1309 EXAM REVIEW

1) $f(x) = x^{-1/2} - \frac{1}{4}x^3$

$f'(x) = -\frac{1}{2}x^{-3/2} - \frac{3}{4}x^2$

$f(x) = (3x+2)(4x-5)$

$f'(x) = (3x+2)(4) + (3)(4x-5)$

$= 12x + 8 + 12x - 15$

$f'(x) = 24x - 7$

$f(x) = \frac{3x+5}{x^2-3}$

$f'(x) = \frac{(x^2-3)(3) - (3x+5)(2x)}{(x^2-3)^2}$

$= \frac{3x^2 - 9 - 6x^2 - 10x}{(x^2-3)^2}$

$f'(x) = \frac{-3x^2 - 10x - 9}{(x^2-3)^2}$

2) $f(x) = 3(x^2-2)^4$

$f'(x) = 12(x^2-2)^3(2x)$

$f'(x) = 24x(x^2-2)^3$

$f(x) = (4x^2-4x+1)^{-1}$

$f'(x) = -1(4x^2-4x+1)^{-2}(8x-4)$

$f(x) = (3x)(2x^2+3)^{1/2}$

$f'(x) = 3x \cdot \frac{1}{2}(2x^2+3)^{-1/2}(4x) + 3(2x^2+3)^{1/2}$

$= 6x^2(2x^2+3)^{-1/2} + 3(2x^2+3)^{1/2}$

$= 3(2x^2+3)^{-1/2}(2x^2 + (2x^2+3))$

$f'(x) = 3(2x^2+3)^{-1/2}(4x^2+3)$

3) $f(x) = 3\ln(x+4)$

$f'(x) = 3\left(\frac{1}{x+4}\right)$

$f'(x) = 3/x+4$

$f(x) = 1 + e^x - e^{-2x}$

$f'(x) = e^x - e^{-2x}(-2)$

$f'(x) = e^x + 2e^{-2x}$

$f(x) = [\ln(x^2+3x)]^4$

$f'(x) = 4[\ln(x^2+3x)]^3\left(\frac{2x+3}{x^2+3x}\right)$

4) $f(x) = (e^{x^2}+3)^5$

$f'(x) = 5(e^{x^2}+3)^4 \cdot e^{x^2} \cdot 2x$

$f'(x) = 10xe^{x^2}(e^{x^2}+3)^4$

$f(x) = \frac{e^x}{e^x-1}$

$f'(x) = \frac{(e^x-1) \cdot e^x - e^x(e^x)}{(e^x-1)^2}$

$= \frac{e^x(e^x - 1 - e^x)}{(e^x-1)^2}$

$f'(x) = -e^x/(e^x-1)^2$

$f(x) = 3x^2\ln(2x)$

$f'(x) = 3x^2 \cdot \frac{2}{2x} + 6x\ln(2x)$

$f'(x) = 3x + 6x\ln(2x)$

5) a) $f_x(x,y) = 6x^2 + 6xy - 5y^2$

$f_y(x,y) = 3x^2 - 10xy - 30y^2$

b) $f_x(x,y) = \frac{4x-3y}{2x^2-3xy+y^2}$

$f_y(x,y) = \frac{-3x+2y}{2x^2-3xy+y^2}$

6) $f(x) = 7 + 11x - 6x^2$ $\quad x=2, y=5$

$f(2) = 7 + 22 - 24$

$= 5$

~~$f'(x) = 11 - 12x$~~

$f'(2) = 11 - 24 = -13$

$y - 5 = -13(x-2)$

$f(x) = \ln x$ At $x=1, y=0$

$f(1) = \ln 1 = 0$

$f'(x) = \frac{1}{x}$

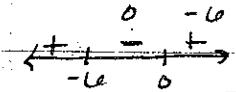
$f'(1) = 1/1 = 1$

$y = 1(x-1)$

7) a) $f(x) = x^3 + 9x^2 + 7$

$f'(x) = 3x^2 + 18x$

$0 = 3x(x+6)$



$\boxed{\text{inc } (-\infty,-6)(0,\infty) \quad \text{dec } (-6,0)}$

$\boxed{\text{local max } (-6,115) \quad \text{local min } (0,7)}$

b) $f(x) = \frac{x+2}{x-3}$ $\quad f'(x) = \frac{\overset{x-3-x-2}{(x-3)(1) - (x+2)(1)}}{(x-3)^2} = -5/(x-3)^2$

$\boxed{\text{dec } (-\infty,3)(3,\infty) \quad \text{no local extrema}}$

8) $f(x) = x^3 - 18x^2 + 10x - 11$

$f'(x) = 3x^2 - 36x + 10$

$f''(x) = 6x - 36$

$0 = 6(x-6)$

6

$\xleftarrow{\quad - \quad | \quad + \quad}$ 6

conc up $(6, \infty)$
conc down $(-\infty, 6)$
Infl. pt $(6, -383)$

$f(x) = 1 - 3x - x^3$

$f'(x) = -3 - 3x^2$

$f''(x) = -6x$

$0 = -6x$

0

$\xleftarrow{\quad + \quad | \quad - \quad}$ 0

conc up $(-\infty, 0)$
conc down $(0, \infty)$
Infl. pt $(0, 1)$

9) $\int (6x^{-4} - 2x^{-3} + 1)\,dx$

$= 6\frac{x^{-3}}{-3} - 2\frac{x^{-2}}{-2} + x + C$

$= \underline{-2x^{-3} + x^{-2} + x + C}$

$\int x(5-2x^2)^{-5}\,dx = \int u^{-5}(-\frac{1}{4}du)$

$u = 5 - 2x^2$

$-\frac{1}{4}du = -4x\,dx$

$= -\frac{1}{4}\,\frac{u^{-4}}{-4} + C$

$= \underline{\frac{1}{16}(5-2x^2)^{-4} + C}$

$\int (x - e^x)\,dx$

$= \underline{\frac{x^2}{2} - e^x + C}$

$\int (x^2 + 2x)e^{x^3+3x^2}\,dx = \int e^u(\frac{1}{3}du)$

$u = x^3 + 3x^2$

$\frac{1}{3}du = (3x^2 + 6x)dx$

$= \frac{1}{3}e^u + C$

$= \underline{\frac{1}{3}e^{x^3+3x^2} + C}$

$\int x^2(2x^3+1)^{1/2}\,dx$

$u = 2x^3 + 1$

$\frac{1}{6}du = 6x^2\,dx$

$\int u^{1/2}(\frac{1}{6}du)$

$= \frac{2}{3}\frac{1}{6}\,\frac{u^{3/2}}{3/2} + C$

$= \underline{\frac{1}{9}(2x^3+1)^{3/2} + C}$

$\int (4x^{-1} + \frac{1}{4}x)\,dx$

$= 4\ln|x| + \frac{1}{4}\frac{x^2}{2} + C$

$= \underline{4\ln|x| + \frac{1}{8}x^2 + C}$

10) $\int_1^2 (2x^{-2} - 3)\,dx = \frac{2x^{-1}}{-1} - 3x \Big|_1^2$

$= [\frac{-2}{2} - 3(2)] - [\frac{2}{-1} - 3]$

$= \boxed{-2}$

$\int_0^1 x(3x^2+2)^{1/2}\,dx$

$u = 3x^2 + 2 \quad x=1: u=5$

$\frac{1}{6}du = 6x\,dx \quad x=0: u=2$

$\int_2^5 u^{1/2}(\frac{1}{6}du) = \frac{1}{6}\,\frac{u^{3/2}}{3/2}\Big|_2^5$

$= \frac{1}{9}(5)^{3/2} - \frac{1}{9}(2)^{3/2}$

$= \boxed{.928}$

$\int_1^4 3x^{-1/2}\,dx = 3\frac{x^{3/2}}{3/2}\Big|_1^4$

$= 2(4)^{3/2} - 2(1)^{3/2}$

$= \boxed{14}$

$\int_1^2 (x+1)(2x^2+4x+4)^{-1}\,dx$

$u = 2x^2 + 4x + 4 \quad x=2: u=20$

$\frac{1}{4}du = (4x+4)dx \quad x=1: u=10$

$\int_{10}^{20} u^{-1}\frac{1}{4}du = \frac{1}{4}\ln u\Big|_{10}^{20}$

$= \frac{1}{4}\ln 20 - \frac{1}{4}\ln 10$

$= \boxed{.173}$

$\int_{-3}^{-1} (-3x^{-2} + x^{-1})\,dx = -3\frac{x^{-1}}{-1} + \ln|x| \Big|_{-3}^{-1}$

$= [3(-1)^{-1} + \ln|-1|] - [3(-3)^{-1} + \ln|-3|]$

$= \boxed{-3.099}$

$\int_0^3 x(x+1)^{1/2}\,dx$

$u = x+1, \ u-1 = x$

$du = dx$

$x=3: u=4$

$x=0: u=1$

$\int_1^4 (u-1)u^{1/2}\,du = \int_1^4 (u^{3/2} - u^{1/2})\,du$

$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\Big|_1^4$

$= [\frac{2}{5}(4)^{5/2} - \frac{2}{3}(4)^{3/2}] - [\frac{2}{5}(1)^{5/2} - \frac{2}{3}(1)^{3/2}]$

$= \boxed{7.733}$

11) $y = 5 - x^2$

$y = 2 - 2x$

$-1 \le x \le 3$

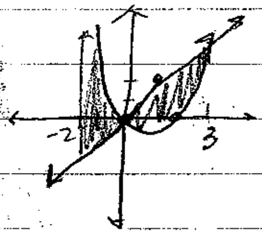
$\int_{-1}^{3} [(5-x^2) - (2-2x)]\,dx = \boxed{32/3}$

$-x^2 + 2x + 3$

$y = x^2 - x$

$y = 2x$

$-2 \le x \le 3$



$\int_{-2}^{0} [(x^2-x) - 2x]\,dx + \int_0^3 [2x - (x^2-x)]\,dx$

$x^2 - 3x \qquad 3x - x^2$

$\boxed{13.167}$

12) $C(x) = 1000 + 100x - 125x^2$

$C'(x) = 100 - 250x$

$C'(50) = -12{,}400$

cost decreasing by
\$12,400

$P(x) = 20x - .02x^2 - 320$

$P'(x) = 20 - .04x$

$P'(40) = 18.40$

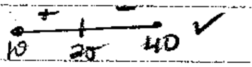
profit increasing
by \$18.40

13) $N(x) = -3x^3 + 225x^2 - 3000x + 17{,}000$
$10 \leq x \leq 40$

$N'(x) = -9x^2 + 450x - 3000$

$N''(x) = -18x + 450$

$0 = -18x + 450$



$x = 25$

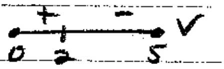
$25,000

$N(t) = 10 + 6t^2 - t^3 \quad 0 < t < 5$

$N'(t) = 12t - 3t^2$

$N''(t) = 12 - 6t$

$0 = 12 - 6t$



$t = 2$

In 2 yrs

14) x = # \$1 inc.

$R(x) = (\overset{\text{price}}{80 + x})(\overset{\text{rooms}}{300 - 3x})$

$R'(x) = (80+x)(-3) + 1(300-3x)$

$0 = -240 - 3x + 300 - 3x \qquad R'' = -6 < 0$ max

$6x = 60$

$x = 10 \qquad$ **price = $90**

x = # additional trees

$y(x) = (\overset{\text{trees}}{30 + x})(\overset{\text{lbs}}{50 - x})$

$y'(x) = (30+x)(-1) + 1(50-x)$

$0 = -30 - x + 50 - x \qquad y'' = -2 < 0$ max

$2x = 20$

$x = 10 \qquad$ **40 trees**

15) a) $P_x(x,y) = 140 - 8x + 2y$

$P_x(15,10) = 140 - 8(15) + 2(10)$

$= 40$

$P_y(x,y) = 200 + 2x - 24y$

$P_y(15,10) = 200 + 2(15) - 24(10)$

$= -10$

wedding cakes

b) $f_x(x,y) = 25x^{-.5}y^{.5}$

$f_x(250,125) = 25(250)^{-.5}(125)^{.5}$

$= 17.678$

$f_y(x,y) = 25x^{.5}y^{-.5}$

$f_y(250,125) = 25(250)^{.5}(125)^{-.5}$

$= 35.355$

capital

16) $4000 = 2000(1 + .12/4)^{4t}$

$2 = 1.03^{4t}$

$\ln 2 = 4t \ln 1.03$

$23.45 = 4t$

$t = 5.862$ yrs

$8000 = 5000e^{.096t}$

$1.6 = e^{.096t}$

$\ln 1.6 = .096t$

4.896 yrs $= t$

17) $120{,}000 = P\left[\frac{(1 + .10/4)^{4 \cdot 15} - 1}{(.10/4)}\right]$

$120{,}000 = P(135.991\ldots)$

$P = \$882.41$

Need \$5,000,000:

$FV = 190{,}000\left[\frac{(1 + .06/2)^{2 \cdot 10} - 1}{(.06/2)}\right]$

$= 190{,}000(26.870\ldots)$

$FV = \underline{5{,}105{,}371.15}$

yes

18) $12{,}000 = P\left[\dfrac{1-(1+.12/12)^{-12\cdot 5}}{(.12/12)}\right]$

$12{,}000 = P(44.955\ldots)$

$\underline{P = \$266.93}$

$PV = 2000\left[\dfrac{1-(1+.084/12)^{-12\cdot 20}}{(.084/12)}\right]$

$= 2000(116.076\ldots)$

$= \$\,\underline{232{,}152.01}$

19) $FV = e^{.05\times 40}\int_0^{40} 3000\, e^{-.05t}\,dt$

$= \underline{\$383{,}343.37}$

$FV = e^{.10\times 5}\int_0^{5} 10{,}000\, e^{.05t}\, e^{-.10t}\,dt$

$= e^{.5}\int_0^{5} 10{,}000\, e^{-.05t}\,dt$

$= \underline{\$72{,}939.17}$

20) $p = D(x) = 50 - .01x$ $\qquad \bar{p} = \$24$ $\qquad p = S(x) = 11 + .05x$

$24 = 50 - .01\bar{x}$

$-26 = -.01\bar{x}$

$\bar{x} = 2600$

$CS = \int_0^{2600} \underbrace{[50 - .01x - 24]}_{26-.01x}\,dx$

$= \$\,\underline{33{,}800}$

$24 = 11 + .05\bar{x}$

$13 = .05\bar{x}$

$\bar{x} = 260$

$PS = \int_0^{260} \underbrace{(24 - 11 - .05x)}_{13-.05x}\,dx$

$= \$\,\underline{1690}$