITION 4:** (i) *The allocation in a fully private equilibrium is (Pareto) efficient given the number of schools; the equilibrium number of schools is not, however, generally*

*efficient.* (ii) *The public–private-sector equilibrium has neither an efficient number of schools, nor an efficient student allocation given the number of schools.*

When fixed costs of schooling are small, the departure from efficiency in a fully private equilibrium will be correspondingly small. Part (i) of Proposition 4 can then be interpreted as making a case for private schooling and the vouchers we study. However, we have some reservations concerning this efficiency result. First, we are sympathetic to the view of many that access to a quality education is a right and serves as a means to limit historical inequities. Second, longer-run externalities from education not considered by private schools, like reduced crime, may be present. For these reasons, we explore the consequences of vouchers on all types instead of just providing aggregate measures. A somewhat distinct concern arises because exact equilibrium exists only in special cases. The interpretation of the efficiency results in the approximate equilibrium we study is discussed in subsection D, below.

### C. Vouchers

We examine tax-financed cash awards to all those attending private school.<sup>22</sup> No role for vouchers is present in the tuition-free public sector. Reformulate the model by everywhere adding the amount of the voucher,  $v$ , to  $y$ , for households that choose a private school. The government’s budget constraint is:

$$(7) \quad \iint_S tyf(b, y) db dy \\ = \iint_{\mathcal{A}_1 \cup \mathcal{A}_2 \cup \dots \cup \mathcal{A}_n} vf(b, y) db dy \\ + \hat{N}[F + V(\hat{k})],$$

This positive externality will tend to cause too little entry. We believe that too many or too few private schools are possible, but we have not proved this.

<sup>22</sup> Our model permits households to retain as income any excess of the voucher amount over the tuition paid to the private school of choice, thereby avoiding considerable complication.

where  $\hat{N}$  and  $\hat{k}$  denote, respectively, the cost-minimizing number and size of schools in the public sector that satisfy demand for public education. Vouchers lower the real price of private education and increase the demand for it. We examine the effects of vouchers in our computational model.

#### D. Existence of Equilibrium and an Approximate Equilibrium

As we discuss in the Appendix, exact equilibrium generally fails to exist due to the integer number of private schools. We examine an approximate equilibrium in our computational analysis. Our ‘‘epsilon-competitive equilibrium’’ requires that no (utility-taking) private school, incumbent or entrant, could increase profits by more than  $\varepsilon$ . Let  $\pi_{\max}$  and  $\pi_{\min}$  denote the maximum and minimum profits earned by incumbent schools [which maximize profits over  $p(b, y)$  and  $\alpha(b, y)$  locally], and replace  $Z\Pi$  in the definition of equilibrium with

$$\text{MAX}[\pi_{\max}, \pi_{\max} - \pi_{\min}, -\pi_{\min}] \leq \varepsilon.$$

Here  $\pi_{\max}$  equals the maximum potential profits to an entrant, and the maximum of the second two terms in the brackets equals the largest feasible profit increase by an incumbent school. The revised definition of equilibrium continues to require UM, PSP, MC, and local profit maximization by incumbent private schools [i.e., (6a)–(6c)]. Last, the number of private schools is the minimum number satisfying these requirements.

The epsilon equilibrium retains all the positive properties of an exact equilibrium except that private schools could gain  $\varepsilon$  in profits via global adjustments. The allocation of students in a fully private equilibrium would then continue to satisfy efficiency condition (i).

### III. Computational Equilibrium Model and Illustrative Results

We develop a computational model to illustrate our results, to examine vouchers, and to explore issues for which comparative-static analysis may yield ambiguous results. We calibrate it to existing empirical evidence so that

the results will provide at least suggestive evidence about the impact of policy interventions. However, scant empirical evidence exists on some important parameters of the model.

#### A. Specification and Calibration

We require specifications for the density of income and ability, the utility and achievement functions, and the cost function for education.

We assume that  $\begin{bmatrix} \ln(b) \\ \ln(y) \end{bmatrix}$  is distributed bivariate normal with mean  $\begin{bmatrix} \mu_b \\ \mu_y \end{bmatrix}$  and covariance matrix

$$\begin{bmatrix} \sigma_b^2 & \rho\sigma_b\sigma_y \\ \rho\sigma_b\sigma_y & \sigma_y^2 \end{bmatrix}.$$

To calibrate the distribution of income, we use mean (\$36,250) and median (\$28,906) income for households from U.S. census data for 1989. With units of income in thousands of dollars, these imply that  $\mu_y = 3.36$  and  $\sigma_y = 0.68$ .

We adopt specification (2) for the combined utility-achievement function. To calibrate the ability distribution we presume that educational achievement determines future earnings and that the benchmark economy is in a steady state. First, define normed achievement,  $a_N$ , as our achievement function raised to the power  $1/\beta$  and multiplied by a constant,  $a_N \equiv Ka^{1/\beta} = K\theta^{\gamma/\beta}b$ .<sup>23</sup> Then, a student with ability  $b$  attending a school with a peer quality of  $\theta$  is presumed to have future annual earnings ( $E$ ) given by  $\ln E = \ln a_N = \ln K + (\gamma/\beta)\ln \theta + \ln b$ . This normalization is such that a percentage change in ability leads to the same percentage change in dollars earned. Henderson et al. (1978) report the change in achievement percentile that results from moving students from classes stratified by ability to mixed

<sup>23</sup> The constant of proportionality,  $K$ , is arbitrary. A convenient scaling is to set  $K = E[b]^{-\gamma/\beta}$ . This scaling has the property that, if all students in the population were to attend the same school (i.e.,  $\theta = E[b]$ ), then normed achievement would equal ability (i.e.,  $a_N = b$ ).

classes. An elasticity of achievement with respect to peer ability that is 30 percent as large as the elasticity with respect to own ability is representative of the results they report. We adopt the somewhat conservative value of  $\gamma/\beta = 0.2$ . To complete the calibration of the distribution of ability, we then assume that the observed household-income distribution is the income distribution that emerges in a steady-state equilibrium in our benchmark model.<sup>24</sup> This yields  $\mu_b = 2.42$  and  $\sigma_b = 0.61$ . Thus, mean and median ability are 13.6 and 11.3, respectively, and the standard deviation of ability is 9.1.<sup>25</sup>

Gary Solon (1992) and David J. Zimmerman (1992) provide evidence on the correlation between father's income and son's income, and they both find that the best point estimate of this correlation is approximately 0.4. Intergenerational correlation in income arises from two sources: correlation between household income and student ability and, for given ability, correlation between income and quality of school attended. Hence, SBI suggests that the intergenerational correlation in incomes is an upper bound on the correlation between parent's income and child's ability. For purposes of sensitivity analysis, we then assume that  $\rho \in [0, 0.4]$ . For our benchmark case, we set  $\rho = 0$ ,

which is particularly convenient for our steady-state calibration of the model. This completes the calibration of  $f(b, y)$ .

We now complete the calibration of preferences. The Cobb-Douglas specification implies unitary price and income elasticities for school quality,  $\theta$ . Given the absence of empirical evidence on the demand for quality, these are plausible focal values and are consistent with estimates of demand for school expenditure (see e.g., Theodore Bergstrom et al., 1982). This function also implies that the marginal rate of substitution between school quality and the numeraire is invariant to own ability. Empirical evidence is mixed about whether an improvement in peer group is more beneficial to high- or low-ability students. Hence, our model's assumption that the effect of peer group is not biased toward either high- or low-ability types seems an appropriate choice for a baseline model. If school quality could be purchased at a constant price per unit of quality, each household's expenditure on education relative to total expenditure on other goods would be  $\gamma/(1 + \gamma)$ . The existing share of aggregate disposable personal income in the United States that is spent on education is approximately 0.056. Hence, we set  $\gamma = 0.06$ . Using  $\gamma/\beta = 0.2$  from above, the calibrated utility-achievement function is then

$$U = (y_t - p)\theta^{0.06}b^{0.30}.$$

We chose a cost function that is quadratic in the percentage of students (or households) a school serves:

$$F + V(k) = 12 + 1,300k + 13,333k^2,$$

with parameters set as follows. Expenditure per student in public schools in 1988 was \$4,222 (*Statistical Abstract*, 1991 p. 434) and there was  $1/2$  student per household (*Statistical Abstract*, 1992 pp. 46, 139). We specified our benchmark case to have four private schools and chose parameter values such that average cost in equilibrium was approximately \$4,200 per pupil.<sup>26</sup> Experimentation indicated that

<sup>24</sup> More precisely, we let the distribution of ability be lognormal, and we approximate by assuming that this generates a lognormal distribution of earnings. We set the first two moments of the distribution of earnings equal to the first two moments of the distribution of income. That is, we choose  $\mu_b$  and  $\sigma_b$  such that our benchmark equilibrium has  $E[a_N] = E[y]/m$  and  $\text{Var}[a_N] = \text{Var}[y]/m^2$ . The constant  $m$  is the ratio of employed workers per household to the number of students per household ( $m = 2.6$  in 1990). The distribution of earnings will not be exactly lognormal because of the discrete difference in schools attended, even though the distribution of ability is presumed to be lognormal. If every student attended public school in the benchmark model, and hence faced the same  $\theta$ , earnings would be exactly lognormal. The approximation is a good one because 90 percent of the students do attend public schools as we will see.

<sup>25</sup> Ability can be related to IQ. Using  $\text{IQ} \sim \mathcal{N}(100, 256)$ , one obtains  $\ln b = -1.38 + 0.038(\text{IQ})$ . In our no-voucher steady state, this implies that a worker with an IQ of 100 has expected income of \$22,074, and a 10-point increase in his IQ increases expected income to \$32,510. See the discussion in what follows relating to Figure 6 and the calculation of expected steady-state income conditional on ability.

<sup>26</sup> We have presented the cost function in terms of the percentage of students served or, equivalently, the per-

equilibrium properties are not very sensitive to the benchmark number of schools, but rather are sensitive to the minimum of the average cost of schooling.

We set  $\varepsilon = 4.2$ . This is the minimum value sufficient to assure existence of epsilon equilibrium for voucher values varying from zero to \$4,200 per student.<sup>27</sup>

### B. Results

For our benchmark equilibrium with no voucher, the public sector has 90 percent of the student population, and the four private schools combined serve the remainder. The actual U.S. percentage of students enrolled in public schools during this period equaled 88 percent. Increasing  $\rho$  from 0 to 0.4 reduces public-sector attendance to 88 percent. Effects on other variables of so changing  $\rho$  are also small, and the results that follow are for  $\rho = 0$ .

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centage of households served,  $k$ . In terms of  $k$ , average cost reaches a minimum at \$2,100, with  $k^* = 0.03$ ; \$2,100 can then be interpreted as the average cost *per household*. There are twice as many households as students in the United States. Letting  $s$  denote the *number* of students and substituting  $s = k/2$ , one sees that the minimum of the average cost *per student* is \$4,200. In our presentation, we focus on per-student measures of tuition and costs; the related per-household measures are simply half those of the per-student values.

<sup>27</sup> This value is about 7 percent of the cost of a school operating at a scale that minimizes cost per student. Relative to fixed cost,  $\varepsilon$  is approximately 35 percent. Of course, a minimal  $\varepsilon$ , however measured, is desirable. We have studied how the minimum  $\varepsilon$  varies as we vary efficient school scale,  $k^*$ , while holding average cost constant. We find that the requisite  $\varepsilon$  to support equilibrium varies approximately proportionately with  $k^*$  if fixed cost is varied proportionately with  $k^*$ . This suggests, as we would expect, that  $\varepsilon$  can be made as small as desired if  $k^*$  is made sufficiently small. We have also investigated increasing fixed cost while holding  $k^*$  and minimum average cost constant. This tends to reduce the ratio of  $\varepsilon$  to fixed cost but increases the absolute magnitude of  $\varepsilon$  required to sustain equilibrium. Our investigation reveals that substantive findings from the computational model are not sensitive to the choice of  $k^*$  or the relative magnitude of fixed to variable cost. Rather, the key aspect of costs is the value of average cost at the minimum, and as discussed in the text, this value is based on observed school costs. The problem with pursuing a calibration that further lowers  $\varepsilon$  is that it leads to a computationally unmanageable number of schools for large vouchers.

Other computational results are presented in Figures 1–6. The upper panel of Figure 1 presents the boundary loci and admission sets in type space, in addition to the equilibrium  $\theta$ 's and  $k$ 's. Here and in some other figures, both absolute and percentile ability scales are provided for perspective. The lower panel displays the allocation for a voucher of \$1,800. The linear boundary loci derive from the Cobb-Douglas specification. For results we present, intersections of boundary loci, if any, occur very near the bounds of the support of type space. Such intersections are insignificant, but we have encountered cases having only a piecewise linear public-private boundary due to meetings of loci well interior to type space. In such cases, some (high-income) students in public-sector schools have other than the lowest- $\theta$  private school as their best alternative.

In addition to illustrating the strict hierarchy of schools, the data in Figure 1 show a negative correlation between school qualities and school size in the private sector. Those who attend lower-quality private schools face closer substitutes, flattening the derived average revenue curves (discussed above) of these schools and increasing their size in equilibrium. The negative correlation of school quality and school size is a testable prediction, and it seems plausible that the most elite schools are smallest.

Figure 2 shows effective marginal costs for the four private schools in equilibrium with no voucher, truncated at the maximum ability of students attending each school. For students on the admission boundaries of their schools, tuition rates equal effective marginal costs. While price discrimination by income as well as ability is practiced on the interior of schools' admission spaces, price is close to marginal cost for almost all students. Hence, as a first approximation, one can interpret the marginal-cost functions in Figure 2 as price functions. High-ability students are seen to receive a tuition discount (financial aid) in all schools. In addition, students of sufficiently high ability pay negative tuition (i.e., a tuition waiver plus a stipend) in the top three schools. For our calibration, the standard deviation of ability is about 9 units. The results imply tuition reductions of roughly \$2,350 and \$3,240

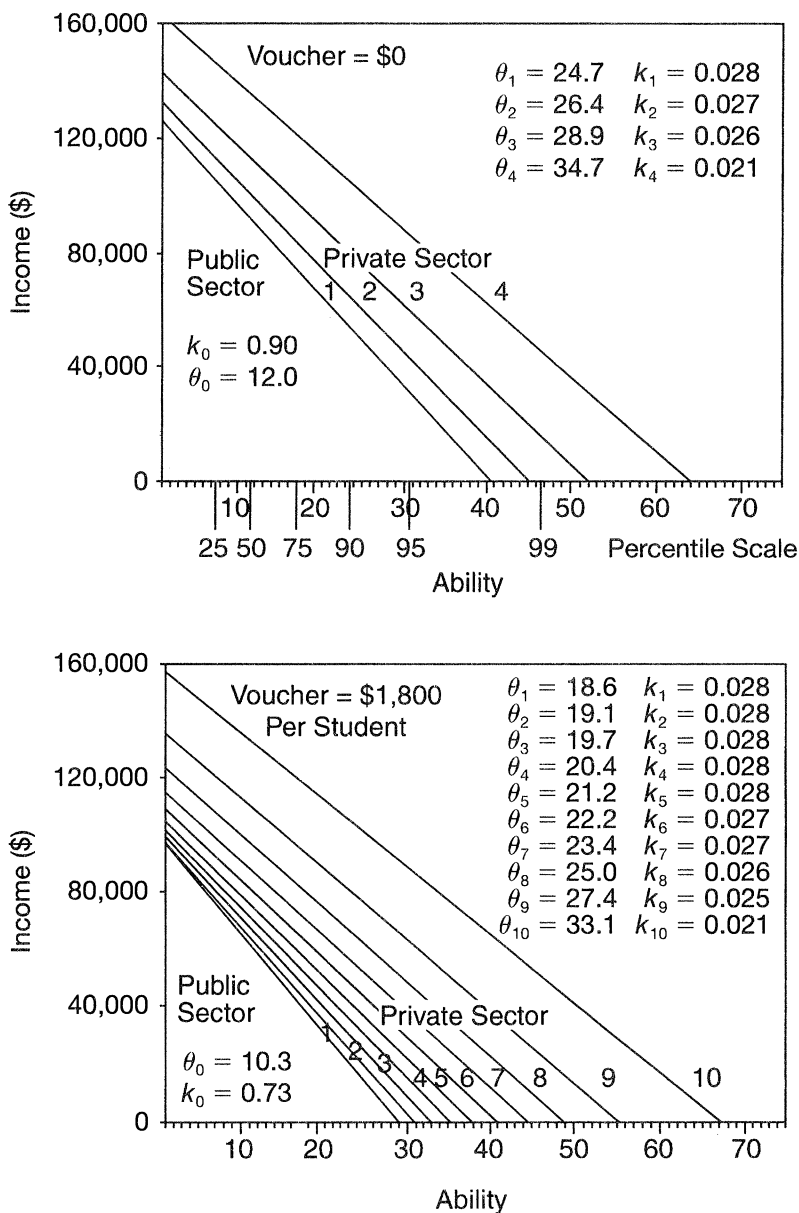


FIGURE 1. BOUNDARY LOCI WITHOUT AND WITH A \$1,800 VOUCHER

for a one-standard-deviation increase in ability in schools 1 and 4, respectively.

The top panel of Figure 3 shows the ability distributions in the public-school sector and in the four private schools in the computed equilibrium. These distributions, all normalized to

have an area of 1, illustrate the hierarchy among schools established in Proposition 1.

We simulated the effect of vouchers in amounts ranging from \$0 to the minimum average cost of educating a student, \$4,200. The effect of these vouchers on the distribution of

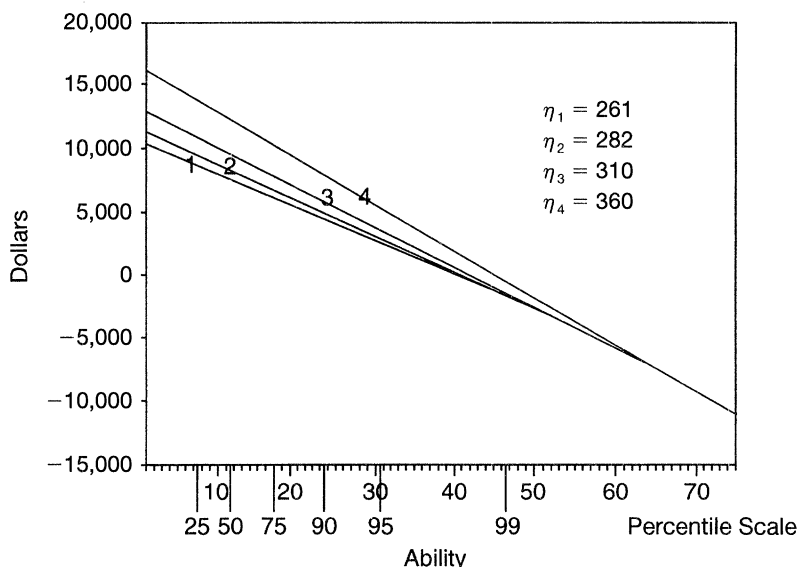


FIGURE 2. MARGINAL-COST FUNCTIONS WITH FOUR PRIVATE SCHOOLS

ability types attending public school is illustrated in the bottom panel of Figure 3. To illustrate the effect of the voucher program on public-school attendance of students of various ability levels, we have not normalized these distributions to have an area of 1. Thus, they represent the “numbers” of students at each ability level that are served by the public-school sector.

Figure 4 illustrates several ways in which equilibrium changes with the voucher. The upper panel shows the decline in the proportion of students attending public school as the voucher increases. The curve showing size during “transition” assumes the parental-income and student-ability distributions discussed above. When a voucher policy is introduced or varied (i.e., during transition), the income distribution of the parent generation will typically differ from the income distribution of the succeeding generation because the new voucher changes the distribution of earnings of a generation relative to that preceding. By contrast, in steady state, the income distribution is unchanging from generation to generation. To obtain the steady state for each value of the voucher, we hold the ability distribution fixed. We find the parent-income dis-

tribution that leads to an earnings distribution of the next generation that replicates the income distribution of the parents. The size of the public sector is virtually the same in transition and steady state (Figure 4), as are other variables that can be compared across the cases. To conserve space, other results presented are for the transition except when stated otherwise.

As expected, the size of the public-school sector declines to zero when the voucher nears the minimum average cost of providing education. The number of private schools at each voucher value is also shown in the top panel of Figure 4, illustrating the entry of new private schools as the voucher is increased. While the graph shows a continuous approximation, the number of schools is, of course, an integer at each voucher level in our computational model. We also calculated for each voucher level the percentage of households that favor that voucher level as compared to a voucher of zero. As the top panel of Figure 4 shows, support for a voucher program increases with the voucher but never reaches a majority. For example, only about 31 percent of the population benefits from a \$2,000 voucher, and this includes the 10 percent who attend private

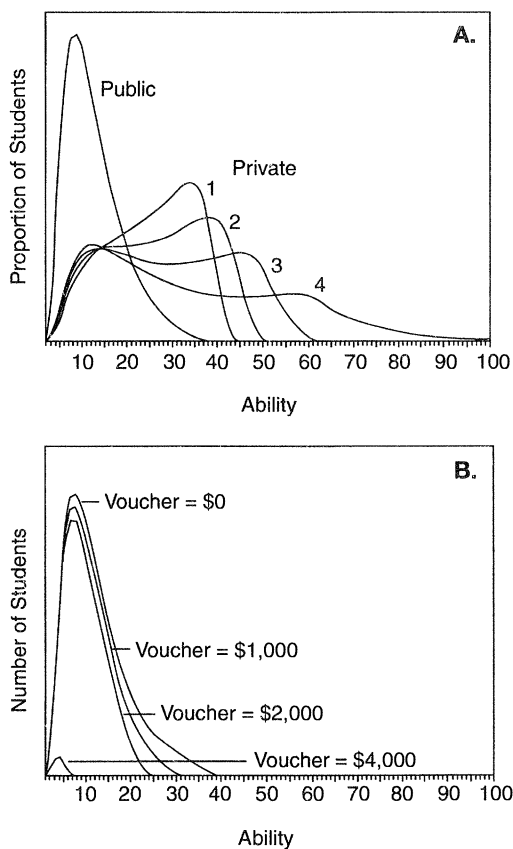


FIGURE 3. (A) ABILITY DISTRIBUTIONS IN PUBLIC AND PRIVATE SCHOOLS (VOUCHER = \$0) AND (B) THE EFFECT OF VOUCHERS ON THE DISTRIBUTION OF ABILITIES OF STUDENTS ATTENDING PUBLIC SCHOOL

school with no voucher. A theme of our computational findings is that, while gains from vouchers result on average, there is a majority with relatively small losses and a minority with relatively large gains.

The bottom panel of Figure 4 also shows the per-student welfare gain, compensating variation plus net profit change (the latter relatively small), associated with the introduction of the voucher. The welfare gain associated with the maximum voucher is a relatively modest 0.5 percent of mean income, but as we will show, the distributional effects are more substantial. Welfare rises with the voucher until about 86 percent of the population attends

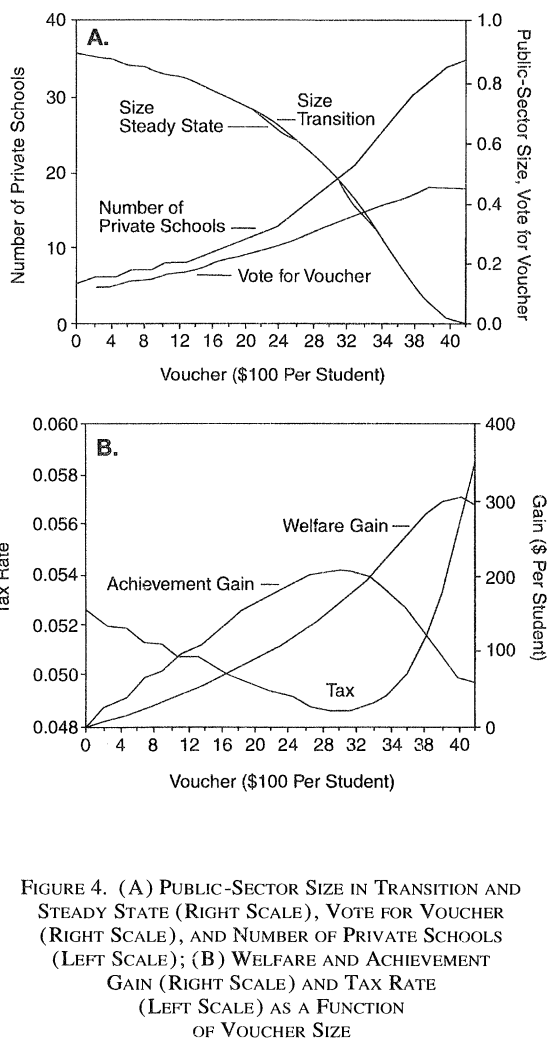


FIGURE 4. (A) PUBLIC-SECTOR SIZE IN TRANSITION AND STEADY STATE (RIGHT SCALE), VOTE FOR VOUCHER (RIGHT SCALE), AND NUMBER OF PRIVATE SCHOOLS (LEFT SCALE); (B) WELFARE AND ACHIEVEMENT GAIN (RIGHT SCALE) AND TAX RATE (LEFT SCALE) AS A FUNCTION OF VOUCHER SIZE

school in the private sector. Apparently, negative entry externalities come to dominate welfare gains beyond this point (see footnote 21), as is manifest in rising average costs of private schools. The bottom panel of Figure 4 shows the tax rate at each voucher level. For low voucher levels, the cost of the voucher is more than offset by the reduction in public-school costs resulting from students who are induced by the voucher to choose private school, and the tax rate falls. Eventually, the tax rate must rise, however, since a sufficiently large voucher covers the cost of education for virtually all students in all schools.

Figure 4 also shows that per-student normed achievement, or equivalently, average future earnings of those who will work, rises and then declines with the voucher. The partition of students that maximizes earnings (gross or net of schooling costs) entails a strict hierarchy of schools with stratification by ability but not by income. This is implied by the complementarity of  $b$  and  $\theta$  in our normed achievement function, and its property that peer effects impact per-student earnings more the higher is a school's  $\theta$ .<sup>28</sup> Maximizing welfare differs from maximizing net earnings in our model because demand for student achievement depends on household income. By one interpretation, our model ascribes a household consumption motive to the educational achievement of the household's child(ren) (see footnote 8). The further partitioning of students caused by increasing the voucher will tend to increase welfare, as we have seen, but it has less-clear effects on ability stratification and, hence, on earnings. One effect of an increased voucher is to increase competition for high-ability students, which causes the boundary loci between incumbent private schools to become flatter (as further discussed below). This decreases the "degree" of ability stratification. Counteracting this is the migration of students into the private sector and the finer partition of students associated with entry of private schools. The latter effect dominates at low vouchers in our computations, and earnings rise initially with the voucher. The former effect comes to dominate for moderately large vouchers, and earnings decline with further increases in the voucher. Hence, while welfare is maximized at a voucher high enough that 86 percent of students attend private schools, earnings maximization necessitates a substantially lower voucher that draws only 47 percent of students to the private sector.

<sup>28</sup> Proof and detailed analysis of these points is in an unpublished Appendix, which is available from the authors upon request. A nascent related literature studies the optimal grouping of workers of varying skills into firms (see Michael Kremer and Eric Maskin, 1995). Every student in a school has his own output (achievement), while workers in a firm have but one output. Hence, the results of the literature on worker grouping do not immediately map into the problem of student grouping.

The aggregate effects on achievement and welfare summarized above are likely to be sensitive to the details of the specification of the achievement function. In our specification of normed achievement, gains from peer ability are proportionate to own ability. A specification of normed achievement in which high-ability students gain even more from high peer quality than do low-ability students would yield greater aggregate gains from sorting students by ability, for example. As we noted earlier, there is little empirical evidence on this issue.

Aggregate achievement is maximized by complete ability stratification in our Cobb-Douglas specification, as observed above. In our computational model, complete ability stratification increases (normed) aggregate achievement by 4.6 percent over complete mixing of students.<sup>29</sup> One-third of this potential gain is achieved in the public/private equilibrium without a voucher. As we illustrated in the bottom panel of Figure 4, aggregate achievement is nonmonotonic in the amount of the voucher.

<sup>29</sup> Normed aggregate achievement with complete mixing is given by

$$\begin{aligned} A_m &\equiv K \int_0^\infty \exp\left[\frac{\gamma}{\beta}(\mu_b + \sigma_b^2/2)\right] b f_b(b) db \\ &= K \exp\left[\left(1 + \frac{\gamma}{\beta}\right)(\mu_b + \sigma_b^2/2)\right] \end{aligned}$$

where  $f_b(b)$  is the lognormal density function. Normed aggregate achievement with complete ability stratification is given by

$$\begin{aligned} A_s &\equiv K \int_0^\infty b^{(\gamma/\beta)+1} f_b(b) db \\ &= K \exp\left[\left(\frac{\gamma}{\beta} + 1\right)\mu_b + \left(\frac{\gamma}{\beta} + 1\right)^2 \sigma_b^2/2\right]. \end{aligned}$$

Hence,

$$\frac{A_s}{A_m} = \exp\left[\frac{\gamma}{\beta} \left(\frac{\gamma}{\beta} + 1\right) \frac{\sigma_b^2}{2}\right] \approx 1.046$$

for our calibration. In steady state with complete mixing, one can show that  $\sigma_b^2$  is invariant to  $\gamma/\beta$ . Hence, the preceding formula can be used to determine how potential gains to ability stratification vary with  $\gamma/\beta$ . Such gains increase with  $\gamma/\beta$  (e.g., to about 45 percent for  $\gamma = \beta$ ).

Maximum achievement occurs at a voucher of \$2,800, and this maximum is 50 percent of the potential achievement gain of moving from complete mixing to complete ability stratification. We emphasize that such aggregate gains can undoubtedly be made either larger or smaller by varying the relative benefit of peer quality to students of different ability levels.

The remaining figures illustrate the distributional effects of the voucher. The effects of the voucher on achievement and welfare can be divided into the impacts on the four groups illustrated in the upper panel of Figure 5. Area A contains students who are in the public schools before and after the introduction of a \$2,000 voucher. Areas B and C combined are students who switch from public to private school when the voucher is introduced. Area D contains students in the private-school sector before and after the introduction of the voucher.

The primary gains in achievement accrue to students who switch from the public- to the private-school sector with introduction of the voucher (areas B and C). They have achievement gains ranging between 12.9 percent and 20.2 percent. The major losses in achievement are experienced by students who remain in the public school after the voucher is introduced. They all experience a 4.9-percent loss. The latter group is, of course, much larger than the former.

The welfare effects (measured by compensating variation) are distributed somewhat differently. Students who remain in the public sector (area A) all experience welfare losses. The quality of the school they attend has deteriorated, and tax changes are small as discussed above. Since public schools charge a price of zero, the voucher does not reduce the cost of education for public-school students.

Paradoxically, some of those who switch from the public to the private sector (area B) are also made worse off. Their alternatives are adversely affected by the voucher. They can either stay in a public-school system of reduced quality, or they can pay tuition at private school. They choose the latter, but the voucher defrays only a portion of the cost. Thus, while they have large achievement gains, those gains are more than offset by the reduction in in-

come net of tuition.<sup>30</sup> For each income level, the largest loss within this group is sustained by the students at the lower boundary of area B. By boundary indifference, the magnitude of loss for a student on this lower boundary is the same as for a comparable student on the upper boundary of region A. Students on the boundary between regions B and C neither gain nor lose from the voucher.

The remaining two groups (C and D) gain from the voucher. The largest gains as a proportion of income accrue to high-ability, low-income households. As the voucher increases the demand for private education, it increases competition for high-ability students and the financial aid they receive.<sup>31</sup> The highest-ability students experience a tuition reduction that is almost twice the amount of the voucher. The greatest gains are thus in the lower-right portion of the upper panel of Figure 5.

The bottom panel of Figure 5 illustrates further the distributional effects of the \$2,000 voucher. Compensating variation as a percentage of income is plotted for four different income levels as a function of ability. This figure demonstrates that the gains accrue to those with high ability, and among high-ability households, the gains are proportionately greater for low-income households. Losses are realized by households that remain in the public-school sector due to the decline in public-school peer quality induced by the voucher. Again, those in area B in the upper panel are also losers. The proportionate loss for the latter two sets of households is low, but they make up a majority of the population.

<sup>30</sup> This result is similar to Benabou's (1996a) finding that equilibrium segregation across communities (and schools) by endowed human capital (ability) can cost high-human-capital types more in housing-price premia than they gain relative to an allocation without segregation.

<sup>31</sup> For example, when the voucher is increased from 0 to \$1,800, the  $\eta$ 's in the top four schools rise from 360, 310, 282, and 261 (dollar discount per unit of ability; see Figure 2) to 398, 351, 326, and 308, respectively. Interestingly, the increased competition for high-ability students actually reduces the quality of the top schools, as entrants bid some of these students away, and the boundary loci between private schools become flatter. Compare the  $\theta$ 's of the top schools in the two panels of Figure 1. This phenomenon persists in the steady state.

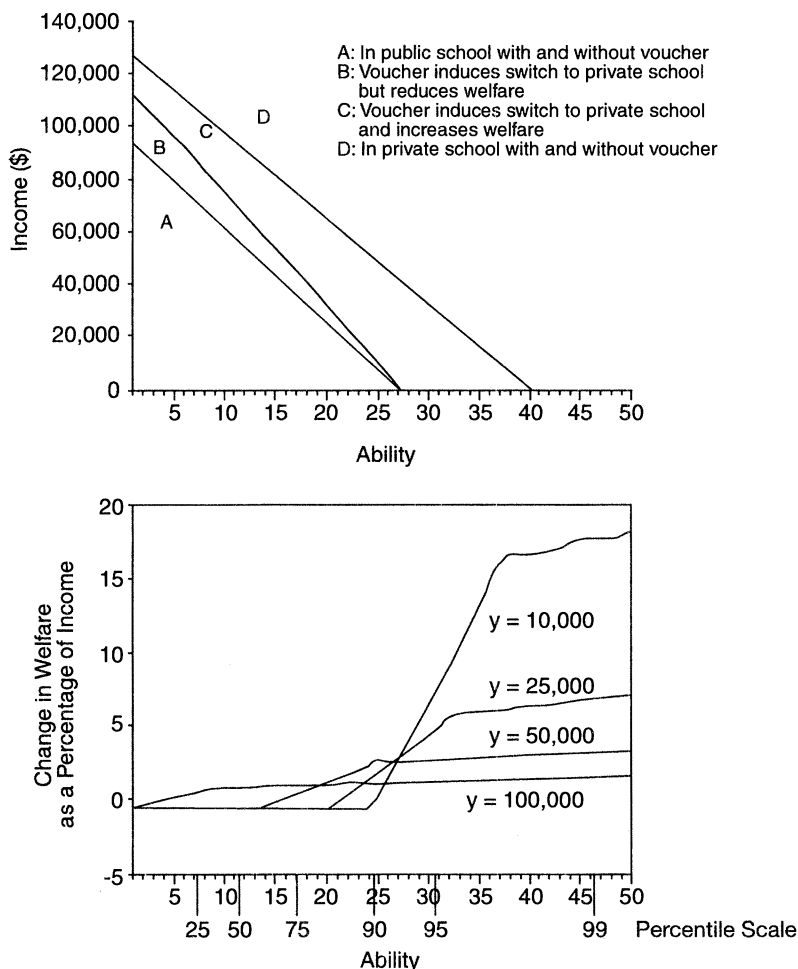


FIGURE 5. BOUNDARY LOCI BETWEEN PUBLIC AND PRIVATE SCHOOLS WITHOUT AND WITH A \$2,000 VOUCHER AND LOCUS OF HOUSEHOLDS INDIFFERENT TO PROVISION OF THE VOUCHER (UPPER PANEL) AND CHANGE IN WELFARE AS A FUNCTION OF ABILITY FOR FOUR DIFFERENT INCOME LEVELS (LOWER PANEL)

Figure 6 illustrates the distributional effects of the voucher in steady state. For each ability and a voucher level, we calculate the expected steady-state income of a student as follows. Given a student ability and knowing the steady-state distribution of parental income, we calculate the probability that a student will attend each of the available schools. Using this, each school's quality, and the student's ability, we calculate expected income. Figure 6 shows the percent-

age change in expected steady-state income relative to the zero-voucher steady state. One would expect gains to accrue to the bulk of the relatively highest-ability students since the relatively highest-ability students are most likely to attend a higher-quality school as a result of the introduction of the voucher. Lower-ability students comprising approximately 70 percent of the population are made worse off because they are likely either to remain in the public sector when the voucher

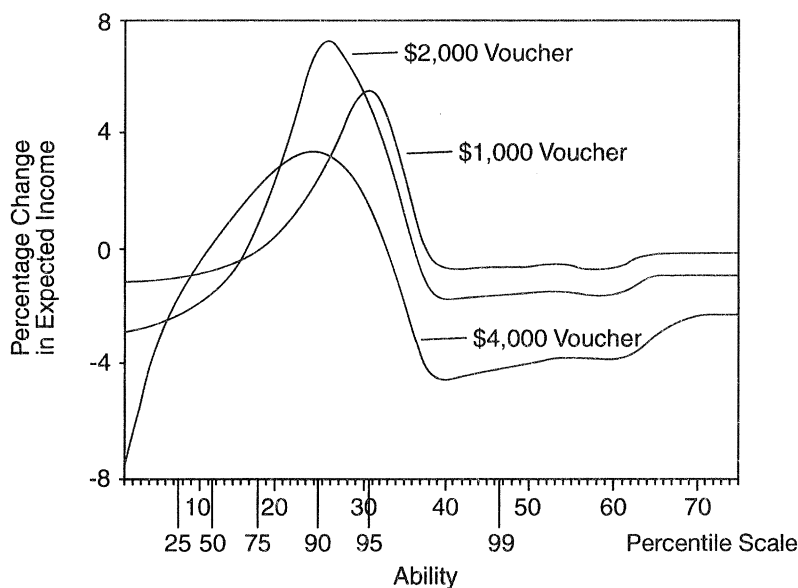


FIGURE 6. PERCENTAGE STEADY-STATE CHANGE IN EXPECTED INCOME FROM INTRODUCING VOUCHERS FOR STUDENTS OF DIFFERING ABILITY LEVELS

is introduced (a public sector of diminished quality) or to enter a low-quality private school. The top 2–3 percent of the ability distribution (abilities 35 and higher) also have lower expected income because the very top schools that they will attend decline somewhat in quality (see footnote 30).

#### IV. Concluding Remarks

A recent Department of Education study reports that nearly half of all adult Americans read and write so poorly that they are unable to function effectively in the workplace, and not surprisingly, that many of them live in poverty (see *Newsweek*, 1993; *Wall Street Journal*, 1993). This depressing statistic, like many others before it, has led to calls for change in the U.S. educational system. There is no shortage of reform proposals. The research challenge is to develop models that provide systematic links connecting preferences, the educational “production” process, costs, and institutional structure to consequences. Such models should develop testable predictions for validating or rejecting the elements of beliefs

that give rise to policy proposals, and they should facilitate systematic comparison and evaluation of policy alternatives.

One goal of our research is to provide a useful foundation both in permitting formal analysis of educational policy issues and in guiding empirical work. We are introducing inputs into the model so that schools can compete for students by varying inputs as well as tuition policy. We (Epple and Romano, 1995) are also enriching the model to contrast equilibrium in an open-enrollment system (as in the model in this paper) to a neighborhood system in which students may only attend the school in the geographic neighborhood in which they reside. With Elizabeth Newlon, we (Epple et al., 1997) have analyzed equilibrium when public schools adopt tracking by ability.<sup>32</sup> These extensions are discussed further below. The model can be extended to allow for multiple peer characteristics. This is likely to lead to an equilibrium with a more diverse set of private schools, diversity seen by pro-

<sup>32</sup> See Gamoran (1992) for an interesting empirical analysis of tracking.

ponents of vouchers as a potential benefit. A further extension lets teachers vary in skill with student achievement increasing in both the school's teacher-student ratio and the mean skill of teachers. If a teacher's utility depends on the abilities of students taught, private schools are advantaged in hiring highly skilled teachers. Allowing a teacher's utility to depend also on the skills of colleagues is a key step toward applying the model to higher education. In analyzing higher education, we also intend to introduce a quasi-public sector (state schools) subsidized by tax dollars but facing egalitarian dictates that constrain admission and tuition policies.

Regarding implications for empirical work, our analysis delivers several predictions summarized in the propositions we have presented. These include a negative correlation between income and ability within each private school, a negative correlation between private-school size and quality, tuitions declining in ability in all private schools, tuitions increasing in income for the best private school in each geographic market, and little dependence of tuition on income in the remaining private schools in each market, a scale of operation in all private schools lower than that which minimizes average cost, a strict hierarchy of schools, stratification by income, and stratification by ability.<sup>33</sup>

As one of our referees stated, what we would like to know about a voucher program is "who gains, who loses, and how does it add up?" Earlier, we discussed aggregate effects in detail. Here we discuss further the redistributive effects. In this paper, we consider an open-enrollment public system in which public and private schools are equally effective in delivering education. Our model implies that a voucher program will result in entry of new private schools and movement of students from the public to the private sector. Students remaining in the public sector are those with relatively low income and low ability, and those students experience losses. Because vouchers in-

crease the premium on ability, the greatest proportionate gains from the voucher accrue to low-income, high-ability students.

How sensitive are our results likely to be to the assumption of a homogeneous open-enrollment public-school system? The conclusion that low-income, high-ability students experience large proportionate gains is likely to hold regardless of the organization of the public-school system. However, the distribution of losses from a voucher program is likely to depend on the organization of public schools. For example, most public schools in the United States place students of differing abilities into different "tracks." Applying the framework developed here to study tracking reveals the following (Epple et al., 1997). The largest proportionate losses from a voucher program are to students who remain in the high-ability public-school track when the voucher is introduced, because the voucher draws the most-able students from the high track into the private sector. In the presence of tracking, the voucher has very little impact on the low-income, low-ability students. Public schools consign these students to a low-ability peer group, and the voucher leads to modest decline in the quality of that peer group. The loss to such students from a voucher program is much smaller than the losses they experience from introduction of public-school tracking.

Consider now a neighborhood public-school system, without tracking, that assigns students to a school based on the neighborhood in which they reside. We have shown (Epple and Romano, 1995) that, even absent differences in expenditure across schools, Tiebout sorting will create a public-school hierarchy deriving from peer-group differences when income and student ability are positively correlated. We anticipate that the introduction of a significant private sector, as supported by vouchers, would again benefit most poor, high-ability students. How poor, low-ability students would be affected relative to the case of open-enrollment public schools is less clear. As with tracking, the latter students begin with a weaker peer group. Thus, unfortunately, those who remain in public school when vouchers covering part of educational costs are introduced may have

<sup>33</sup> The specific properties of the tuition functions are developed in Epple and Romano (1993).

little to lose. On the other hand, their most-able peers will be the first to enter the private sector. This could imply even further losses to those who remain behind. We think this is an important issue for research, since neighborhood school systems continue to dominate in the United States.

It is often argued that large central-city public-school systems in the United States are ineffective in delivering education, and that the failings of these schools are visited primarily on disadvantaged students who reside in central cities. Clearly, if vouchers lead private schools to supplant ineffective public schools or inspire better performance from such ineffective schools, then a voucher program would lead to widely distributed gains. In the presence of public-school tracking, for example, such gains, even if relatively modest, might be sufficient to offset the effects of diminished peer quality experienced by low-income, low-ability students. Such gains would need to be more substantial, however, to offset the losses experienced by students who remain in the high-ability public-school track after vouchers are introduced.

Whether such gains in technical efficiency from voucher programs would be substantial is an open question (see footnote 4). Our paper stresses the allocative effects of vouchers and shows that vouchers could have significant distributional consequences. It indicates a need for continued effort to quantify the effects of private-sector competition in education and to quantify the effects of school quality on students of differing ability levels. There are also implications for the design of voucher systems intended to promote alternative goals. If vouchers are intended to improve technical efficiency without increasing ability segregation, then less-able students will need more financial assistance (or equivalent controls must be enacted). If ability segregation is to be further promoted to increase aggregate achievement, then vouchers will need to be income dependent. Hopefully, our framework will facilitate the investigation of such voucher-design issues, as well as the investigation of other proposals affecting the entry and exit of schools or student access to schools.

## APPENDIX

### PROOF OF PROPOSITION 1:

We show first that the public school has the worst peer group. Suppose to the contrary that

$$\theta_0 \geq \theta_1 \in \underset{i \in \{1, 2, \dots, n\}}{\text{MIN}} \theta_i.$$

Any student who would have to pay a positive price to attend private school 1 would prefer the free public school. Hence, private school 1 could not be profitable.

Showing that a strict hierarchy of private schools characterizes equilibrium is considerably more involved. The proof is by contradiction, so assume that  $\theta_i = \theta_j$  for some  $i \neq j$ ,  $i, j = 1, 2, \dots, n$ . We show that this implies

$$\begin{aligned} \text{(A1)} \quad p_i(b, y) &= p_j(b, y) = \text{MC}_i(b) \\ &= \text{MC}_j(b) \quad \forall (b, y) \end{aligned}$$

with  $\alpha_i(b, y) > 0$  and/or  $\alpha_j(b, y) > 0$ .

We will go on to show by construction that (A1) implies that school  $i$  (or  $j$ ) can increase profits by admitting and expelling certain students, contradicting profit maximization.

Condition (6a) implies  $p_i(b, y) = p_j(b, y) \quad \forall (b, y)$  since  $\theta_i = \theta_j$ . Condition (6b) and market clearance imply that  $\text{MC}_i(b) \geq p_i(b, y) = p_j(b, y) \geq \text{MC}_j(b)$  for students who attend school  $j$  and, analogously,  $\text{MC}_j(b) \geq p_i(b, y) = p_j(b, y) \geq \text{MC}_i(b)$  for students who attend school  $i$ . The linearity of  $\text{MC}(b)$  implies: (a)  $\text{MC}_j(b) = \text{MC}_i(b) \quad \forall b$ ; (b)  $\text{MC}_i(b) < \text{MC}_j(b) \quad \forall b$  (or equivalently, the reverse); or (c)  $\text{MC}_i(b) > (=) (<) \text{MC}_j(b)$  as  $b > (=) (<) b'$ , for some  $b'$  (again, reversing  $i$  and  $j$  provides an equivalent). Case (b) precludes school  $j$  from admitting any students and can be rejected. Case (c) precludes  $\theta_i = \theta_j$ , since school  $i$  would admit only student types with  $b \leq b'$ , and school  $j$  would admit only student types with  $b \geq b'$ . This leaves case (a), which then implies (A1).

We now show that student types  $(b_1, y_1)$  and  $(b_2, y_2)$  exist, with  $\alpha_i(b_1, y_1) \in (0, 1]$  and  $\alpha_i(b_2, y_2) \in (0, 1]$ , such that school  $i$  can increase profits by admitting the same number of  $(b_1, y_1)$  types as it expels of  $(b_2, y_2)$  types. School  $i$  will increase profits then by admitting some students who initially attend  $j$  but expelling the same number of its own students. Note that, since tuitions equal effective marginal costs in school  $i$  for both such types of students, it is a local nonconcavity of profits that must drive this result. Hence, we will consider higher-order derivatives.

Proceeding, we have

$$(A2) \quad \frac{\partial \pi_i}{\partial [\alpha_i(b_1, y_1)f(b_1, y_1)]} \\ = p_i^*(b_1, y_1, \theta_i) - V'(k_i) \\ - (\theta_i - b_1) \frac{1}{k_i} \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f db dy.$$

Denote the expression (A2) with  $\pi_1^i$  [the subscript since the first variation is for type  $(b_1, y_1)$ ];  $\pi_2^i$  is analogous. Using (5c) and (5d), one obtains

$$(A3) \quad \pi_{11}^i \equiv \frac{\partial^2 \pi_i}{\partial^2 [\alpha_i(b_1, y_1)f(b_1, y_1)]} \\ = -V''(k_i) + 2 \left( \frac{b_1 - \theta_i}{k_i} \right) \\ \times \left[ \frac{\partial p_i^*(b_1, y_1, \theta_i)}{\partial \theta_i} \right. \\ \left. - \frac{1}{k_i} \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f db dy \right] \\ + \left[ \left( \frac{b_1 - \theta_i}{k_i} \right)^2 \right. \\ \left. \times \iint_S \frac{\partial^2 p_i^*}{\partial \theta_i^2} \alpha_i f db dy \right]$$

and

$$(A4) \quad \pi_{12}^i \equiv \frac{\partial^2 \pi_i}{\partial [\alpha_i(b_1, y_1)f(b_1, y_1)] \partial [\alpha_i(b_2, y_2)f(b_2, y_2)]} \\ = -V''(k_i) + \frac{\partial p_i^*(y_1, b_1, \theta_i)}{\partial \theta_i} \left( \frac{b_2 - \theta_i}{k_i} \right) \\ + \frac{\partial p_i^*(y_2, b_2, \theta_i)}{\partial \theta_i} \left( \frac{b_1 - \theta_i}{k_i} \right) \\ - \left( \frac{b_2 + b_1 - 2\theta_i}{k_i} \right) \frac{1}{k_i} \iint_S \frac{\partial p_i^*}{\partial \theta_i} \alpha_i f db dy \\ + \frac{(b_1 - \theta_i)(b_2 - \theta_i)}{k_i^2} \iint_S \frac{\partial^2 p_i^*}{\partial \theta_i^2} \alpha_i f db dy.$$

The expression for  $\pi_{22}^i$  is analogous to (A3). Let  $\Delta_i$  equal the change in the number (density) of types  $(b_i, y_i)$  enrolled in school  $i$ . For  $\Delta_i$ 's sufficiently small, Taylor's theorem implies the sign of the change in school  $i$ 's profit,  $\Delta \pi_i$ , will be the same as the sign of

$$(A5) \quad \pi_1^i \Delta_1 + \pi_2^i \Delta_2 + \frac{1}{2} [\pi_{11}^i (\Delta_1)^2 \\ + \pi_{22}^i (\Delta_2)^2 + 2\pi_{12}^i \Delta_1 \Delta_2].$$

We know that  $\pi_1^i = \pi_2^i = 0$  since prices equal effective marginal costs. We will consider admission changes such that  $\Delta_1 = -\Delta_2$ . Hence, (A5) simplifies to  $\frac{1}{2} (\Delta_1)^2 [\pi_{11}^i + \pi_{22}^i - 2\pi_{12}^i]$ . Substituting and simplifying yields

$$(A6) \quad \text{sign } \Delta \pi_i \\ = \text{sign} \left\{ \frac{2}{k_i} (b_1 - b_2) \left[ \frac{\partial p_i^*(y_1, b_1, \theta_i)}{\partial \theta_i} \right. \right. \\ \left. \left. - \frac{\partial p_i^*(y_2, b_2, \theta_i)}{\partial \theta_i} \right] \right. \\ \left. + \left( \frac{b_1 - b_2}{k_i} \right)^2 \iint_S \frac{\partial^2 p_i^*}{\partial \theta_i^2} \alpha_i f db dy \right\}.$$

Suppose that  $y_1$  is greater than  $y_2$  but very close to it, and that  $b_1$  is greater than  $b_2$  but closer still to  $b_2$ , formally, of lower-order

difference. In a moment, we show that school  $i$  can substitute students this way. From (6a),

$$\frac{\partial p_i^*}{\partial \theta_i} = \frac{\partial U / \partial \theta_i}{\partial U / \partial y_i}.$$

Lemma 5 in the Appendix to Epple and Romano (1993) shows that  $p_i^*(b, y, \theta)$  is continuous in  $(b, y)$ . By SCI and this lemma (keeping in mind that  $b_1$  is extremely close to  $b_2$ ), the first term in (A6) is positive. Moreover, it will dominate the second term due to the lower order of the difference  $(b_1 - b_2)$  and, again, using the lemma.

The result then follows so long as such a substitution of students by school  $i$  is feasible. Such a substitution is clearly feasible if there is a positive measure of type space over which both schools  $i$  and  $j$  admit students. Suppose, however, that no such overlap exists. School  $i$  accomplishes an equivalent substitution of students in what can be thought of as two steps. It first expels half of all of its students and admits half of all of the students who attend school  $j$  initially. Since  $\theta_i = \theta_j$  and  $MC_i = MC_j$ , this step has no effect on  $i$ 's profits or  $\theta_i$ ,  $k_i$ , and  $\eta_i$  (hence  $MC_i$ ). Now the profit-increasing substitution is feasible, which constitutes the second step. In this case, the variation that increases profits is just more involved.

#### PROOF OF PART (ii) OF PROPOSITION 3:

Household choice is as though tuitions equal effective marginal cost by part (iii) of Proposition 2; that is, type  $(b, y)$  chooses school  $i$  over  $j$  if and only if  $U(y_i - MC_i(b), a(\theta_i, b)) \geq U(y_i - MC_j(b), a(\theta_j, b))$ . Keep in mind our convention that  $MC_0(b) \equiv 0 \forall b$  and let  $\eta_0 \equiv 0$ . Also, let  $p_i^d \equiv V'(k_i) + \theta_i \eta_i$ ,  $i = 1, \dots, n$ , and  $p_0^d \equiv 0$ , so that the first argument in the utility function is  $y_i + \eta b - p^d$ . Choice over schools by type  $(b, y)$  facing tuition  $MC_i(b)$  is equivalent to choice over schools by type  $(b, y_i + \eta_i b)$  facing *type-independent* tuitions  $p_i^d$ . We map the school-choice problem into  $(\theta, p^d)$ -space and show that a standard single-crossing argument implies SBA.

Since  $0 = \eta_0 < \eta_1 \leq \eta_2 \leq \dots \leq \eta_n$ , an infinite number of nondecreasing and differentiable functions  $\hat{\eta}(\theta)$  exist satisfying  $\hat{\eta}(\theta_i) = \eta_i$ ,  $i = 0, 1, \dots, n$ . Choose any such function. Choice of schools is equivalent to

$$(A7) \quad \text{MAX}_{i \in \{0, 1, \dots, n\}} [U(y_i + \hat{\eta}(\theta_i)b - p_i^d, a(\theta_i, b))].$$

Indifference curves in  $(\theta, p^d)$ -space are upward sloping:

$$(A8) \quad \left. \frac{dp^d}{d\theta} \right|_{U(\cdot) = \bar{v}} = b\hat{\eta}' + \frac{(\partial U(\cdot)/\partial a)(\partial a/\partial \theta)}{\partial U(\cdot)/\partial y_i}$$

where  $U(\cdot)$  has the same arguments as in (A7). The first term in (A8) is nonnegative since  $\hat{\eta}(\theta)$  is nondecreasing, and the second term is obviously positive. These indifference curves also have slopes that are strictly increasing in  $b$  for  $\theta > \theta_0$  and weakly increasing at  $\theta = \theta_0$ . Differentiation of (A8) with respect to  $b$  yields:

$$(A9) \quad \hat{\eta}' \frac{\partial}{\partial y_i} \left[ \frac{(\partial U(\cdot)/\partial a)(\partial a/\partial \theta)}{\partial U(\cdot)/\partial y_i} \right] + \hat{\eta}' + \frac{\partial}{\partial b} \left[ \frac{(\partial U(\cdot)/\partial a)(\partial a/\partial \theta)}{\partial U(\cdot)/\partial y_i} \right]$$

where  $b$  in the first argument of  $U(\cdot)$  is held constant in the latter term. The first term in (A9) is strictly (weakly) positive by SCI for  $\theta > (=) \theta_0$ , and the second two terms are weakly positive by  $\hat{\eta}' \geq 0$  and W-SCB, respectively. Given that slopes of indifference curves increase with  $b$ , SBA follows by a standard single-crossing argument.

*Discussion of Existence.*—Here we describe more fully the existence problem, potential resolutions of it, and the related literature on club theory. Consider a quasi-equilibrium satisfying the exact-equilibrium requirements UM, PSP, and MC, but substituting an exogenously determined number of private schools for ZII and replacing PIM with a requirement

of local profit maximization [i.e., (6a)–(6c)] for all private schools. Proposition 1 would continue to apply, implying that the private schools serve different niches of students. Unless the distribution of types is “just so,” the profitability of serving the different niches will vary. Each private school is maximizing profits locally, but the profit function (which the utility-taking schools share) has multiple peaks (in the space of functions  $[p(b, y), \alpha(b, y)]$ ), and only one school is at the global maximum. The other private schools are at lower-profit peaks and would have an incentive to mimic the tuition/admission policy of the highest-profit school. This problem persists regardless of the (finite) number of schools, so that  $Z\Pi$  cannot generally be satisfied.

The existence problem is fundamental to club economies (see below), and no easy resolution presents itself. The epsilon equilibrium described in the text keeps private schools at their local maxima and, roughly, allows entry until the profit peaks and their differences are minimized. One could make  $\varepsilon$  arbitrary low by reducing the fixed costs, but this presents computational difficulties and sacrifices realism. Related to this, one might also assume constant costs of schooling. This would lead to an infinite number of schools serving infinitely refined peer groups. We have made some progress on such a specification in a special case with the help of an anonymous referee. We believe exact equilibrium exists in such specifications, but this has not been proved. While this model is extremely interesting, it is quite complex and not yet tractable.

The natural game-theoretic specification also encounters existence problems. Suppose that, in two stages, private schools first commit to enter (e.g., pay fixed costs) and then play a Nash game in tuition functions. We believe that the second-stage profit functions will continue to be multi-peaked, and Bertrand-like lack of pure-strategy equilibrium will arise. Scotchmer (1985) shows existence of equilibrium in pure strategies in the case of clubs with *anonymous* crowding using a game-theoretic approach. The difference is that incumbent clubs earn the same level of profits, a property that our model will not generally have, because the variety of types induces differentiated “clubs.”

The problem we encounter is familiar to club economies with *nonanonymous* crowding. Either economies of scale or indivisibility of club members generally leads to existence problems in such economies, both causes detailed in Scotchmer and Wooders (1987) and Scotchmer (1997). Our continuum of agent types also implies a continuum of each type, eliminating the latter existence problem as in Scotchmer (1986). But the first existence problem remains. In lieu of exact equilibria, epsilon-competitive equilibria are also studied in this literature, with the difference that it is club members rather than club owners who do not fully optimize. A correspondence between exact equilibrium and the exact core and between epsilon equilibrium and an epsilon core is demonstrated in this literature. This normative focus highlights consumer behavior rather than “firm” behavior, and consumers are the natural candidates for deviation from full optimization. Our positive focus provides a more salient role for firms (private-school owners), and they are more convenient candidates for deviation than the infinity of households. Hence, our epsilon equilibrium has households fully optimize but not private schools.

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