REGRESSION DISCONTINUITY

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Presentation

- What is regression discontinuity
  - Background
  - Definition
  - Uses in educational research
  - Design, analysis, model
  - Assumptions and considerations
- Regression discontinuity analysis in R
  - Example 1: No statistical significance
  - Example 2: Main effect
  - Example 3: Interaction effect
  - Example 4: Main and interaction effect
  - Example 5: Multiple groups

Background

- Brought to forefront by Thistlethwaite and Campbell (1960)
  - Mimicking effects of randomized selection without bias
  - Developing methods for statistical analysis

- Used within other fields of research, such as medicine and economics, before popularity grew in education. (Cook, 2008)

Definition

- Quasi-experimental research design
  - Unique assignment method
  - Cut-off criterion to select groups
    - Pre-determined
    - Completely known
- An alternative to randomized control trial experiments
  - Assign participants to groups based on an observed variable
    - Ex. Standardized assessment score, GPA
- A method used for program evaluation
  - Compare outcomes of treatment group determined by cutoff criterion to outcomes of comparison group

Education Research

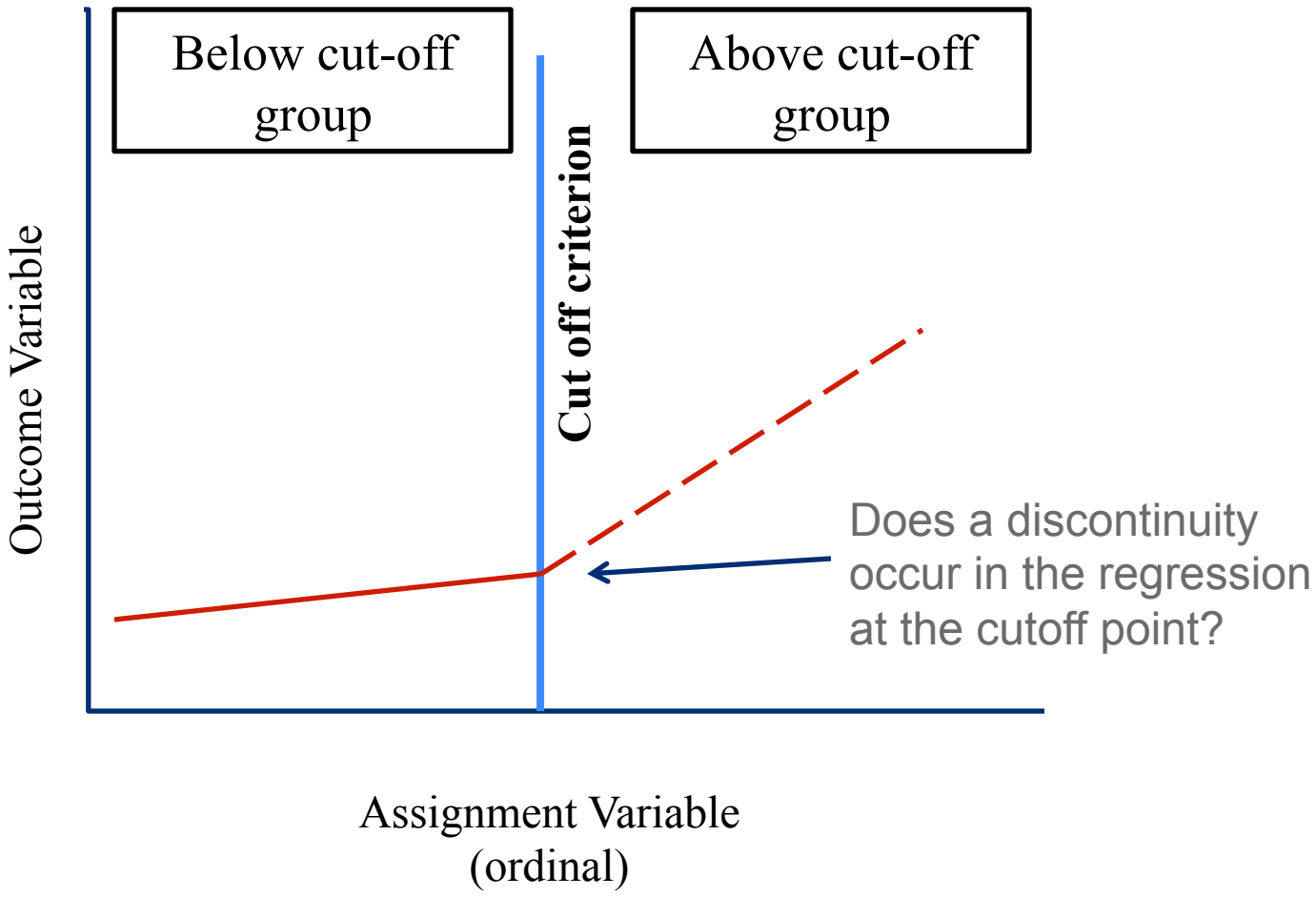
Treatment Assignation

- Skipping class in college and exam performance: An RD class experiment (Dobkin, Gil, & Marion, 2009)
  - *Students below median on midterm assigned mandatory attendance policy.*
  - *Students above median were not.*
  - *Results on final exam of two groups analyzed.*

Program Evaluation

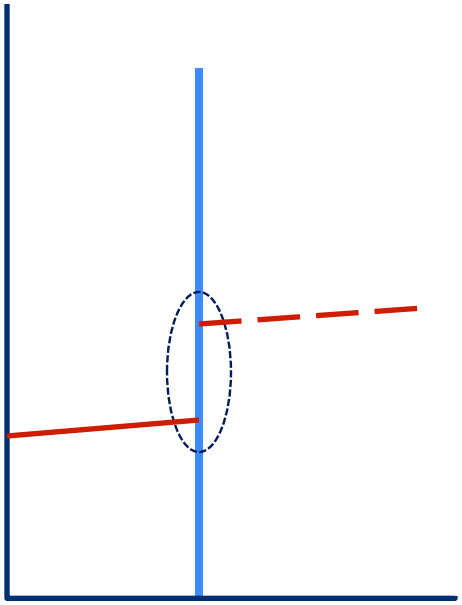
- Impact of Texas top 10% law on college enrollment (Niu & Tienda, 2010)
  - *Studied minority enrollment trends at UT and TAMU as predicted by class rank*
  - *Comparison group were students ranking at or below 10%.*
  - *Treatment group were students ranking above 10%.*

Design


Below cut-off group
Above cut-off group
Cut off criterion
Outcome Variable
Does a discontinuity occur in the regression at the cutoff point?
Assignment Variable (ordinal)

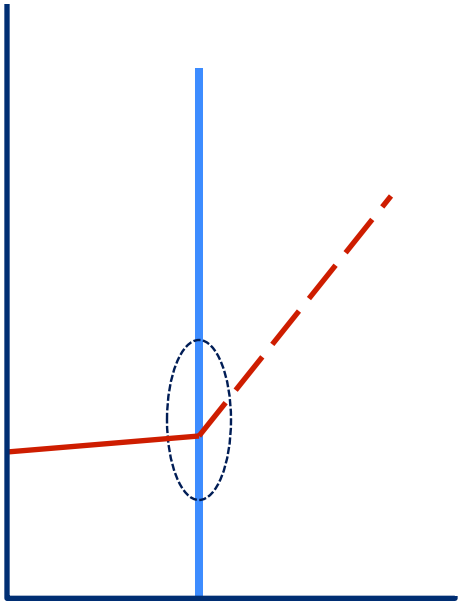
Analysis

- Does a discontinuity occur in the regression at the cutoff point?



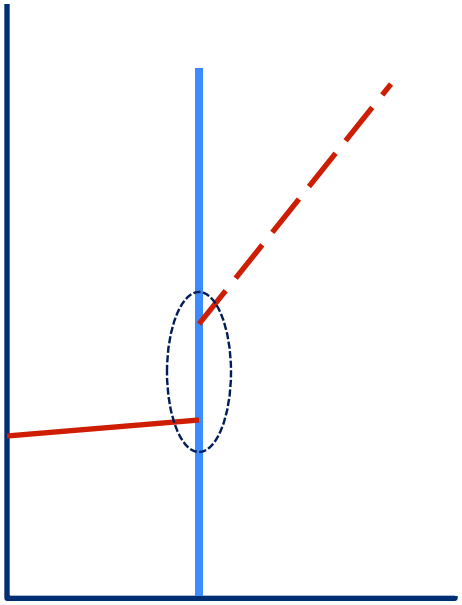
Main Effect

Change in intercept



Interaction Effect

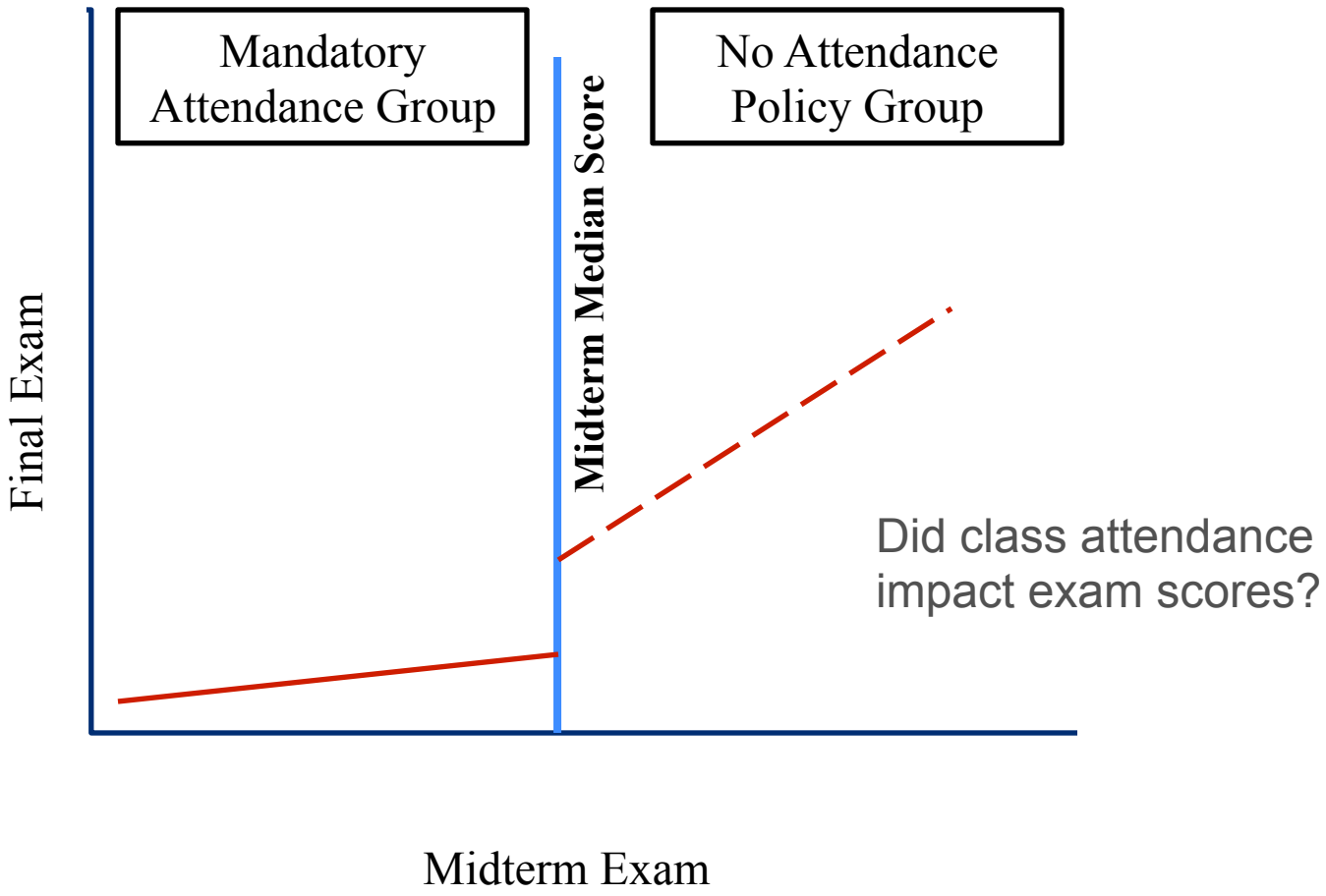
Change in slope



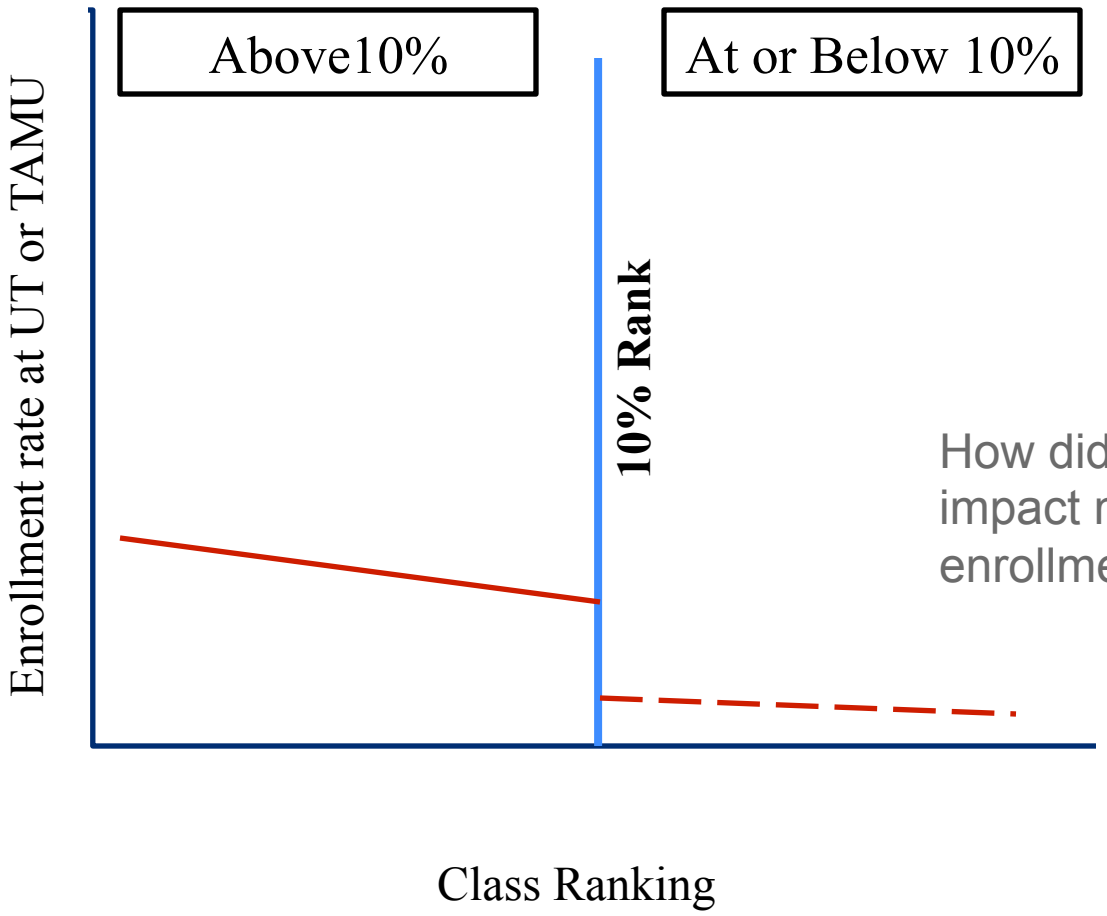
Main & Interaction Effect

Change in intercept & slope

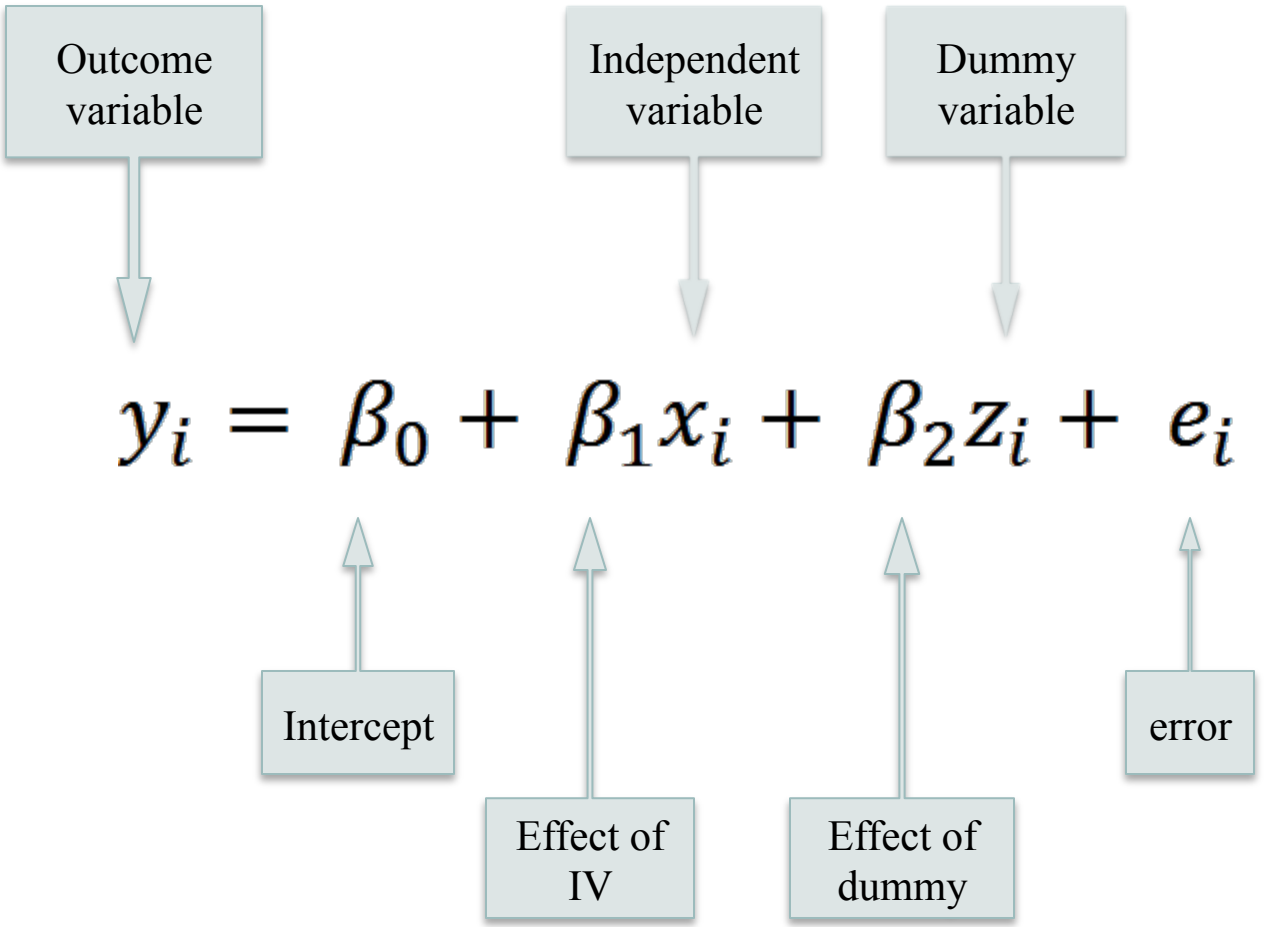
Design: Skipping Class Study



Design: Texas Top 10% Law



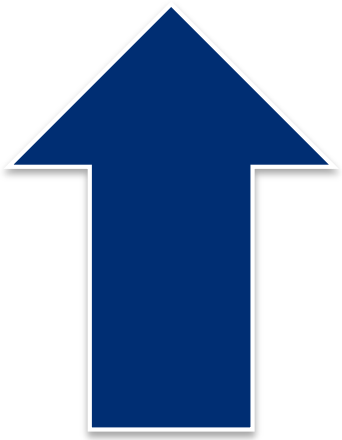
Model


Outcome variable
Independent variable
Dummy variable
$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + e_i$
Intercept
Effect of IV
Effect of dummy
error

Assumptions

Cutoff Criterion	• Must be followed without exception
Pre-post distribution	• Must not be better explained with a curve
Comparison group pretest variance	• Must have sufficient number of values
Continuous pretest distribution	• Both groups came from same distribution
Equal treatment	• Treatment administered in same way for all participants

Considerations



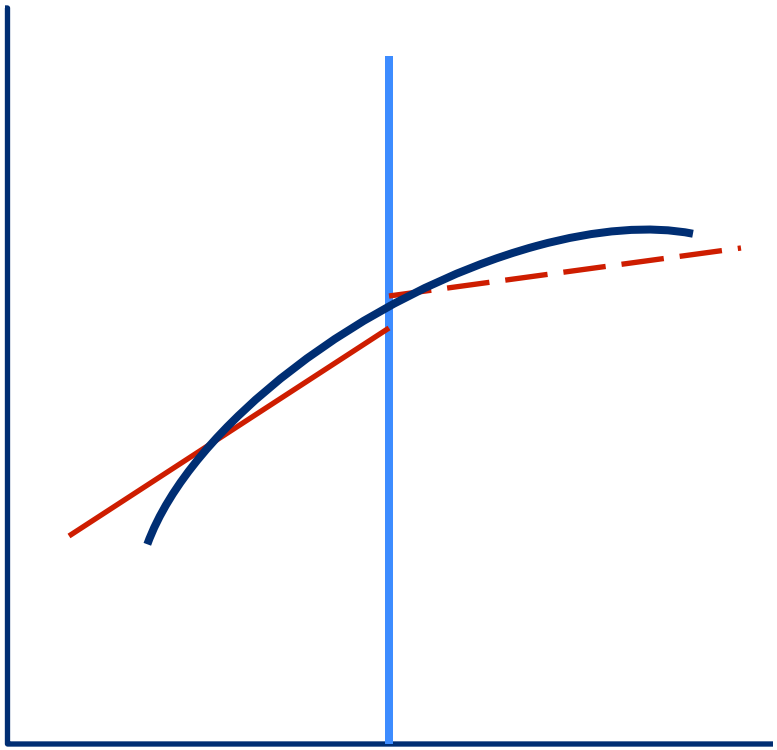
Strengths

- Unbiased assignation
- Unbiased estimate of effects of intervention
- Good alternative to RCT
- Practical method for school settings

Limitations

- Need to meet many assumptions (fuzzy designs)
- Need to model results correctly
- Need ~3x sample size of RCT
- Not as statistically powerful
- Discontinuity could be curvilinear relationship

Curvilinearity



- When the discontinuity is better explained by a curvilinear relationship instead of regression lines
- Best way to avoid this—specify model correctly.

Computing in R

library (lattice)

1. Transform the pre-treatment value (x).
   - Subtract cutoff score from pre-treatment score. $x_T = x_i - x_c$
2. Dummy code the assignment variable.
   - 0 = pre-treatment, 1 = post-treatment
3. Run a regression on dependent variable with transformed *x* and dummy variable.

Example 1: No statistical significance

```
library (lattice)
```

DATA SET 1

```
group1<-data.frame(time1=1:20,score1=(c(10,15,20,25,30,35,40,45,50,55,
60,65,70,75,80,85,90,95,100,105)),interv1=(factor(rep(0:1,each=10))))
group1$score1<-jitter(group1$score1,factor=12)

DETERMINE CUTOFF CRITERION. (10.5)
```

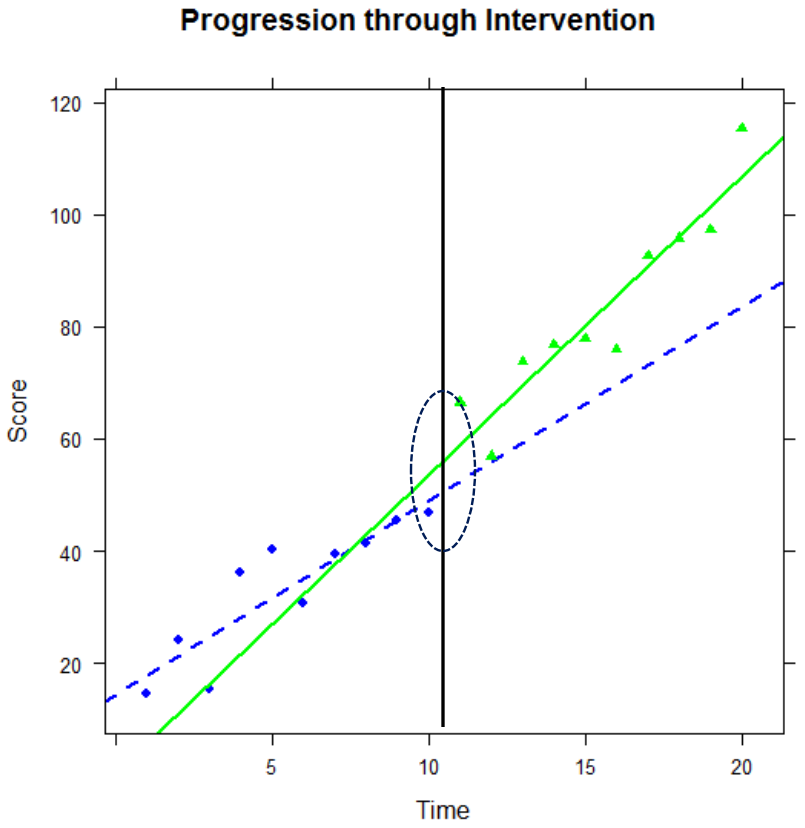
VISUAL ANALYSIS

```
xyplot(score1~time1,group1,xlab="Time",ylab="Score",
main="Progression through Intervention",pch=c(19,17),groups=interv1,
col=c("blue","green"),type=c("p","r"),lwd=2,lty=c(2,1))
```

STATISTICAL ANALYSIS

```
analysis1<-lm(score1~I(time1-10.5)*interv1,group1)
summary(analysis1)
```

Example 1: No statistical significance




```
 analysis1<-lm(score1~I(time1-10.5)*interv1,group1)
> summary(analysis1)

Call:
lm(formula = score1 ~ I(time1 - 10.5) * interv1, data = group1)

Residuals:
    Min      1Q  Median      3Q     Max
-9.5498 -3.5464 -0.2439  3.2496  8.4917

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                50.7682     3.8568  13.163 5.33e-10 ***
I(time1 - 10.5)             3.4603     0.6689   5.173 9.24e-05 ***
interv11                    5.4994     5.4544   1.008    0.328
I(time1 - 10.5):interv11    1.8591     0.9459   1.965    0.067 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.075 on 16 degrees of freedom
Multiple R-squared: 0.9634,     Adjusted R-squared: 0.9565
F-statistic: 140.2 on 3 and 16 DF,  p-value: 1.069e-11
```

Example 1: No statistical significance

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 x_i z_i + e_i$$

y = 50.77 + 3.46(time1-10.5) + 5.50(interv1) + 1.86(time-10.5)(interv1)

```
 analysis1<-lm(score1~I(time1-10.5)*interv1,group1)
> summary(analysis1)

Call:
lm(formula = score1 ~ I(time1 - 10.5) * interv1, data = group1)

Residuals:
    Min      1Q  Median      3Q     Max
-9.5498 -3.5464 -0.2439  3.2496  8.4917

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F-statistic: 140.2 on 3 and 16 DF,  p-value: 1.069e-11
```

PRE-CUTOFF GROUP

y = 50.77 + 3.46 (time1-10.5) + 5.50(0) + 1.86(time1-10.5)(0)

y = 50.77 + 3.46 (time1-10.5)

POST-CUTOFF GROUP

y = 50.77 + 3.46 (time1-10.5) + 5.50(1) + 1.86(time1-10.5)(1)

y = 56.27 + 5.32 (time1-10.5)

Example 2: Main Effect

```
library (lattice)
```

DATA SET 2

```
group2<-data.frame(time2=1:20,score2=c(5,10,15,20,25,30,35,40,45,50,
100,105,110,115,120,125,130,135,140,145),interv2=(factor(rep(0:1,each=10))))
group2$score2<-jitter(group2$score2,factor=10)

DETERMINE CUTOFF CRITERION. (10.5)
```

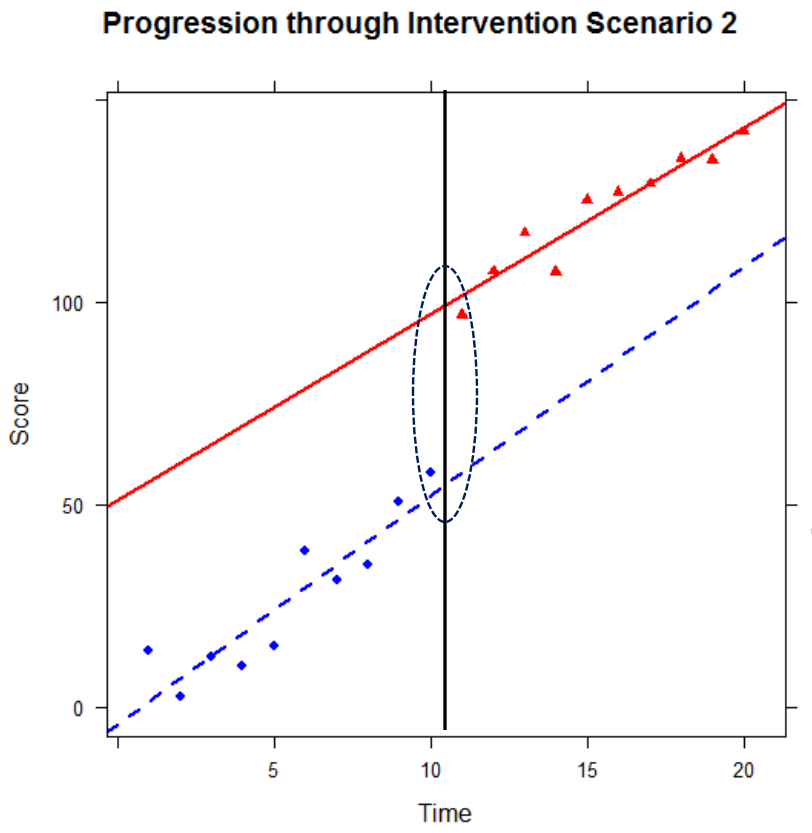
VISUAL ANALYSIS

```
xyplot(score2~time2,group2,xlab="Time",ylab="Score",
main="Progression through Intervention Scenario 2",pch=c(19,17),
groups=interv2,col=c("blue","red"),type=c("p","r"),lwd=2,lty=c(2,1))
```

STATISTICAL ANALYSIS

```
analysis2<-lm(score2~I(time2-10.5)*interv2,group2)
summary(analysis2)
```

Example 2: Main Effect




```
 analysis2<-lm(score2~I(time2-10.5)*interv2,group2)
> summary(analysis2)

Call:
lm(formula = score2 ~ I(time2 - 10.5) * interv2, data = group2)

Residuals:
    Min      1Q  Median      3Q     Max
-8.8572 -4.5041 -0.1747  4.3713 12.5243

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                55.0152     4.1225  13.345 4.35e-10 ***
I(time2 - 10.5)             5.6650     0.7149   7.924 6.28e-07 ***
interv21                   44.2711     5.8301   7.594 1.08e-06 ***
I(time2 - 10.5):interv21   -1.0723     1.0111  -1.061    0.305
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.494 on 16 degrees of freedom
Multiple R-squared: 0.9867,     Adjusted R-squared: 0.9842
F-statistic: 395.6 on 3 and 16 DF,  p-value: 3.253e-15
```

Example 2: Main Effect

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 x_i z_i + e_i$$

y = 55.02 + 5.67(time2-10.5) + 44.27(interv2) – 1.07(time2-10.5)(interv2)

```
 analysis2<-lm(score2~I(time2-10.5)*interv2,group2)
> summary(analysis2)

Call:
lm(formula = score2 ~ I(time2 - 10.5) * interv2, data = group2)

Residuals:
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-8.8572 -4.5041 -0.1747  4.3713 12.5243

Coefficients:
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Multiple R-squared: 0.9867,     Adjusted R-squared: 0.9842
F-statistic: 395.6 on 3 and 16 DF,  p-value: 3.253e-15
```

PRE-CUTOFF GROUP

y = 55.02 + 5.67 (time2-10.5) + 44.27(0) – 1.07(time2-10.5)(0)

y = 55.02 + 5.67 (time2-10.5)

POST-CUTOFF GROUP

y = 55.02 + 5.67 (time2-10.5) + 44.27(1) – 1.07(time2-10.5)(1)

y = 99.29 + 4.60 (time2-10.5)

Example 3: Interaction Effect

```
library (lattice)
```

DATA SET 3

```
group3<-data.frame(time3=1:20,score3=c(50:59,60,65,70,75,80,85,90,95,100,
105),interv3=(factor(rep(0:1,each=10))))

group3$score3<-jitter(group3$score3,factor=10)


DETERMINE CUTOFF CRITERION. (10.5)
```

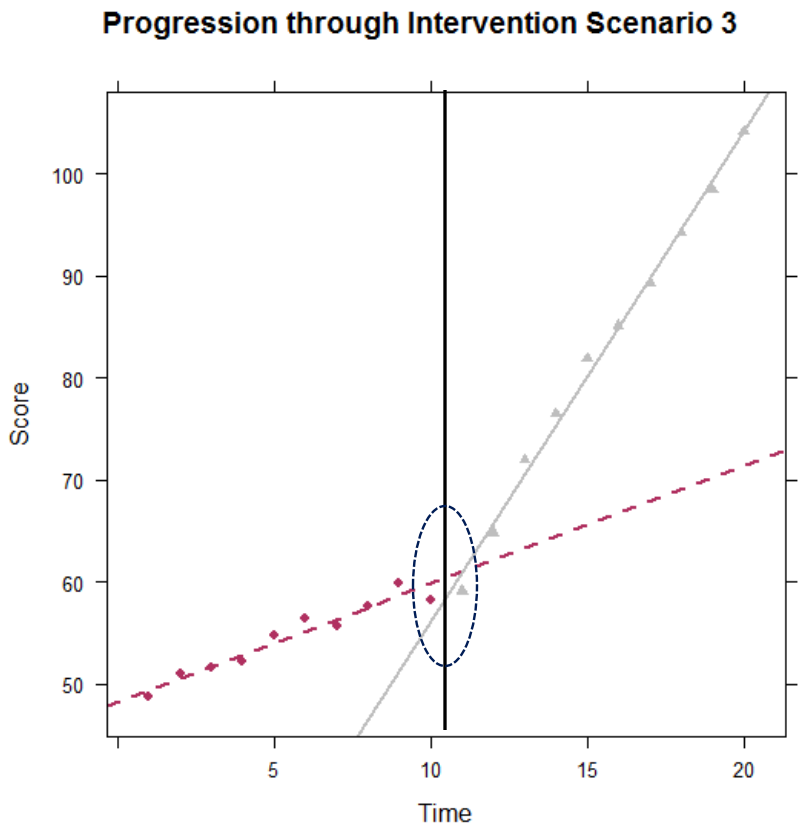
VISUAL ANALYSIS

```
xyplot(score3~time3,group3,xlab="Time",ylab="Score",
main="Progression through Intervention Scenario 3",pch=c(19,17),
groups=interv3,col=c("maroon","grey"),type=c("p","r"),lwd=2,lty=c(2,1))
```

STATISTICAL ANALYSIS

```
analysis3<-lm(score3~I(time3-10.5)*interv3,group3)
summary(analysis3)
```

Example 3: Interaction Effect




```
 analysis3<-lm(score3~I(time3-10.5)*interv3,group3)
> summary(analysis3)

Call:
lm(formula = score3 ~ I(time3 - 10.5) * interv3, data = group3)

Residuals:
    Min      1Q  Median      3Q     Max
-1.7942 -0.6819 -0.1268  0.8217  1.7521

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                60.4268     0.6981  86.564  < 2e-16 ***
I(time3 - 10.5)             1.1573     0.1211   9.560 5.12e-08 ***
interv31                   -1.9138     0.9872  -1.939   0.0704 .
I(time3 - 10.5):interv31    3.6504     0.1712  21.322 3.56e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.1 on 16 degrees of freedom
Multiple R-squared: 0.9967,     Adjusted R-squared: 0.9961
F-statistic:  1630 on 3 and 16 DF,  p-value: < 2.2e-16
```

Example 3: Interaction Effect

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 x_i z_i + e_i$$

y = 60.43 + 1.16(time3-10.5) – 1.91(interv3) + 3.65(time3-10.5)(interv3)

```
 analysis3<-lm(score3~I(time3-10.5)*interv3,group3)
> summary(analysis3)

Call:
lm(formula = score3 ~ I(time3 - 10.5) * interv3, data = group3)

Residuals:
    Min      1Q  Median      3Q     Max
-1.7942 -0.6819 -0.1268  0.8217  1.7521

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)               60.4268     0.6981  86.564  < 2e-16
    ***
I(time3 - 10.5)            1.1573     0.1211   9.560 5.12e-08
    ***
interv31                  -1.9138     0.9872  -1.939   0.0704 .
I(time3 - 10.5):interv31   3.6504     0.1712  21.322 3.56e-13
    ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.1 on 16 degrees of freedom
Multiple R-squared: 0.9967,     Adjusted R-squared: 0.9961
F-statistic:  1630 on 3 and 16 DF,  p-value: < 2.2e-16
```

PRE-CUTOFF GROUP

y = 60.43 + 1.16 (time3-10.5) – 1.91(0) + 3.65(time3-10.5)(0)

y = 60.43 + 1.16 (time3-10.5)

POST-CUTOFF GROUP

y = 60.43 + 1.16 (time3-10.5) – 1.91(1) + 3.65(time3-10.5)(1)

y = 58.52 + 4.81 (time3-10.5)

Example 4: Main & Interaction Effect

```
library (lattice)
```

DATA SET 4

```
group4<-data.frame(time4=1:20,score4=c(50:59,
80,85,90,95,100,105,110,115,120,125),interv4=(factor(rep(0:1,each=10))))

group4$score4<-jitter(group4$score4,factor=12)

DETERMINE CUTOFF CRITERION. (10.5)
```

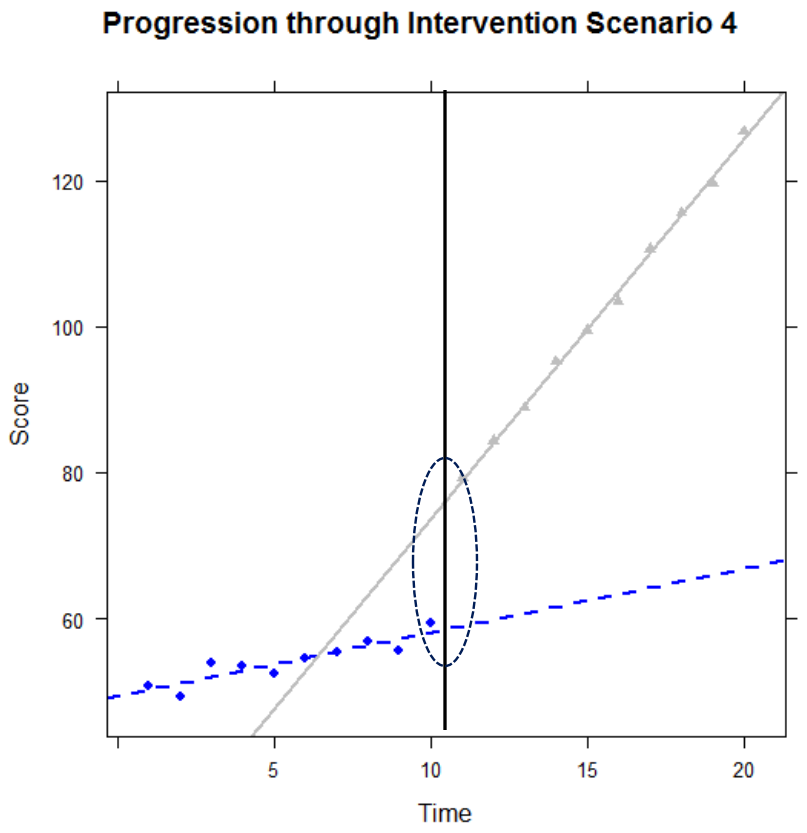
VISUAL ANALYSIS

```
xyplot(score4~time4,group4,xlab="Time",ylab="Score",
main="Progression through Intervention Scenario 4",pch=c(19,17),
groups=interv4,col=c("blue","grey"),type=c("p","r"),lwd=2,lty=c(2,1))
```

STATISTICAL ANALYSIS

```
analysis4<-lm(score4~I(time4-10.5)*interv4,group4)
summary(analysis4)
```

Example 4: Main & Interaction Effect




```
analysis4<-lm(score4~I(time4-10.5)*interv4,group4)
> summary(analysis4)

Call:
lm(formula = score4 ~ I(time4 - 10.5) * interv4, data = group4)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8244 -0.5385  0.2426  0.5778  1.9935

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                58.5511     0.6854  85.432  < 2e-16
    ***
I(time4 - 10.5)             0.8715     0.1189   7.332 1.68e-06
    ***
interv41                   17.7430     0.9692  18.306 3.73e-12
    ***
I(time4 - 10.5):interv41    4.3282     0.1681  25.750 1.89e-14
    ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.08 on 16 degrees of freedom
Multiple R-squared: 0.9987,     Adjusted R-squared: 0.9984
F-statistic:  3964 on 3 and 16 DF,  p-value: < 2.2e-16
```

Example 4: Main & Interaction Effect

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 x_i z_i + e_i$$

y = 58.55 + 0.87(time4-10.5) + 17.74(interv4) + 4.33(time4-10.5)(interv4)

```
analysis4<-lm(score4~I(time4-10.5)*interv4,group4)
> summary(analysis4)

Call:
lm(formula = score4 ~ I(time4 - 10.5) * interv4, data = group4)

Residuals:
    Min      1Q  Median      3Q     Max
-1.8244 -0.5385  0.2426  0.5778  1.9935

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)               58.5511     0.6854  85.432  < 2e-16
    ***
I(time4 - 10.5)            0.8715     0.1189   7.332 1.68e-06
    ***
interv41                  17.7430     0.9692  18.306 3.73e-12
    ***
I(time4 - 10.5):interv41   4.3282     0.1681  25.750 1.89e-14
    ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.08 on 16 degrees of freedom
Multiple R-squared: 0.9987,     Adjusted R-squared: 0.9984
F-statistic:  3964 on 3 and 16 DF,  p-value: < 2.2e-16
```

PRE-CUTOFF GROUP

y = 58.55 + 0.87 (time4-10.5) + 17.74(0) + 4.33(time4-10.5)(0)

y = 58.55 + 0.87 (time4-10.5)

POST-CUTOFF GROUP

y = 58.55 + 0.87 (time4-10.5) + 17.74(1) + 4.33(time4-10.5)(1)

y = 76.29 + 5.20 (time4-10.5)

Example 5: Multiple Groups

```
library (lattice)
```

DATA SET 5

```
group5<-data.frame(time5=1:30,score5=c(50:59,
60,65,70,75,80,85,90,95,100,105,106:115),interv5=(factor(rep(0:2,each=10))))
group5

group5$score5<-jitter(group5$score5,factor=12)

DETERMINE CUTOFF CRITERION. (10.5)
```

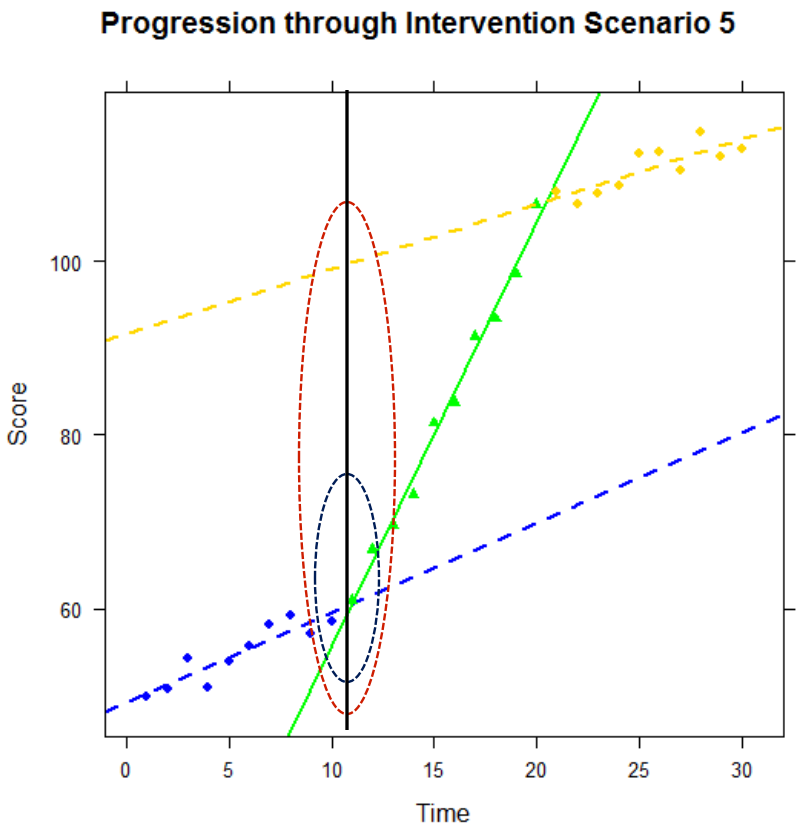
VISUAL ANALYSIS

```
xyplot(score5~time5,group5,xlab="Time",ylab="Score",
main="Progression through Intervention Scenario 5",pch=c(19,17),groups=interv5,
col=c("blue","green","gold"),type=c("p","r"),lwd=2,lty=c(2,1))
```

STATISTICAL ANALYSIS

```
analysis5<-lm(score5~I(time5-10.5)*interv5,group5)
summary(analysis5)
```

Example 5: Multiple Groups




```
 analysis5<-lm(score5~I(time5-10.5)*interv5,group5)
> summary(analysis5)

Call:
lm(formula = score5 ~ I(time5 - 10.5) * interv5, data = group5)

Residuals:
    Min      1Q  Median      3Q     Max
-2.3839 -1.2123 -0.4481  1.4422  2.3807

Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)                60.0638     1.0027  59.901  < 2e-16 ***
I(time5 - 10.5)             1.0397     0.1739   5.979 3.59e-06 ***
interv51                   -1.9882     1.4181  -1.402    0.174
interv52                   39.3363     2.8388  13.857 6.02e-13 ***
I(time5 - 10.5):interv51    3.8519     0.2459  15.663 4.22e-14 ***
I(time5 - 10.5):interv52   -0.2932     0.2459  -1.192    0.245
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.579 on 24 degrees of freedom
Multiple R-squared: 0.9966,     Adjusted R-squared: 0.9959
F-statistic:  1414 on 5 and 24 DF,  p-value: < 2.2e-16
```

Example 5: Multiple Groups

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 z_i + \beta_4 x_i z_i + \beta_5 x_i z_i + e_i$$

y = 60.06 + 1.04 (time5-10.5) – 1.99 (interv5) + 39.34 (interv5) + 3.85 (time5-10.5)(interv5) – 0.29 (time5-10.5)(interv5)

```
 analysis5<-lm(score5~I(time5-10.5)*interv5,group5)
> summary(analysis5)

Call:
lm(formula = score5 ~ I(time5 - 10.5) * interv5, data = group5)

Residuals:
    Min      1Q  Median      3Q     Max
-2.3839 -1.2123 -0.4481  1.4422  2.3807

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)            60.0638     1.0027  59.901  < 2e-16
    ***
I(time5 - 10.5)         1.0397     0.1739   5.979 3.59e-06
    ***
interv51               -1.9882     1.4181  -1.402    0.174
interv52               39.3363     2.8388  13.857 6.02e-13
    ***
I(time5 - 10.5):interv51   3.8519     0.2459  15.663 4.22e-14
    ***
I(time5 - 10.5):interv52  -0.2932     0.2459  -1.192    0.245
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.579 on 24 degrees of freedom
Multiple R-squared: 0.9966,     Adjusted R-squared: 0.9959
F-statistic:  1414 on 5 and 24 DF,  p-value: < 2.2e-16
```

PRE-CUTOFF GROUP

y = 60.06 + 1.04 (time5-10.5) – 1.99 (0) + 39.34 (0) + 3.85 (time5-10.5)(0) – 0.29 (time5-10.5)(0)

y = 60.06 + 1.04 (time5-10.5)

POST-CUTOFF GROUP 1

y = 60.06 + 1.04 (time5-10.5) – 1.99 (1) + 39.34 (1) + 3.85 (time5-10.5)(1) – 0.29 (time5-10.5)(1)

y = 97.41 + 4.60 (time5-10.5)

POST-CUTOFF GROUP 2

y = 60.06 + 1.04 (time5-10.5) – 1.99 (2) + 39.34 (2) + 3.85 (time5-10.5)(2) – 0.29 (time5-10.5)(2)

y = 134.76 + 8.16 (time5-10.5)

Conclusion

- Good option when unable to do RCT, especially in educational settings.
- Try to use as large a sample size as possible.
- Analyze data visually and statistically.
- Meet all assumptions—sharp is better than fuzzy.
- Beware of the curve.
- Specify your model correctly, and err on the side of overspecifying.

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