Planned Contrast ANOVA

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Introduction to Planned Contrast ANOVA

- In a typical ANOVA, we are sometimes encumbered by the fact that we must test all possible mean comparisons.
- It may be that we would rather test some specific hypotheses.
- In order to do this, we need can use linear regression techniques (although we have not covered this yet) to compare the differences between pairs of means.
- These contrasts can only compare two means at once, but we can combine multiple means from different levels to compute mean pair tests (e.g., students in Programs 1 and 2 vs. students in Programs 3 and 4)

Hypothesis Test(s) in Planned Contrast ANOVA

- Recall that the null hypothesis for a one-way ANOVA can be written as:
  \[ H_0: \bar{X}_1 = \bar{X}_2 = \bar{X}_3 \]

- For the planned contrast ANOVA, we can test any specific contrast that we wish to test, but may only test a total of \( K - 1 \) contrasts.
- In the case of a one-way ANOVA with three levels, we could test a total of \( K - 1 = 2 \) contrasts.
- These hypotheses could potentially be:
  \[ H_{0,C1}: \bar{X}_{P1} = \bar{X}_{P2} \]
  \[ H_{0,C2}: \bar{X}_{P1andP2} = \bar{X}_{P3} \]

Making Contrasts

- In creating the contrasts, we assign new values to the groups that we want to test with each contrast.
- In the case of the study in the previous slide, we would setup the contrasts as follows:

<table>
<thead>
<tr>
<th>Program</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>
Simple Heuristic Example in R

```r
> pcdata <- data.frame(program = factor(rep(1:3, each = 2)),
+   score = c(9, 10, 11, 10, 12, 13))
> pcdata
   program score
1       1     9
2       1    10
3       2    11
4       2    10
5       3    12
6       3    13
> anova(aov(score ~ program, pcdata))
Analysis of Variance Table
Response: score
           Df Sum Sq Mean Sq  F value Pr(>F)
program     2 9.3333 4.6667 9.33333  0.0515
Residuals   3 1.5000 0.5000
```

Setting Up the Contrasts

```r
> pcdata$c1 <- rep(c(1, -1, 0), each = 2)
> pcdata$c2 <- rep(c(1, 1, -2), each = 2)
> pcdata
   program score c1 c2
1       1     9 1  1
2       1    10 1  1
3       2    11 -1  1
4       2    10 -1  1
5       3    12  0 -2
6       3    13  0 -2
> anova(lm(score ~ c1 + c2, pcdata))
Analysis of Variance Table
Response: score
           Df Sum Sq Mean Sq F value Pr(>F)
c1          1 1.0000 1.0000  2.000 0.2522
C2          1 8.3333 8.3333 16.667 0.0265
Residuals   3 1.5000 0.5000
```

Running the PC ANOVA

```r
> anova(lm(score ~ c1 + c2, pcdata))
Analysis of Variance Table
Response: score
           Df Sum Sq Mean Sq  F value Pr(>F)
c1          1 1.0000 1.0000  2.0000  0.252
C2          1 8.3333 8.3333 16.667  0.0265
Residuals   3 1.5000 0.5000
```

Examples of Orthogonal Contrasts

- In the past example, we saw that the contrasts were perfectly uncorrelated, which means that they perfectly carve up the \(SS_B\) into equal shares.
- Here are some other examples for \(K = 4\).

<table>
<thead>
<tr>
<th>Ex 1</th>
<th>Ex 2</th>
<th>Ex 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>C2</td>
<td>C3</td>
</tr>
<tr>
<td>C1</td>
<td>C2</td>
<td>C3</td>
</tr>
<tr>
<td>C1</td>
<td>C2</td>
<td>C3</td>
</tr>
</tbody>
</table>

- Note that for this study, the two \(SS\) for C1 and C2 exactly equal the \(SS_B\) in the one-way ANOVA.
- This is because the two contrasts are orthogonal.

```r
> cor(pcdata$c1, pcdata$c2)
[1] 0
```
An Example with Non-Orthogonal Contrasts

> pcdata$c3 <- rep(c(1, 0, -1), each = 2)
> anova(lm(score ~ c1 + c3, pcdata))

Analysis of Variance Table
Response: score
  Df Sum Sq Mean Sq F value Pr(>F)
  c1 1  1.0000 1.0000  2.000 0.25222
  c3 1  8.3333 8.3333 16.667 0.02655
  Residuals 3  1.5000 0.5000

> anova(lm(score ~ c3 + c1, pcdata))

Analysis of Variance Table
Response: score
  Df Sum Sq Mean Sq F value Pr(>F)
  c3 1  9.0000 9.0000 18.000 0.02398
  c1 1  0.3333 0.3333  0.667 0.47402
  Residuals 3  1.5000 0.5000

In-Class Homework

- Get the dataset at http://faculty.smu.edu/kyler/courses/7311/pcdata2.txt
- Look at each of the means and try and determine the best planned contrasts to run.
- Run the PC ANOVA after examining the means and SDs. Also, you might want to look at the boxplots for the study.

In-Class Homework 2

- Get the dataset at http://faculty.smu.edu/kyler/courses/7311/pcdata3.txt
- Look at each of the means and try and determine the best planned contrasts to run.
- Run the PC ANOVA after examining the means and SDs. Also, you might want to look at the boxplots for the study.