SORTING WITH MOTIVATED AGENTS: IMPLICATIONS FOR SCHOOL COMPETITION AND TEACHER INCENTIVES

Timothy Besley  Maitreesh Ghatak
London School of Economics  London School of Economics

Abstract
This paper proposes a simple framework to study incentives and matching in the market for teachers. The framework is used to investigate the consequences of introducing incentive pay for teachers when contracts have both a matching and an incentive effect. Our analysis suggests that school competition and teacher incentives cannot be studied in isolation from one another. (JEL: D73, I20, J45, L31)

1. Introduction
There are two key aspects of education reform on the current agenda: (1) efforts to increase competition between schools and (2) using explicit incentives to motivate teachers. Both have been studied extensively but mainly in isolation from one another.

In the area of school competition, much of the discussion concerns the consequences for competition of pupil mobility.\(^1\) There is comparatively less interest in how competition affects the labour market for teachers.\(^2\) This is true even though there is strong evidence that the quality of teachers matters to pupil attainment.\(^3\)

On the issue of teacher incentive pay, the introduction of such pay remains controversial and has been resisted (especially by unions) in a number of places.\(^4\)

Acknowledgments: Financial support from the ESRC through research grant RES-000-23-0717 is gratefully acknowledged.
E-mail addresses: Besley: t.besley@lse.ac.uk; Ghatak: m.ghatak@lse.ac.uk
1. See the studies in Hoxby (2003a) for an overview of the evidence from the U.S. For U.K. evidence see Burgess et al. (2004).
2. However, notably exceptions include Hoxby (2002) and Hanushek and Rivkin (2003) who show that that school competition affects the demand for teachers and hence has an impact on the labour market for teachers.
3. See, for example, Rivkin, Hanushek and Kain (2005).
4. In the U.S. a number of teacher incentive programmes have been introduced in public schools. According to a 1997 study 12% of school districts were using merit pay in some way but the amount of incentive pay in these districts amounted to around 2% of base pay. In contrast in private schools incentive pay is considerably higher (Dee and Keys 2005).
That said, the debate is often confused and yields somewhat contradictory arguments. It is sometimes said that incentive pay is unnecessary as teachers are motivated enough already. However, other arguments seem to hinge on the possibility that incentives work too well and distort teacher efforts toward socially unproductive activities such as “teaching to the test”. There also appear to be concerns that incentive pay has unpalatable general equilibrium implications—for example, by increasing inequality.

This paper investigates these issues beginning with three basic features of schooling. First, teachers are motivated agents in the sense of Besley and Ghatak (2005)—they have non-pecuniary objectives which include preferences over the kind of students that they teach and the way in which they are taught. Second, there is evidence that teachers respond to monetary incentives.\footnote{These incentive schemes typically take the form of rewards based on pupil attainment—see, for example, Atkinson et al. (2004) and Lavy (2002). The evidence on teachers does not seem consistent with motivational crowding-out discussed in the literature on intrinsic motivation (see Frey 1997).} Third, there is evidence that competition has an impact on how schools operate.\footnote{See, for example, Hoxby (2003b).}

We develop a model where heterogeneous teachers and schools are matched with schools picking contracts to motivate teachers. These contracts must clear the market for teachers as well as creating incentives.\footnote{Lazear (2003) also discusses matching issues and their importance in the context of teacher incentives. However, he does not develop a model of it.}

The model delivers three main insights. First, it shows that, even if feasible, it is not always optimal to implement incentive pay. We look at where in the distribution of schools incentive pay will be used. Second, we show that improved sorting and allowing incentive pay tend to increase productivity (inducing higher levels of teacher effort). However, the distribution of gains is uneven in the “quality” distribution of schools. We also show that this allocates more rents to good teachers.

The paper is organized as follows. In Section 2, we lay out the model. Section 3 studies contracts and Section 4 matching for a specific example intended to illustrate how the model works. Section 5 draws out implications from the model.

2. The Model

A school consists of a group of parents—the “principal”—and a risk neutral agent—the teacher. Schooling quality is of two possible levels, high ($Y_H = 1$), or low ($Y_L = 0$). The probability of high quality is equal to the effort supplied by the teacher, $e$, at a cost $c(e) = e^2/2$. Effort is unobservable and hence non-contractible. We assume that the teacher has no wealth that can be pledged as a performance bond. Thus, a limited liability constraint operates which implies that the agent has to be given a minimum consumption level of $w \geq 0$ every
period, irrespective of performance. Because of the limited-liability constraint, the moral hazard problem has bite. This is the only departure from the first-best in our model. We assume that the school has sufficient resources to finance any required salary package and that there is an outside option other than teaching which yields a reservation utility of zero. We also assume that the principal must make a non-negative payoff.

The mapping from effort to the outcome is the same for all schools. There are two types of principals, indexed \( i \in \{1, 2\} \), and two types of agents, indexed \( j \in \{1, 2\} \). The types of the principals and the agents are perfectly observable. A principal of type \( i \) matched with an agent of type \( j \) receives a payoff of \( \pi_{ij} > 0 \) if quality is high and zero otherwise. An agent of type \( j \) matched with a principal of type \( i \) receives a payoff of \( \theta_{ij} > 0 \) if quality is high and zero otherwise.

**Assumption 1.** \( \pi_{11} \geq \pi_{22} \) and \( \theta_{11} \geq \theta_{22} \) and \( \pi_{12} = \pi_{21} = \pi \) and \( \theta_{12} = \theta_{21} = \theta \).

To ensure an interior solution for effort we assume \( \pi_{11} + \theta_{11} < 1 \). We concentrate on the case of vertical matching where

\[
\begin{align*}
\pi_{11} &> \pi_{22} \\
\theta_{11} &> \theta_{22}.
\end{align*}
\]

This says that type 2 principals and agents are inferior in a well-defined sense. Moreover, these lower types would rather be matched with type 1’s if they could. Here, the interpretation is in terms of good and bad schools/teachers. Vertical matching occurs where good teachers are more motivated when they teach good students. All students do better when they are taught by good teachers. However, the highest payoff to teachers and students is attained when good teachers teach good students.

Contracts between principals and agents have two components: a fixed wage \( w_{ij} \), which is paid regardless of the school quality, and a bonus \( b_{ij} \), which the agent receives if the outcome is \( Y_H \).

3. Contracts

As a benchmark, consider the first-best case, where effort is contractible. This results in effort being chosen to maximize the joint expected payoff of the principal and the agent. This effort level depends on agent motivation and hence the principal-agent match. However, the contract offered to the agent plays no allocative role in this case. Thus, although matching may raise efficiency, it has no implications for incentives in the first best. It is straightforward to calculate that the first-best effort level in a pairing between a principal of type \( i \) and an agent is type \( j \) is \( \pi_{ij} + \theta_{ij} \). The expected joint surplus is \( (\pi_{ij} + \theta_{ij})^2 / 2 \).
In the second-best, effort is not contractible. The principal’s optimal contracting problem under moral hazard solves

$$\max_{b_{ij},w_{ij}} u^p_{ij} = (\pi_{ij} - b_{ij})e_{ij} - w_{ij}$$

subject to the following constraints.
1. **Limited liability constraint**, requiring that the agent be left with at least \(w\):
   $$b_{ij} + w_{ij} \geq w, \quad w_{ij} \geq w.$$  
2. **Participation constraint** of the agent:
   $$u^a_{ij} = e_{ij}(b_{ij} + \theta_{ij}) + w_{ij} - \frac{1}{2}e_{ij}^2 \geq u_j.$$  
3. **Incentive-compatibility constraint**, which stipulates that the effort level maximizes the agent’s private payoff given \((b_{ij}, w_{ij})\):
   $$e_{ij} = \arg \max_{e_{ij} \in [0, 1]} \left( e_{ij}(b_{ij} + \theta_{ij}) + w_{ij} - \frac{1}{2}e_{ij}^2 \right).$$

We restrict attention to the range of reservation payoffs for the agent in which the principal earns a non-negative payoff. The incentive-compatibility constraint can be simplified to

$$e_{ij} = b_{ij} + \theta_{ij},$$

so long as \(e_{ij} \in [0, 1]\). Let \(\bar{u}_j\) be the value of the reservation payoff of an agent of type \(j\). For simplicity, we work with the case where \(w = 0\). Although this is unrealistic, it simplifies the algebra without affecting the substance of the argument. Let \(Y_{ij} = \max\{(\pi_{ij} + \theta_{ij})/2, \theta_{ij}\}\). Then, the following proposition characterizes the optimal contract.

**Proposition 1.** (Besley and Ghatak 2005) Suppose that \(\pi_{11} + \theta_{11} < 1\). An optimal contract \((b^*_{ij}, w^*_{ij})\) between a principal of type \(i\) and an agent of type \(j\), given a reservation payoff \(\bar{u}_j \in [0, (\pi_{ij} + \theta_{ij})^2/2]\), exists and has the following features.

1. The fixed wage is set at the subsistence level: \(w^*_{ij} = 0\).
2. The bonus payment is characterized by

   $$b^*_{ij} = \begin{cases} 
   \max[0, (\pi_{ij} - \theta_{ij})/2] & \text{if } \bar{u}_j < \frac{1}{2}Y_{ij}^2, \\
   \sqrt{2\bar{u}_j - \theta_{ij}} & \text{if } \frac{1}{2}Y_{ij}^2 \leq \bar{u}_j \leq (\pi_{ij} + \theta_{ij})^2/2.
   \end{cases}$$

3. The optimal effort level is given by \(e^*_{ij} = b^*_{ij} + \theta_{ij}\).
As emphasized in Besley and Ghatak (2005), the bonus scheme depends on the relative motivation of principals and agents. We refer to the case where $\theta_{ij} > \pi_{ij}$ as when the agent is strongly motivated. In this case, bonus payments are not needed to incentivize agents.

To study matching, we write down the payoff of the principal at the optimal contract. Then define

$$S(\pi_{ij}, \theta_{ij}, z) = \begin{cases} 
\pi_{ij} \theta_{ij} & \text{if } \pi_{ij} < \theta_{ij} \text{ and } z < \frac{1}{2} \{Y_{ij}\}^2, \\
(\pi_{ij} + \theta_{ij})^2/4 & \text{if } \pi_{ij} \geq \theta_{ij} \text{ and } z < \frac{1}{2} \{Y_{ij}\}^2, \\
\sqrt{z}(\pi_{ij} + \theta_{ij} - \sqrt{2z}) & \text{if } \frac{1}{8} \{Y_{ij}\}^2 \leq z \leq (\pi_{ij} + \theta_{ij})^2/2 
\end{cases}$$

as the surplus of a principal whose motivation is $\pi_{ij}$ when he employs an agent whose motivation is $\theta_{ij}$ at reservation utility level $z$.

4. Matching

The previous section focused on the standard incentive role of contracts given the need to elicit effort in the second-best. In this section, we look at the matching process. The recent results of Legros and Newman (2003) provide useful tools for studying this model for contractual environments where the transferable utility assumption is not valid.

Observe first that $S(\pi, \theta, z)$ is increasing in $\pi$ and $\theta$, with $\partial^2 S/\partial \theta \partial \pi > 0$. More specifically, it satisfies the differentiable version of the generalized increasing differences condition of Legros and Newman (2003, Proposition 3). Specifically:

$$\frac{\partial^2 S(\pi, \theta, S(\pi, \theta', z))}{\partial \pi \partial \theta} + \frac{\partial^2 S(\pi, \theta, S(\pi, \theta', z))}{\partial \pi \partial z} \cdot \frac{\partial S(\pi, \theta', z)}{\partial z} \geq 0$$

for $\theta' \leq \theta$. This implies that all matches are assortative.

To pin down the actual contracts that “support” the matched outcome requires further assumptions that determine the allocation of surplus between principals and agents when matching takes place competitively. Let $n_{ja}^a$ be the number of agents of type $j$ and $n_{ja}^p$ be the number of principals of type $j$. We work with the following example.\(^8\)

**Assumption 2.** (i) $n_{ja}^a > n_{ja}^p$ for $j \in \{1, 2\}$; (ii) $n_{ja}^a < n_{ja}^p + n_{ja}^p$.

This says that there is a surplus of agents of both kinds relative to principals, but there are fewer type 1 agents overall than there are principals.

\(^8\) This is for illustration. Other cases could be studied in the framework.
We also focus on the case where the agents of all varieties are strongly motivated, that is, do not require incentives in any principal-agent match where their voluntary participation constraint is not binding. The following assumption guarantees this.

**Assumption 3.** $\theta_{22} \geq \pi_{11}$.

Thus $\theta_{11} > \theta > \theta_{22}$ and $\pi_{11} > \pi > \pi_{22}$. We refer to principals and agents of type 1 as “good” and those of type 2 as “bad”. Every school now wants to hire a good teacher. However, there are not enough such teachers to go around. The competition among bad schools for good teachers bids up those teachers’ utility. Because there is under-supply of effort in the model due to non-contractible effort, this comes in the form of higher bonuses paid to good teachers in bad schools. Good teachers (who are scarce overall) are the beneficiaries of this. The utility level of a good teacher in a bad school is given by solving

$$S(\pi_{22}, \theta_{22}, 0) = S(\pi_{21}, \theta_{21}, \hat{u}).$$

Using the expression for the principal’s expected payoff, we obtain

$$\hat{u} = \frac{1}{8} \left( \pi + \theta + \sqrt{(\pi + \theta)^2 - 4\pi_{22}\theta_{22}} \right)^2.$$

From Proposition 1, we know that $b_{22} = 0$, because bad teachers in bad schools face an outside option of zero. Correspondingly, $e_{22} = \theta_{22}$. Proposition 1 also tells that the bonus pay of good teachers in bad schools is $b_{21} = \sqrt{2\hat{u}} - \theta_{21} > 0$. Bonus pay here is used to clear the market for good teachers. However, given the underlying incentive problem, it also boosts effort, which would not be the case in a standard competitive model where fixed wages are used to clear the market.9

Correspondingly, $e_{21} = \sqrt{2\hat{u}} > \theta_{21} > \theta_{22}$. Therefore, the productivity gap between bad schools with bad teachers compared to bad schools with good teachers is now greater.

The competition between bad schools for good teachers may also have an impact on the behaviour of good schools with good teachers. This depends on whether the participation constraint of teachers in those schools binds at $\hat{u}$. In particular, we have

$$b_{11} = \begin{cases} 
0 & \text{if } \frac{1}{2}(\theta_{11})^2 \geq \hat{u}, \\
\sqrt{2\hat{u}} - \theta_{11} > 0 & \text{if } \frac{1}{2}(\theta_{11})^2 < \hat{u}.
\end{cases}$$

In the first case, we only see the use of bonus pay in middle ranking schools. The effort level is $e_{11} = \theta_{11}$. In the second case, competition among the low-quality

---

9. See Section 5.3 for more on this.
schools spills over into high-quality schools. The effort level is $e_{11} = \sqrt{2}u$. Thus, the possibility of bonus pay increases the productivity of the middle-ranking and good schools, but has no impact on the worst schools. The first case is relevant if

$$\theta_{11} - \theta > \pi - \frac{\theta_{22}}{\theta_{11}};$$

the second case is relevant if

$$\theta_{11} - \theta < \pi - \frac{\theta_{22}}{\theta_{11}}.$$

We are in the second case if the agent is indifferent about whom they teach, because the left hand side is zero. Then all schools that employ good teachers pay a bonus and offer the same contract to teachers.

To understand this condition better when a principal of type $i$ is matched with an agent of type $j$, let us introduce a parameter $\gamma_{ij}$ capturing the match-specific productivity, where (i) $\gamma_{11} \geq \gamma_{22}$ and (ii) $\gamma_{12} = \gamma_{21} = \gamma$. We write $\theta_{ij} = \theta(\gamma_{ij})$ and $\pi_{ij} = \pi(\gamma_{ij})$, where $\theta(\cdot)$ and $\pi(\cdot)$ are monotonically increasing functions. Now

$$\theta_{11} - \theta = \theta(\gamma_{11}) - \theta(\gamma)$$

and

$$\pi - \pi_{22} \frac{\theta_{22}}{\theta_{11}} = \pi(\gamma) - \pi(\gamma_{22}) \frac{\theta(\gamma_{22})}{\theta(\gamma_{11})}.$$

The larger is $\gamma_{11} - \gamma$ relative to $\gamma - \gamma_{22}$, the more likely is the first case; the opposite holds when $\gamma_{11} - \gamma$ is small relative to $\gamma_{12} - \gamma$. This condition captures whether the gap between the best and the mid-ranking schools (“upper tail inequality”) is more important than the gap between mid-ranking and bad schools (“lower tail inequality”). In the former case, good teachers are much more productive in high-quality schools than in low-quality schools. Therefore, to attract and motivate them, low-quality schools have to offer a bonus. In contrast, because the threat of competition from low-quality schools for good teachers is weak, the outside option of good teachers in high-quality schools is weak and they are not paid a bonus. An analogous interpretation holds for the latter case.

5. Implications

5.1. Comparison with Random Matching

The model allows us to think about the implications of different ways of allocating teachers to schools. Suppose that a bureaucrat were to randomly match teachers to
schools and that there is no scope for rematching. A teacher can refuse to work for a school, in which case her outside option is to be unemployed (i.e., $u_j = 0$ for $j = 1, 2$). Because we focus on the case of strongly motivated agents, all teachers get $w_{ij} = w$ and $b_{ij} = 0$.

Now consider what happens if we allow schools to recruit teachers freely and offer any compensation package (subject to voluntary participation). First, observe that this does not affect the pay or productivity of bad teachers in bad schools. Good teachers in bad schools now get paid a bonus and the productivity of these schools is higher than before. Also, if lower tail inequality is important, then the keenness of bad schools to hire good teachers results in good teachers in good schools getting paid a bonus and raises productivity compared to the case with random matching.

Under assortative matching, average pay and productivity are higher. However, free matching of teachers and schools raises pay inequality among teachers compared to before. Also, inequality in terms of school productivity goes up. However, the productivity of bad schools with bad teachers does not change and so, if one uses a maximin social welfare criterion, assortative matching is preferable.

5.2. Comparison with Horizontal Matching

Sorting is horizontal if $\pi_{11} = \pi_{22}$ and $\theta_{11} = \theta_{22}$. In this case, the principal–agent pairs are equally productive under efficient matching. Besley and Ghatak (2005) motivates this by considering schools with heterogeneous missions. These could be different ways of teaching or different educational curricula. Parents value education more when they have their preferred curriculum and teachers are motivated when they can teach according to their preferred curriculum.

If Assumptions 2 and 3 hold, there is assortative matching in the horizontal matching case as shown in Besley and Ghatak (2005). Employed agents earn a surplus. However, they are not be paid any incentive pay and put in effort equal to $\theta_{ij}$. All schools are equally productive (Besley and Ghatak 2005, Proposition 4).

With horizontal sorting, the objectives of both efficiency and equity are better attained than when there is random matching. The analysis suggests, therefore, that the implications of school competition and teacher incentives in terms of efficiency and equity differ substantially, depending on whether we believe horizontal or vertical sorting to be more important.

---

10. Random matching might sound implausible but what we have in mind is matching by criteria that are orthogonal to $\pi_{ij}$ and $\theta_{ij}$ (e.g., seniority).

11. This reflects the idea that there could be many different ways for a school to be good (see Ferrero 2004). This could be based on preferences over alternative pedagogical or curricular methods.
5.3. Introduction of Incentive Pay

Suppose that, in the absence of incentive pay, wages have to clear the labour market for teachers. Fixed-wage premia are then be used to achieve sorting. The principal of a bad school is indifferent between hiring a good and bad teacher if

$$\theta \pi - w_{21} = \theta_{22} \pi_{22}.$$

Then the payoff of a good teacher in a bad school is

$$\frac{1}{2} (\theta)^2 + \theta \pi - \theta_{22} \pi_{22}.$$

We can now compare this with \( \hat{u} \) above. Straightforward algebra shows that good teachers are better off under a system that allows for incentive pay. Bad teachers are as well off as before. Also, from our previous analysis incentive pay unambiguously increases productivity. Yet teacher unions strongly oppose the use of teacher incentives saying it would make teachers worse off.

Evidence suggests that teachers move from one school to another driven mainly by school and student characteristics and appear to be relatively unresponsive to salary levels (Hanushek, Kain, and Rivkin 2004). This tends to leave disadvantaged, low achieving students with relatively inexperienced teachers. Given our framework, bonuses may work here. The greatest performance gains are possible in low-performing schools. Therefore, if such schools offer bonuses, they might attract more motivated and/or able teachers as well as motivate existing teachers. This is not pure conjecture. Tennessee, under the initiative of a philanthropic organization (The Benwood Foundation), offers incentives to teachers from high-performing schools to move to low-performing schools, which includes an annual bonus based on significant test gains.

12. Because of the incentive problem, using fixed wages to clear labour markets is inefficient compared to using bonus schemes.

13. There is also the issue of whether rents can be extracted by all good teachers. This is not true if \((\theta_1)^2/2 > (\theta)^2/2 + \theta \pi - \theta_{22} \pi_{22}\).

14. For example, reacting to U.K. government’s proposal to introduce teacher incentives, Doug McAvoy, General Secretary of the National Union of Teachers, said in March 2004 that “The extension of performance related pay based on pupil progress to the main scale will further demoralise and demotivate teachers and make the profession less attractive. All teachers are disadvantaged.” (www.teachers.org.uk/story.php?id=3039).

15. In the paper we have implicitly focused on output-based incentive schemes. One problem of offering output-based incentive schemes is that much of the output of the teacher is not observed until many years after the student has had her course. However, incentive schemes can be input-based as well. For example, in India where absence rates for teachers are over 24%, a programme that gave incentives to teachers to improve their attendance immediately led to significant improvements—from 36% in comparison schools to 18% in treatment schools (Duflo and Hanna, 2005). Our framework can be easily modified to think about input-based incentive schemes by interpreting the outcome as a signal concerning the level or quality of the input.
5.4. Competition and Incentive Pay

School competition raises demand for teachers with characteristics that enable a school to attract better students (Hoxby 2002). One way to interpret this in terms of our model is that greater competition increases $\pi_{11} - \pi_{21}$ and $\pi_{21} - \pi_{22}$ while holding the $\theta$'s constant. Naturally, this leads to greater incentive pay for good teachers in both high-quality schools and low-quality schools. To the extent the increase in $\pi_{11}$ is significant, good teachers are not paid bonuses in high-quality schools but receive bonuses in low-quality schools. In general, our model predicts that greater competition leads to bonus pay for teachers in medium- and low-quality schools.

6. Concluding Comments

This paper has proposed a simple model for thinking about how sorting works in the labour market for teachers. The paper has illustrated an approach where studying the interaction of contracts and matching is essential. An important implication of this is that issues of school competition and incentive pay for teachers cannot be studied in isolation from one another. We have derived implications for both equity and efficiency in school performance for the case of vertical matching. The paper has underlined the importance of taking a general equilibrium approach to contracts and matching. It also emphasizes the importance of integrating the labor market for teachers into our understanding of school performance.

References


