This article proposes a novel mechanism whereby larger markets increase competition and facilitate process innovation. Larger markets, in the sense of more people or more open trade, support a larger variety of goods, resulting in a more crowded product space. This raises the price elasticity of demand and lowers markups. Firms, therefore, become larger to break even. This facilitates process innovation, as larger firms can amortize R&D costs over more goods. We demonstrate this mechanism in a standard model of process and product innovation. In doing so, we question some important results in the new trade and endogenous growth literatures.

1. INTRODUCTION

A large body of empirical work suggests that greater competition enhances productivity. For example, Nickell (1996) finds that U.K. manufacturing firms facing a larger number of competitors experienced higher productivity growth, and Galdón-Sánchez and Schmitz (2002) show that increased competitive pressure in the iron ore industry during the 1980s can explain productivity increases of up to 100 percent in some countries. Often, greater competition and higher productivity are linked to market size. For example, Syverson (2004) documents in a study of the U.S. cement industry that firms in larger cities are more productive, and Luzio and Greenstein (1995) and Lewis (2004) document substantial increases in productivity following a reduction in trade barriers in the Brazilian computer and automobile industries. Despite this empirical support, the question of how larger markets and greater competition facilitate innovation is very much an open one.

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2 See Blundell et al. (1999) and Zitzewitz (2003) for further evidence on the positive relation between competition and innovation. For quantitative evidence on the negative effects of monopoly and entry barriers on TFP, see Herrendorf and Teixeira (2007).

3 For a survey of the empirical evidence on the relation between trade liberalization and productivity, see Tybout (2003).
This article proposes a novel mechanism whereby larger markets lead to more competition and facilitate the adoption of more advanced technologies. The mechanism works by changing the price elasticity of demand. Larger populations or greater openness allow for more substitution between goods, thereby raising the price elasticity of demand. As a result, markups fall and competition toughens. With lower markups firms must sell more goods to break even. This increase in firm size is essential for innovation. As larger firms are able to amortize the fixed costs of R&D over a greater number of goods, they find it more profitable to adopt more advanced technologies.

The idea that firm size facilitates process innovation has a long history in economics, going back as far as Schumpeter (1942). There is much empirical evidence supporting this view. For example, Atack et al. (2008) find that larger firms were more likely to use steam power in the 19th century. Hannan and McDowell (1984) reach a similar conclusion when analyzing the relationship between the size of banks and the adoption of ATMs in the 1970s. In terms of R&D expenditures, Cohen and Klepper (1996) find that they rise with firm size, with a greater share being allocated to process innovation. The novelty of our article, therefore, is not its emphasis on the importance of firm size for innovation, but rather the establishment of a general equilibrium link between market size, firm size, and innovation through the price elasticity of demand.

We make price elasticity dependent on market size by embedding Lancaster (1979) preferences into an otherwise standard model of product and process innovation. The Lancaster construct, which is based on Hotelling’s (1929) spatial model of horizontal differentiation, assumes that each consumer has an ‘ideal variety’, identified by his location on the unit circle. By having all varieties located on the unit circle, the product space is bounded. This boundedness of the product space underlies the positive relationship between market size and the price elasticity of demand. A larger market, in the sense of a larger population, leads to more varieties being produced, implying a more crowded product space and more substitution between goods. A larger market, in the sense of lower trade costs, does not increase the number of varieties being produced, but does make goods more substitutable as foreign produced varieties become more affordable for home consumers.

Allowing the elasticity to depend on the market size, as we do here, implies a positive scale effect on process innovation, thus overturning the conventional wisdom in the trade and growth literatures. The standard view in these literatures is that once one endogenizes the number of varieties, there is no longer a positive effect of population or trade liberalization on process innovation. The absence of such a positive scale effect reflects the dominance of the Spence (1976) and Dixit and Stiglitz (1977) preference construct. With Spence–Dixit–Stiglitz there is no elasticity effect because the product space is unbounded. Hence, as the market expands, there is no change in elasticity, markups, and firm size. As a result, larger markets do not make it easier to bear the fixed costs of innovation. In effect, with Spence–Dixit–Stiglitz preferences, the additional rents associated with an increase in market size are completely dissipated by a proportional increase in the number of varieties, leaving no room for process innovation. This point was made by Grossman and Helpman (1991a, chapter 9) in the context of trade liberalization and by Young (1998) in the context of eliminating the growth rate scale effect present in the first generation endogenous growth models, which did not allow for product innovation. As we show here, making the elasticity depend on the market size causes this positive scale effect to reemerge.

The empirical evidence is far more supportive of Hotelling–Lancaster than of Spence–Dixit–Stiglitz. Barron et al. (2008) compute price elasticities in U.S. gasoline markets and find that larger

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4 Salop (1979) is a similar construct based on Hotelling (1929).
6 One notable exception of a model that generates an elasticity effect using the Spence–Dixit–Stiglitz is Holmes and Schmitz (2001). However, they accomplish this in an artificial manner. In particular, they assume that domestic firms can affect the manufacturing price index but foreign firms cannot. This leads to the odd results that the effect of taking two equally sized countries in autarky and going to free trade is different from the effect of doubling the population of a given country.
markets are associated with more elastic demand; Hummels and Lugovskyy (2008) document that import demand in larger markets is more responsive to changes in trade costs; Tybout (2003), in reviewing the literature on trade liberalization, concludes that markups fall with import competition; Campbell and Hopenhayn (2005) report a positive relationship between market size and firm size for a number of retail industries across U.S. cities; and Hummels and Klenow (2005) find that the number of varieties increases less than proportionally with the size of the market across a wide variety of industries and countries.7 Although others, such as Hummels and Klenow (2005) and Hummels and Lugovskyy (2008), have made the point that the Hotelling–Lancaster construct is more consistent with certain empirical regularities, we are the first to consider its relevance for technological innovation. In doing so, we show that the difference between Hotelling–Lancaster and Spence–Dixit–Stiglitz is not only empirically relevant, it is also theoretically important.

Indeed, one key theoretical implication of our model is that the productivity gains associated with larger markets are the result of innovations by established firms. Although there is ample evidence that established firms innovate more when trade is liberalized, this result is not easily obtained in general equilibrium models.8 With the exception of Atkeson and Burstein (2007), the literature on trade and productivity does not consider process innovation by existing firms, but instead emphasizes firm selection that favors the ‘survival of the fittest.’9 Although Atkeson and Burstein (2007) are a notable exception, there are some key differences with our model. Whereas their model requires firm heterogeneity and fixed export costs, our model requires neither. Moreover, in Atkeson and Burstein (2007) the positive effect on innovation only arises for marginal decreases in trade costs. That is, an increase in market size, in the sense of either going from autarky to free trade or increasing the population size, does not affect process innovation in their model. In contrast, we find positive effects on process innovation both when there is a marginal decrease in trade costs and when we compare autarky to free trade.10

The rest of the article is organized as follows: Section 2 describes a one-period, two-country model. Section 3 defines the equilibrium for this open economy and characterizes it along several dimensions. Section 4 examines the equilibrium properties of the model, in particular, how process innovation depends on the size of the market. Section 5 concludes.

2. THE MODEL ECONOMY

The model consists of two identical countries, referred to as Home and Foreign, and indexed by $i = H, F$. Each country contains a business sector and a household sector. The business sector is monopolistically competitive and produces a set of differentiated goods. Each differentiated good producer acts as a monopolist and chooses its price, quantity, and production process. The sole input in each production process, or technology, is labor. Individual technologies differ in the marginal product of labor and the fixed operating cost. The household sector supplies labor to the business sector and uses its income to buy the differentiated goods. Households are heterogeneous in that each has a different variety of the good it prefers above all others. In contrast to households, goods can be moved across countries, although at some cost. We study a one-period world because the effect of larger markets on innovation can be shown without introducing dynamics. The sectors are described in detail in what follows.

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7 Of course, not every case study supports Hotelling–Lancaster. For instance, Konings et al. (2001) do not find lower markups following pro-competitive reforms.
8 See Schmitz (2005) for an example where trade liberalization led to more innovation by established firms. Pavcník (2002) provides an example where there are both selection effects and within-plant productivity gains in an industry, following a reduction in trade barriers.
9 See, for example, Bernard et al. (2003), Melitz (2003), and Melitz and Ottaviano (2008).
10 Note that the urban and regional literatures also generate efficiency gains from market size for existing firms. However, they rely on externalities to do so.
2.1. Household Sector.

Endowments and Preferences. In each country there is a continuum of measure $L$ of households uniformly distributed around the unit circle. Each household is endowed with one unit of time that it supplies inelastically to the business sector. Household preferences are of the Hotelling–Lancaster type, so that each household has one variety identified by its location on the unit circle that it prefers above all others. The farther away a particular variety, $v$, lies from a household’s ideal variety, $\tilde{v}$, the lower the utility derived from a unit of consumption of $v$. Let $d_v^\beta$ denote the shortest arc distance between variety $v$ and the household’s ideal variety $\tilde{v}$. Following Hummels and Lugovskyy (2008), the utility a type $\tilde{v}$ household residing in country $i = H, F$ derives from consuming $c_v$ units of variety $v$ is

$$\frac{c_v^\gamma}{1 + d_v^{\beta\tilde{v}}},$$

where $1 + d_v^{\beta\tilde{v}}$ is Lancaster’s compensation function, i.e., the quantity of variety $v$ that gives the household the same utility as one unit of its ideal variety $\tilde{v}$. The parameter $\beta > 0$ determines how fast a household’s utility diminishes with the distance from its ideal variety.

Utility Maximization. Let $V$ denote the set of varieties produced in the world. The utility function (1) implies that each household buys only one variety. In particular, a household will buy the variety $v'$ that minimizes the cost of a quantity equivalent to one unit of its ideal variety $\tilde{v}$. For a given variety $v$, the quantity equivalent to one unit of the household’s ideal variety is $(1 + d_v^\beta)$. Its cost then is $p_v^i(1 + d_v^\beta)$, where $p_v^i$ is the price of variety $v$ in country $i = H, F$. Thus, the household with ideal variety $\tilde{v}$ buys the variety $v'$ that satisfies

$$v' = \arg\min_{v \in V} [p_v^i(1 + d_v^\beta)].$$

Let $w^i$ denote the wage earnings of a household residing in country $i = H, F$. As we subsequently explain, this is the only source of household income in the economy. It follows from the household’s budget constraint that a household in $i = H, F$ that consumes variety $v'$ does so in a quantity given by

$$c_v^i = \frac{w^i}{p_v^i},$$

This is the demand for variety $v'$ by an individual household with ideal variety $\tilde{v}$. Its demand for all other varieties $v \in V$ is zero.

2.2. Business Sector.

Technology. The business sector in each country is monopolistically competitive and produces a set of differentiated goods. These goods can be traded internationally, but at a cost. Trade costs are of the iceberg type; to deliver one unit of a given variety overseas requires a shipment of $\tau \geq 1$ units. There is free entry and exit of firms. Each firm is located at a specific point on the unit circle, corresponding to the variety it produces. As in Lancaster (1979), firms can costlessly relocate on the circle.

A firm can choose between a continuum of increasing returns to scale technologies indexed by the letter $\gamma \geq 0$ to produce its differentiated good. Labor is the sole input to each technology. The technologies differ in the marginal product of labor and the fixed cost to using the technology. More specifically, the marginal product of labor associated with technology $\gamma$ is $A(1 + \gamma)$ and the fixed labor cost of operating technology $\gamma$ is $k e^{\phi\gamma}$. Let $Q_v^i$ be the quantity of variety $v$ produced by a firm using technology $\gamma$ in country $i = H, F$ and let $L_v^i$ denote the units of labor it employs. Then,

$$Q_v^i = A(1 + \gamma)[L_v^i - ke^{\phi\gamma}].$$
We assume that $\phi > 0$ and $\kappa > 0$ so that the fixed cost is an increasing, convex function of the technology. Subsequently, we refer to $\gamma = 0$ as the “benchmark” technology.

**Profit Maximization.** The fixed labor cost implies that each variety, regardless of the technology used, will be produced by a single firm. In maximizing their profits, firms behave noncooperatively, taking the choices of other firms in both countries as given. They also take all aggregate variables as given. Each firm chooses the price and quantity of its good to be sold in the Home country, the price and quantity of its good to be sold in the Foreign country, the number of workers to hire, and the technology to be operated.

For reasons of space, we only present the profit maximization problem facing Home firms. (Expressions for Foreign firms can be derived by analogy.) Before writing the expression for a firm’s profit, we introduce some additional notation. In particular, we use a double superscript so as to distinguish between the production location and the consumption location of a given variety, where the first superscript refers to the production location and the second refers to the consumption location. For example, $C^{HF}$ denotes the Foreign consumption of a Home-produced variety, and $C^{HH}$ denotes the Home consumption of a Home-produced variety. In light of this additional notation, we suppress the subscript $v$, in the analysis that follows.

Using the production function (3) together with the fact that the firm’s total output meets the demand of Home and Foreign consumers, namely,

\[ Q^H = C^{HH} + \tau C^{HF}, \]

a Home firm’s profits can be written as

\[ \Pi^H = p^{HH}C^{HH} + p^{HF}C^{HF} - w^H \left[ e^{\phi\gamma} + \frac{C^{HH} + \tau C^{HF}}{A(1 + \gamma)} \right], \]

where $p^{HH}$ and $p^{HF}$ are the prices of a Home-produced variety in the Home and the Foreign markets. A Home firm chooses $(p^{HH}, p^{HF}, \gamma)$ to maximize the above equation, subject to demand in the Home market and demand in the Foreign market, taking the wage, $w^H$, as given. As in the standard monopoly problem, the profit maximizing price in each market is a markup over the marginal unit cost $w^H/(A(1 + \gamma))$, so that

\[ p^{HH} = \frac{w^H}{A(1 + \gamma)} \frac{e^{HH}}{e^{HH} - 1} \]

11 We assume a fixed cost, instead of a sunk cost, because it ensures zero profits in equilibrium. The distinction is not critical for the results we wish to establish, however. We think of the higher fixed cost as being associated not only with operating the technology but also with developing it.

12 This is the standard assumption in models of monopolistic competition, and in principle it requires firms to be of measure zero. Because in our model only a finite number of varieties will be produced in equilibrium, this condition is certainly not satisfied. However, it is easy to see how we could make firms to be of measure zero, without changing any of our results. Instead of having one business sector with a finite number of firms located around the unit circle, assume there is a continuum of business sectors on the interval $[0, 1]$. Each business sector, indexed by $s$, has a finite number of firms located around a unit circle, also indexed by $s$. In that case, each producer becomes infinitesimally small relative to the overall market, and thus takes all aggregate variables as given. To model this, the only difference we would need to introduce is in the preference expression (1). Instead of choosing the variety $v$ that maximizes (1), a household in country $i$ located at point $\tilde{v}^i$ on unit circle $s$ now chooses in each sector $s$ the variety $v^s$ that maximizes

\[ \int_0^1 \log \left( \frac{\epsilon^s_v}{1 + \delta^{i+v}} \right) ds. \]

The rest of the model would be exactly the same. Because of the already cumbersome notation of the open economy, we do not introduce this further complication in the main model.
\[
\frac{p^{HF}}{\tau} = \frac{w^H}{A(1 + \gamma)} \frac{\varepsilon^{HF}}{\varepsilon^{HF} - 1},
\]

where \(\varepsilon^{HH}\) and \(\varepsilon^{HF}\) are the price elasticities of demand for variety \(v\) in the Home country and in the Foreign country. Namely,

\[
\varepsilon^{HH} = -\frac{\partial C^{HH}}{\partial p^{HH}} \frac{p^{HH} C^{HH}}{p^{HH}},
\]

\[
\varepsilon^{HF} = -\frac{\partial C^{HF}}{\partial p^{HF}} \frac{p^{HF} C^{HF}}{p^{HF}}.
\]

The first order necessary condition associated with the choice of technology, \(\gamma\), is

\[
-\phi \kappa e^{\phi \gamma} + \frac{C^{HH} + \tau C^{HF}}{A(1 + \gamma)^2} \leq 0,
\]

where the inequality in the above expression corresponds to a corner solution, i.e., \(\gamma = 0\).

3. EQUILIBRIUM

As is standard in this literature, we only focus on symmetric Nash equilibria. In such an equilibrium, all firms use the same technology, and all goods are equally spaced along the unit circle. Moreover, each Home-produced variety is surrounded by two Foreign-produced varieties. Before defining a symmetric Nash equilibrium, we derive the aggregate Home demand and the aggregate Foreign demand for the Home produced variety, \(v^H\). (For reasons of space, we do not derive the aggregate demands for a Foreign-produced variety, but this can easily be done by analogy.)

Because in a symmetric Nash equilibrium all varieties produced in the world are equally spaced along the unit circle, aggregate demand for a given Home variety depends only on the locations and the prices of its closest neighbors to its right and its left on the unit circle, which are both Foreign-produced varieties. As these two Foreign-produced neighboring varieties are each located at the same distance \(d\) from the Home-produced variety, we do not need to differentiate between them, and thus denote each by \(v^F\).

Figure 1 is instructive for deriving the aggregate demand for variety \(v^H\) produced in the Home country. We first determine the total demand by households in the Home country. Denote the

\[d^{HH}\]
prices of varieties \( v^H \) and \( v^F \) in the Home market by \( p^{HH} \) and \( p^{FH} \). The Home household who is indifferent between buying varieties \( v^H \) and \( v^F \) is the one whose cost of a quantity equivalent of one unit of its ideal variety in terms of the Home-produced good equals the cost of a quantity equivalent of its ideal variety in terms of the Foreign-produced good. Thus, the Home household that is indifferent between \( v^H \) and \( v^F \) is the one located at distance \( d^{HH} \) from \( v^H \) shown in Figure 1, where

\[ p^{FH} [1 + (d - d^{HH})^\beta] = p^{HH} [1 + (d^{HH})^\beta]. \]

(9) Given this indifference condition applies to households both to the right and to the left of \( v^H \), a share \( 2d^{HH} \) of Home households consumes variety \( v^H \). As Home households are uniformly distributed along the unit circle and each household spends its entire wage earnings on a single variety, it follows that Home households will consume \( C^{HH} \) units of variety \( v^H \), where

\[ C^{HH} = \frac{2d^{HH}w^H L}{p^{HH}}. \]

(10) This is the Home demand for \( v^H \).

Next we analogously derive the demand for variety \( v^H \) by Foreign households. Denote the prices of \( v^H \) and \( v^F \) in the Foreign market by \( p^{HF} \) and \( p^{FF} \). If trade costs are strictly positive, these prices differ from their counterparts in the Home country, i.e., \( p^{HF} \neq p^{HH} \) and \( p^{FF} \neq p^{FH} \). By the same reasoning as above, the Foreign household that is indifferent between buying \( v^H \) and \( v^F \) is located at distance \( d^{HF} \) from \( v^H \), where

\[ p^{FF} [1 + (d - d^{HF})^\beta] = p^{HF} [1 + (d^{HF})^\beta]. \]

(11) Again, because this indifference condition applies to households both to the right and to the left of \( v^H \), a share \( 2d^{HF} \) of Foreign households consumes variety \( v^H \). The total amount of variety \( v^H \) consumed in the Foreign market is thus

\[ C^{HF} = \frac{2d^{HF}w^F L}{p^{HF}}. \]

(12) This is the Foreign demand for \( v^H \).

With these demands in hand, we can solve for the price elasticities in a symmetric Nash equilibrium. This can be done in two steps. Recall that \( d^{HH} \) is the shortest arc distance between the firm and the indifferent Home customer, and \( d \) is the shortest arc distance between the firm and its nearest competitor. First, it is easy to derive from Home demand (10) and (4) that

\[ \frac{\partial C^{HH}}{\partial p^{HH}} \frac{p^{HH}}{C^{HH}} = 1 - \frac{\partial d^{HH}}{\partial p^{HH}} \frac{p^{HH}}{d^{HH}}. \]

(13) Next, we solve for the partial derivative \( \partial d^{HH} / \partial p^{HH} \) by taking the total derivative of the indifference Equation (9) with respect to \( p^{HH} \). This yields

\[ \varepsilon^{HH} = 1 + \frac{[1 + (d^{HH})^\beta]p^{HH}}{p^{HH} \beta(d^{HH})^{\beta-1} + p^{HH} \beta(d - d^{HH})^{\beta-1} d^{HH}}. \]

(14) By analogy, the elasticity faced by a Home firm in the Foreign market is

\[ \varepsilon^{HF} = 1 + \frac{[1 + (d^{HF})^\beta]p^{HF}}{p^{HF} \beta(d^{HF})^{\beta-1} + p^{HF} \beta(d - d^{HF})^{\beta-1} d^{HF}}. \]

(15)
The equations that characterize a Home firm’s profit maximizing decisions are (6), (7), (8), (14), and (15), and the equations that characterize utility maximization associated with a Home variety are (9), (10), (11), and (12). In addition to utility maximization and profit maximization, the market for each variety clears in equilibrium, as expressed in (4).

The labor market must also clear in each country. As $d$ is the shortest-arc distance between any two varieties on the unit circle, it follows that the number of varieties produced in the world is $1/d$. Thus, each country produces $1/(2d)$ varieties. Given the production function (3), each Home firm employs $\kappa e^{\phi \gamma} + (C_{HH} + \tau C_{HF})/(A(1 + \gamma))$ units of labor, so that labor market clearing in the Home country requires

$$L = \frac{1}{2d} \left[ \kappa e^{\phi \gamma} + \frac{C_{HH} + \tau C_{HF}}{A(1 + \gamma)} \right].$$

There is a final equilibrium condition that must be satisfied in each country: the zero profit condition. This is a consequence of free entry and exit. The zero profit condition of a firm located in the Home country is

$$p_{HH} C_{HH} + p_{HF} C_{HF} - w H \left[ \kappa e^{\phi \gamma} + \frac{C_{HH} + \tau C_{HF}}{A(1 + \gamma)} \right] = 0.$$ 

The zero profit condition determines the number of varieties produced in the Home country. Analogous expressions for (4) and (6)–(17) exist for the Foreign country.

We are now ready to define a symmetric equilibrium.

3.1. Definition of Symmetric Equilibrium. A Symmetric Equilibrium is a vector of elements $(p_{ii}^*, \epsilon_{ii}^*, p_{ij}^*, \epsilon_{ij}^*, w_i^*, d_{ii}^*, d_{ij}^*, Q_i^*, C_{ii}^*, C_{ij}^*, \gamma_i^*)$, where $i, j \in \{H, F\}$, $i \neq j$, and $x_{ii}^* = x_{ij}^*$, $x_{ij}^* = x_{ji}^*$, and $x_{i}^* = x_{j}^*$ for any variable $x^*$, that satisfies conditions (4) and (6)–(17).

4. EQUILIBRIUM PROPERTIES

The purpose of this section is to examine how the choice of technology depends on the size of the market. We do this in two ways. First, we study the effect of an increase in population size. Next, we study the effect of a decrease in trade costs.

4.1. Population Size and Technology Choice. In this subsection, we analyze how technology adoption depends on population size. For this purpose, it would be intuitive to focus on a closed economy model. However, having developed the notation and equilibrium conditions for an open economy world, it is sufficient to analyze the zero transportation cost case and vary the size of the total world population. This is because in our model a one-country closed economy with population $2L$ is equivalent to a two-country open economy with zero iceberg costs and a population of $L$ in each country.

We start by proving that for any population size there is a unique symmetric equilibrium. This is followed by a proof that shows that the equilibrium value of $\gamma$ is increasing in the size of the population.

**Proposition 1.** For each population size there is a unique symmetric equilibrium.

**Proof.** With the exception of the technology choice, our model is identical to the one studied in Hummels and Lugovskyy (2008). They show that for a given technology and population size, the symmetric equilibrium is uniquely determined by Equations (4) and (6)–(17), that is, by all but condition (8) in the definition of a Symmetric Equilibrium. Thus, once we endogenize the technology process, we only need to show that there is a unique $\gamma$ that satisfies the first order condition with respect to technology choice (8). To do this we simplify (8). Note that with zero iceberg costs, $C_{ii} = C_{ij} = C$. The first step in this simplification is to insert the price expressions (6) and (7) into the zero profit condition (17). This yields $2C = A(1 + \gamma)(\epsilon - 1)\kappa e^{\phi \gamma}$. The next
Equilibrium Technology

\[
\text{Elasticity (Population 150)}
\]

\[
\text{Elasticity (Population 250)}
\]

**Figure 2**

**EQUILIBRIUM TECHNOLOGY**

step is to insert this expression into Equation (8), so that the first order necessary condition with respect to \( \gamma \) becomes

\[
\kappa e^{\phi \gamma} \left( \frac{\varepsilon - 1}{1 + \gamma} - \phi \right) \begin{cases} 
0 & \text{if } \gamma > 0 \\
< 0 & \text{if } \gamma = 0.
\end{cases}
\]

(18)

Condition (18) can be further simplified to

\[
\varepsilon \begin{cases} 
= 1 + (1 + \gamma)\phi & \text{if } \gamma > 0 \\
< 1 + (1 + \gamma)\phi & \text{if } \gamma = 0.
\end{cases}
\]

(19)

Proving that there is a unique equilibrium amounts to showing that there is a unique \( \gamma \) that satisfies (19). We denote this \( \gamma \) by \( \gamma^* \). To demonstrate this, it suffices to show that \( \varepsilon \) is a strictly decreasing function of \( \gamma \) and that \( 1 + (1 + \gamma)\phi \) is a strictly increasing function of \( \gamma \). These two relations are represented in Figure 2.\(^{14}\) The fact that \( 1 + (1 + \gamma)\phi \) is increasing in \( \gamma \) is immediate. Regarding \( \varepsilon \), under symmetry and zero iceberg costs the elasticity expressions (14) and (15) both simplify to

\[
\varepsilon = 1 + \frac{1}{2\beta} \left( \frac{2}{d} \right)^\beta + \frac{1}{2\beta}.
\]

(20)

Because the total production of a firm is \( \kappa e^{\phi \gamma} A(1 + \gamma)(\varepsilon - 1) \), and the total population is \( 2L \), the total number of firms in the world is \( n = 2L/(\kappa e^{\phi \gamma} \varepsilon) \), where \( n = 1/d \). Substituting into (20) gives

\[
\varepsilon = 1 + \frac{1}{2\beta} \left( \frac{4L}{\kappa e^{\phi \gamma} \varepsilon} \right)^\beta + \frac{1}{2\beta}.
\]

(21)

\(^{14}\) Figure 2 is based on the parameter values of Table 1, which will be used for our numerical experiments on trade liberalization.
Now rewrite (21) as $2\beta \varepsilon^{\beta+1} - (2\beta + 1)\varepsilon^{\beta} - (4L/k\varepsilon^{\phi\gamma})^\beta = 0$ and take the total derivative of this expression with respect to $\gamma$. This yields

$$
\frac{\partial \varepsilon}{\partial \gamma} = -\frac{\beta(4L)^\beta \kappa^{-\beta} \phi \varepsilon^{-\phi\gamma^\beta}}{2\beta(\beta + 1)\varepsilon^{\beta} - (2\beta + 1)\beta \varepsilon^{\beta - 1}}.
$$

From (14) and (15) we know that $\varepsilon > 1$, so that this derivative (22) is strictly negative. The fact that $\varepsilon$ is decreasing in $\gamma$, and the fact that $1 + (1 + \gamma)\phi$ is increasing in $\gamma$, implies that there is exactly one value of $\gamma$ that satisfies $\varepsilon = 1 + (1 + \gamma)\phi$. Denote this value of $\gamma$ by $\gamma'$. If $\gamma' > 0$, then the equilibrium $\gamma^*$ in (19) is equal to $\gamma'$. If $\gamma' \leq 0$, then $\gamma^*$ in (19) is zero. There is thus a unique $\gamma$ satisfying the first order condition (19). This, together with the result of Hummels and Lugovskyy (2008), proves there is a unique symmetric equilibrium for each population size.

Having shown that there is a unique symmetric equilibrium, we now turn to the issue of how the equilibrium value of $\gamma$ depends on the size of the population. We first demonstrate that the price elasticity of demand, $\varepsilon$, is an increasing function of the population.\(^{15}\) This implies that the downward sloping schedule in Figure 2 shifts up when the population increases. Because the upward sloping schedule in Figure 2 is independent of the population, this allows us to conclude that the equilibrium technology choice $\gamma^*$ and the equilibrium elasticity $\varepsilon^*$ both increase with the size of the population. In what follows we prove this.

**Proposition 2.** In a symmetric equilibrium with zero iceberg costs, $\gamma$ is increasing in the size of the population.

**Proof.** The proof amounts to showing that the downward sloping graph in Figure 2 shifts out in response to an increase in population. This is equivalent to showing that the derivative of the elasticity expression (21) with respect to $L$ is positive. For this purpose, we again rewrite (21) as $2\beta \varepsilon^{\beta+1} - (2\beta + 1)\varepsilon^{\beta} - (4L/k\varepsilon^{\phi\gamma})^\beta = 0$ and totally differentiate with respect to $L$ while holding the value of $\gamma$ fixed. This gives

$$
\frac{\partial \varepsilon}{\partial L} = \frac{(4/k\varepsilon^{\phi\gamma})^\beta \beta L^{-1}}{2\beta(\beta + 1)\varepsilon^{\beta} - (2\beta + 1)\beta \varepsilon^{\beta - 1}}.
$$

Because $\varepsilon > 1$, the above partial derivative is strictly positive, so that an increase in $L$ leads to a greater elasticity of demand for any given $\gamma$. We are now ready to complete the proof. From Proposition 1 we know that for each population size there is a unique $\gamma^*$ that is the solution to (19). We have shown in the first part of the proof that $\varepsilon$ is increasing in $L$. Thus, the left-hand side of (19) is increasing in $L$, whereas the right-hand side does not depend on $L$. Because the left-hand side is decreasing in $\gamma$, and the right-hand side is increasing in $\gamma$, this implies that $\gamma^*$ is increasing in $L$.

As the intersection of the two schedules in Figure 2 also gives the equilibrium value of $\varepsilon$, it is obvious that it, too, is increasing in the size of the population. In contrast to (23), this takes into account that $\gamma$ is endogenous. This result is stated in the following Corollary to Proposition 2.

**Corollary 1.** In a symmetric equilibrium with zero iceberg costs, $\varepsilon$ is increasing in the size of the population.

The intuition for these results is straightforward. For the moment, ignore the technology choice of firms. As the population increases, more firms enter the variety space. As a result, the price elasticity of demand increases and markups fall. Thus, to break even, firms must sell more goods. Indeed, because the equilibrium number of workers per firm is $k\varepsilon^{\phi\gamma}e$, and $\varepsilon$ and is an increasing function of $L$, greater population size leads to larger firms both in terms of

\(^{15}\) A similar result is shown in Hummels and Lugovskyy (2008).
goods produced and employment. This implies that the number of firms increases less than proportionally with population size. Indeed,

\[ \frac{L \partial n}{n \partial L} = 1 - \frac{L \partial \varepsilon}{\varepsilon \partial L}, \]

where, as shown in Hummels and Lugovskyy (2008),

\[ \frac{L \partial \varepsilon}{\varepsilon \partial L} = \left(1 + 2\varepsilon \left(\kappa e^{\phi \gamma} \frac{\varepsilon}{4L}\right)^{-\beta}\right)^{-1}. \]

Because this expression is less than one, the percentage change in the number of firms resulting from a one percentage change in the population is less than one as well.

The resulting larger firms endogenously choose higher values of \( \gamma \). To see this, note that the first order condition (8) with respect to \( \gamma \) has two effects: An increase in \( \gamma \) raises a firm’s fixed cost and it lowers its marginal cost. The former (negative) effect is independent of firm size, whereas the latter (positive) effect is increasing in firm size. This explains why larger firms choose higher values of \( \gamma \), giving rise to a positive relation between population size and technological progress.

The elasticity channel is critical for these results. This is apparent from (19), which is the first order condition for a firm’s technology choice. Because the price elasticity schedule shifts up in response to a higher population, the equilibrium value of \( \gamma \) consistent with (19) is an increasing function of population. If, instead, the elasticity channel were to be shut off somehow, and \( \varepsilon \) were thus to be independent of \( L \), then the value of \( \gamma \) consistent with (19) would no longer depend on \( L \). In that case, an increase in market size would have no effect on innovation. This is the result obtained by Young (1998) when using Spence–Dixit–Stiglitz preferences.

Of course, the price elasticity of demand is a market concept, so it should be possible to understand our results at the more basic level of preferences and technology. Toward this goal, consider how a social planner would run the economy. Because of household heterogeneity, there is the issue of what is a reasonable objective function for the planner. Given that the equilibrium for the decentralized economy is characterized by equal consumption, assume the planner maximizes the average utility of households subject to the constraint that every household consumes the same quantity.\(^\text{16}\)

For now take the technology as given, and consider what would happen if the planner were to increase the number of varieties proportionally with an increase in the population. Because this brings households on average closer to their ideal varieties, the average utility would increase. However, this positive effect would become increasingly smaller as the variety space gets filled up. This is easy to see. If, say, the number of varieties doubles from 4 to 8, the average distance between a household and its closest variety drops from 1/16 to 1/32, whereas when the number of varieties doubles from 8 to 16, that average distance only goes down from 1/32 to 1/64. The positive utility effect from increasing the number of varieties is thus decreasing. This explains why the planner would choose not only to increase the number of varieties when the population increases but also to increase firm size, as larger firms result in greater consumption per household on account of the fixed operating cost. As a result, for a given technology, the planner of a more highly populated economy would create not only more firms but also larger firms.\(^\text{17}\)

It is then a short step to understand why the planner would prefer more productive technologies in bigger markets. With larger firms, the share of labor used to cover the fixed cost associated with a given technological upgrade is lower. Thus, the planner would choose more innovation.

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\(^{16}\) A mathematical analysis of this problem is available from the authors upon request.

\(^{17}\) With Spence–Dixit–Stiglitz preferences this is not the case: The positive utility effect of increasing the number of varieties in proportion to the population is constant, instead of decreasing. The optimal firm size in the planner’s solution is thus independent of the size of the population.
4.2. Trade Liberalization and Technology Choice. In this subsection we interpret market size as trade liberalization, and explore how lower trade costs affect technology adoption. We note that if trade liberalization is interpreted as going from autarky to free trade, then this is equivalent to an increase in the size of the population, analyzed in the previous subsection. We therefore interpret trade liberalization as a decrease in positive, but nonprohibitive, iceberg costs. When doing so, the analytical expressions no longer simplify. For this reason we explore the importance of trade liberalization on innovation numerically.

For the numerical analysis we use the parameter values listed in Table 1. The preference parameter, $\beta$, associated with Lancaster’s compensation function, has been assigned a value consistent with the empirical regularity of a positive relation between trade liberalization and markups (Tybout, 2003).\(^{18}\) The exact values of the technology parameters $\kappa$, $A$, and $\phi$, as well as the size of the population $L$, are qualitatively unimportant, except that all should be positive. For these parameter values, we compute the symmetric equilibrium for trade costs ranging from a high of $\tau = 1.25$ to a low of $\tau = 1.0$ (free trade).

The effect of trade liberalization on innovation can be seen in Figure 3. To facilitate interpretation, the horizontal axis is ordered from less trade openness to more trade openness. Once again, market size stimulates innovation. For $\tau$ below 1.12, trade is not open enough for technology adoption to occur, so that $\gamma^* = 0$. Once $\tau$ rises above this threshold, market access through trade liberalization is sufficient for innovation to take off. Moreover, technological progress increases as the market becomes more liberalized.

Although the elasticity channel is once again at work, its origin is different from the case of an increase in the population. In the case of an increase in the population, the variety space becomes

\(^{18}\) When there are positive trade costs, the elasticities faced by a firm in the Home market and the Foreign market are different. To guarantee that a drop in trade costs leads to an increase in the weighted average of the elasticities, a value of $\beta$ less than 1 is needed.
more crowded. This is not true in the case of trade liberalization. Here, a decrease in trade costs intensifies the competition between neighboring Home and Foreign varieties. The effect of this stronger competition is to eliminate some varieties. To see this, Table 2 reports the number of firms and the price elasticity of demand that would exist in an equilibrium without technology choice.\footnote{Because the elasticities faced by a firm differ across markets, the elasticity reported in Table 2 is the weighted average, i.e., $\epsilon = \epsilon_i (C_{ii}/(C_{ii} + C_{ij})) + \epsilon_j (C_{ij}/(C_{ii} + C_{ij}))$.} In other words, it reports the elasticity and the number of firms that satisfy all but Equation (8) in the definition of the symmetric equilibrium for $\gamma = 0$. As can be seen, lower trade costs are associated with greater elasticity and fewer firms. Therefore, whereas population growth leads to more varieties, trade liberalization leads to less varieties. Nevertheless, in both cases the underlying reason for the positive relation between market size and innovation are the same: The greater elasticity makes firms larger and leads to a bigger effect on profits when the marginal cost drops. This makes innovation more attractive.

As in the case of population size, if one were to shut off the elasticity channel, there would no longer be a relation between trade liberalization and technology adoption. This is shown by Atkeson and Burstein (2007) in a model with Spence–Dixit–Stiglitz preferences.

5. CONCLUDING REMARKS

This article has proposed a novel mechanism whereby larger markets lead to the adoption of more advanced technologies. By increasing the number of varieties, larger markets allow for more substitution between varieties, thus raising the price elasticity of demand. As a result, markups fall, and firms become larger to break even. Larger firms imply more innovation, as they can spread the fixed costs of R&D over more units. The idea that firm size is important for innovation is not new. What is clearly new, however, is that we make firm size dependent on market size through the price elasticity of demand, and that we do so in a tractable general equilibrium model.

We generate the elasticity effect by embedding Hotelling–Lancaster preferences into an otherwise standard model of product and process innovation. Preferences based on Hotelling (1929) and Lancaster (1979), although sufficient, are not necessary to give rise to a positive relation between market size and elasticity. For example, this effect is generated by Ottaviano et al. (2002) via a quasi-linear utility function with a quadratic subutility and by Feenstra (2003) via a translog expenditure function. The same effect also arises in oligopoly models, such as the one studied by Gali and Zilibotti (1995). We conjecture that the prediction that larger markets imply more innovation would be preserved using any of these alternative constructs.\footnote{We used the alternative Ottaviano et al. (2002) preferences and found our results to be unchanged.}
There are a number of virtues associated with our approach. First, the mechanism we put forth is supported by a large body of empirical work that relates market size to markups, elasticity, and firm size. Second, our model generates the result that trade liberalization leads to innovation by established firms, without the need of imposing any additional restrictions. Third, going from autarky to free trade and increasing the domestic population give identical results. In addition, trade liberalization always increases innovation, whether the greater openness takes the form of a marginal decrease in trade costs or a shift from autarky to frictionless trade. Although this may be natural to expect, surprisingly many growth and trade models do not have this feature.

Implicitly, in pointing out the virtues of our approach, we challenge the Spence–Dixit–Stiglitz paradigm that dominates both the growth and trade literatures and question many of the conclusions of these literatures. One conclusion, in particular, that we question is the one made by Young (1998), Aghion and Howitt (1998), Dinopolous and Thompson (1998), and Peretto (1998), who showed that models based on Spence–Dixit–Stiglitz did not predict a positive relationship between market size and the balanced path growth rate when both process and product innovation were endogenous. They argued that endogenizing both types of innovation is a plausible way to reconcile endogenous growth theory with Jones' (1995) finding that there was no acceleration in the U.S. growth rate despite a large increase in the number of researchers.

Although making our model dynamic goes beyond the scope of this article, it is clear that, if we were to use the dynamic framework of Young (1998), we would obtain a positive effect of market size on the balanced path growth rate. To see this, recall that Young (1998) assumes complete intertemporal knowledge spillovers. That is, the fixed cost required to improve the technology by a certain proportion is the same in each period, independently of the initial level of the technology. If, as we have shown, a larger market leads to more process innovation in a one-period static model, complete intertemporal knowledge spillovers imply that in a dynamic model it would lead to more process innovation every period.

To the extent that the existence of such a growth scale effect is viewed as an undesirable property of a model, our article suggests that it takes more than adding product innovation to process innovation to eliminate the effect. Instead, our article suggests the alternative approach of assuming incomplete spillovers proposed by Jones (1995), Kortum (1997), and Segerstrom (1998), known as semi-endogenous growth, is more plausible. Studying the balanced growth path properties of a dynamic model with Hotelling–Lancaster preferences, process and product innovation, and incomplete knowledge spillovers is clearly an issue for future research.

REFERENCES


