All-or-Nothing Payments*

Bo Chen†

January 30, 2012

Abstract

We develop a general principal-agent framework to study optimal incentive schemes where agents are hired to work on multiple heterogeneous and interdependent projects. The incentive schemes can be based on output measures, interpreted as the principal’s payoffs, as well as input measures, regarded as observation of some of the agents’ efforts. We identify that a unifying feature of the optimal incentive schemes, called all-or-nothing payments, arises in three natural scenarios of the general framework: unobservable inputs, verifiable inputs, and observable but unverifiable inputs. Our framework and results embed and generalize several previous studies on multitask principal-agent problems with a limited liability constraint.

Keywords: All-or-nothing payments; Moral hazard; Multitask incentive contracts; Verifiability.

JEL classification: C70, D23, D86, M52

1 Introduction

One of the central problems organizations typically face is how to design optimal incentive schemes in the presence of multiple aggregated and disaggregated performance measures.1 Additional complications that commonly arise in such complex contract design problems include verifiability issues of different measures, multiple tasks, and the possibility of relative performance incentive schemes with multiple agents. We

---

*I am grateful to two anonymous referees and an associate editor for various enlightening and constructive suggestions. I am also grateful to Hideshi Itoh, Patrick Schmitz, Larry Samuelson, Zaifu Yang, and Rui Zhao for helpful discussions, and to seminar and conference participants at GETA 2009, Kyoto University, and Yokohama National University for useful comments. All remaining errors are mine.

†Department of Economics, Southern Methodist University, 3300 Dyer Street, Suite 301, Umphrey Lee Center, Dallas TX 75275-0496 (email: bochen@smu.edu; tel.: +1 214 768 2715; fax: +1 214 768 1821).

1See Raith (2008) and references therein, for various illustrations and examples.
take up such issues and analyze the design of optimal incentive schemes in a general multitask agency framework where the principal employs several agents and observes multiple performance measures for each agent. In several natural contracting scenarios, arising from the availability and verifiability of the performance measures, we find that a common crucial feature linking all the optimal incentive compensations is a bonus scheme that rewards only a high level of performance on all the tasks an agent performs. We term such a feature as “all-or-nothing payments”.

Specifically, we consider a contracting environment between a principal and several risk-neutral agents with limited liability, each working on a project that consists of multiple activities or tasks. There are two key elements in our framework. First, the multiple (heterogenous) tasks for an agent are interdependent in the sense that the agent’s efforts in the multiple activities are complementary for the principal. Indeed, in reality, complementarities among the tasks may be the first reason for task clustering and may arise by sharing resources among tasks, the economy of scale or other informational or technological synergy. The second key element is the existence of multiple performance measures: The principal may compensate an agent on both verifiable output performance measures and input performance measures. The output measures may be profits, sales revenue, the security of an entire system of computer networks, while the input measures can range from verifiable information such as standard routines, working hours or service quality from customer satisfaction surveys, to subjective observation such as supervisors’ subjective evaluations, dedication or support for the organization, etc.

In such a multitask multi-agent framework, we characterize the principal’s optimal incentive schemes in three natural contracting environments: unobservable input measures, verifiable input measures, and unverifiable but observable input measures. We find that in all three optimal incentive schemes, the crucial element is an all-or-nothing payment scheme, where the principal rewards the agents only if an “all success” outcome (on both the input and output measures) is observed.

For illustration of the framework and the results on optimal incentive schemes, consider the problem where a business owner, who hires several salespersons, designs the best compensation scheme. Each salesperson’s job involves multiple activities which are complements for the owner’s overall profit: searching and landing new accounts, up-selling existing customers, technical support and customer service, etc. A salesperson’s compensations can be based on the profit he generates for the owner, and possibly on some observable input measures, such as hours worked, the number of new accounts landed, period reviews, etc. Our results provide a rationalization for the common use of an optimal compensation in a general environment: The optimal incentive scheme takes the form of a straight commission-like payment scheme — a salesperson is only paid a positive reward after a high profit level, as well as

\[2\text{See MacDonald and Marx (2001) for interesting real-world examples on such complementarity.}\]

\[3\text{This is equivalent to the contracting environment where input measures are observable but unverifiable, and a contract based on relative performance of the agents is not feasible.}\]
satisfactory input performance measures mentioned above.\textsuperscript{4}

This paper is most closely related to Laux (2001), Zhao (2008) and Chen (2010). Laux (2001), an important precursor to this study, first formally shows that in a multitask with limited liability context, there are gains from multitasking. The key insight is that task clustering allows to punish agents for failures on some tasks by lowering the rewards on successful tasks, thus slackening the limited liability constraint. One contribution of this paper is to extend Laux’s insight to a general setting where tasks/activities can be \textit{asymmetric and interdependent}.\textsuperscript{5} With a more general framework, Laux’s insight can also be understood and interpreted in a broader sense (see Section 4 for a detailed discussion).

Laux’s somewhat specific setting, however, does not allow for input monitoring. Analyzing a similar setting but with input monitoring, Zhao (2008) finds that the principal is strictly better off by conditioning the wage scheme only on the noisy output signals, as long as not all effort choices are observable, hence an \textit{“all-or-nothing monitoring”} result. Lifting a limitation on the incentive schemes, Chen (2010) shows that (in Zhao’s setting) with verifiable outputs and inputs, the optimal contract that exploits the risk-neutrality of the agent (with limited liability) to its full extent indeed uses both measures and entails a lower wage cost. Another contribution of this study is to extend the result in Chen (2010) to a general framework with asymmetric interdependent tasks and multiple performance measures.

Most importantly, this study unites the results in Laux (2001), Zhao (2008) and Chen (2010) by providing a framework that embeds and generalizes all these models. In particular, our framework permits to study what happens if input measures are observable but not verifiable. We show that the rent squeezing insight, which Laux formulated when inputs are not observable, not only extends to the case of observable and verifiable inputs (Zhao (2008) and Chen (2010)), but also to a situation where inputs are observable but not verifiable.

The remainder of the paper is organized as follows: Section 1.1 provides additional literature review. Section 2 introduces the model. Our results on the principal’s optimal contracts for the three cases mentioned previously are presented in Section 3. Section 4 contains concluding remarks. All proofs are relegated to an Appendix.

\section{1.1 Additional Related Literature}

This paper straddles two strands of the literature: multitask agency problems and relative performance incentive schemes. The first formal study on multitask agency problems was Holmström and Milgrom (1991), where (task-specific) linear piece rates

\textsuperscript{4}Admittedly, designing the best compensation plan for salespeople in reality is much more complicated than the environment considered in this paper and has been a highly controversial topic (see the review article Bergen, Dutta and Walker (1992)).

\textsuperscript{5}In our setting, the outputs are \textit{supermodular} and the effort costs are \textit{submodular} in the agents’ effort choices, which generalizes the settings in the papers above.
are studied and it is found that low-powered incentive schemes can be optimal when there are differences in measurement accuracy on the tasks. Bond and Gomes (2009) study a class of multitask agency problems with symmetric and independent tasks and continuous effort choices. With two contracting constraints (the budget constraint and the monotonicity constraint), the authors characterize the optimal incentive schemes, which are generally “fragile”, and a new source of allocative inefficiency across tasks is also identified. While some similarities exist, our setting and that in Bond and Gomes (2009) do not overlap completely as our setting allows asymmetric and interdependent tasks. Moreover, we consider the issues of verifiability and multiple performance measures, which are absent in Bond and Gomes (2009).

An additional complementary literature on multitasking, instead of taking task assignment as given, studies optimal task assignment/division. This includes the job design model in Holmström and Milgrom (1991), the project allocation issue in Laux (2001), optimality of task separation in Dewatripont and Tirole (1999), joint or individual accountability for performance in Corts (2007), and beneficial effects of labor division in Ratto and Schnedler (2008).

On the literature of relative performance incentive schemes, a classic reference is Lazear and Rosen (1981) who show that with risk-neutral agents, both rank-order incentive schemes and individual-based incentive schemes allocate resources efficiently. Subsequent studies have shown that correlation across agents in the randomness affecting output signals enables the principal to gain by basing an agent’s reward on other agents’ outputs. In another set of articles on contracting with multiple agents (Bhattacharya (1983), Malcomson (1984, 1986)), it was shown that an additional attractive feature of rank-order tournaments is their correction of a potential moral hazard of the principal when agents’ output performances are not verifiable. In characterizing the optimal incentive schemes with observable but unverifiable inputs (Section 3.3), we use a similar idea to design a relative performance contract that uses multiple performance measures with mixed verifiability and we identify another equivalence result. We postpone a more detailed discussion on comparisons with the

---

6The multitasking literature has somewhat evolved since the 1990s, mostly happening in accounting (e.g., Feltham and Xie (1994), Datar, Kulp and Lambert (2001)) and labor economics (e.g., Baker (2000, 2002), Schnedler (2008)). Other less related papers include Holmström and Milgrom (1994), Itoh (1991, 1994) and MacDonald and Marx (1991). Dewatripont, Jewitt and Tirole (2000) provide a survey on various contributions of multitask agency theory and applications.

7Specifically, with multiple tasks and a continuous effort choice on each task, an effort vector is incentive compatible if a local incentive constraint and an extremal incentive constraint hold simultaneously. Hence, small shocks on fundamentals (e.g., effort costs) may result in a total collapse of the agent’s effort. This “fragility” contrasts with the standard contracting problem where only local incentive constraint bind. Effort misallocation arises when the agent works only on a subset of tasks while the principal prefers the agent to “diversify” his effort choices among all the tasks.

8Raith (2008) studies the problem of designing optimal compensations when multiple performance measures exist, from an angle where the agent has private information on his productivity.

literature on relative performance measures to Section 3.3.

2 Model

Consider a situation where a principal is endowed with multiple identical and independent one-time projects. The principal can hire \( m \) agents (\( m \in \mathbb{N}, m \geq 2 \)), one for each project, to complete these projects. The basic elements of each principal-agent relationship are described as follows:

**Effort and Production:** Each project contains \( n > 1 \) heterogenous and possibly interdependent tasks, which are collected in the set \( N = \{1, 2, \ldots, n\} \). For each task, the agent makes the choice of an effort level, 0 or 1 (representing *shirk* and *work* respectively). For notational simplicity, denote a generic effort choice of the agent as \( E \subseteq N \), indicating that the agent works in all tasks in \( E \), and shirks in all tasks in \( N \setminus E \). Given this, an effort vector that features working (resp., shirking) in all tasks is simply \( N \) (resp., \( \emptyset \)).

Given effort vector \( E \subseteq N \), output signal \( y \), which can be interpreted as profits, sales revenues or customer satisfaction surveys, is generated stochastically according to the probability distribution \( P(y|E) \), where \( y \in Y \). \( Y \), being the set of all signals, is finite and contains at least two elements. In our setting, we allow the case where a separate output signal is available for each task, as well as the scenario where there is an aggregate output signal for all \( n \) tasks. We assume that distribution \( P \) has full support, or \( P(y|E) \in (0, 1) \) for all \( y \) and \( E \). In addition, conditional on effort choices, the output signals are independent across agents.

**Performance Measures:** The principal can use several performance measures to monitor the agent’s effort. The first is the standard *output-performance measure* — the output signal \( y \), which throughout the paper is assumed to be verifiable. Besides the imperfect output signals, an input-performance measure may also be available — the principal can costlessly and perfectly observe some, but not all, effort levels in the \( n \) tasks, where the number of such tasks (called *observable tasks* hereafter) is \( k \), \( k \in \{1, 2, \ldots, n-1\} \). For the analysis in the sequel, we consider cases where the observation of the \( k \) effort levels is verifiable (i.e., standard routines, administrative/clerical tasks or simply working hours), as well as cases where such observation has the nature of “I know it only when I see it” and is thus difficult and costly to be verified to a court (i.e., professionalism, dedication, or subjective evaluation of a supervisor). Hereafter, we will use terminology “output signals” to denote “output performance measures”, and “perfect observation on the \( k \) effort choices” to denote “input performance measures”.

**Payoffs:** In our setting, both the principal and the agent are risk-neutral. We also assume that the agent has a reservation utility level of zero and is protected by limited liability, i.e., payments from the agents to the principal are never feasible.\(^{10}\)

\(^{10}\)The case of a generic reservation utility level \( u \) can be easily adapted without qualitatively
For simplicity, we assume that the principal’s objective is to motivate the agents to exert effort on all $n$ tasks at a minimal wage payment.\footnote{Although natural and important in our setting, this assumption also implicitly implies that effort costs be invariably small compared to the project profits. In addition, maintaining this assumption prevents us from analyzing issues such as the agent’s effort mis-allocation (as in Holmstrom and Milgrom (1991) and Bond and Gomes (2009)). If we adopt a standard cost-and-benefit analysis for the principal, then implementing partial effort might be optimal, but then one might argue why the principal would assign all $n$ tasks to a single agent (such an argument is admittedly specific for our setting and not useful in general). We discuss on this issue further in Section 4.} Given a wage $w$, interpreted as incentive payments, the agent’s payoff is

$$u(w, E) = w - C(E),$$

where $C(E)$ represents the agent’s cost of exerting effort $E$, and $C: 2^N \rightarrow \mathbb{R}$. We assume that the cost function $C(E)$ is submodular, i.e., for any $i, j \in N$, $i \neq j$, and $E \subseteq N \setminus \{i, j\}$,:\footnote{For a function defined on a product of ordered set, supermodularity is equivalent to increasing differences (Vives (2000)). An alternative but equivalent definition is $P(s|E) + P(s|E') \leq P(s|E \cap E') + P(s|E \cup E')$, for all $E, E' \subseteq N$.}

$$0 < C(E \cup \{i, j\}) - C(E \cup \{j\}) \leq C(E \cup \{i\}) - C(E).$$

(1)

We normalize the cost of shirking in all tasks to be zero, or $C(\emptyset) = 0$. Our specification of $C(E)$ allows for cases where costs in exerting effort in different tasks are asymmetric or non-linear, subsuming the symmetric and linear effort cost specification in Laux (2001) and Zhao (2008) as a special case. Submodularity of $C(E)$ can be intuitively interpreted as that efforts are complements for each agent as working in one task helps reducing the cost of exerting effort in another task.

**Assumptions on $P(y|E)$:** Denoting the signal profile that features an overall success as $s$, the output function $P(s|E)$ is assumed to be supermodular for any $E \subseteq N$. Formally, the function $P(s|E)$ is supermodular in $E$ if and only if

$$P(s|E \cup \{i, j\}) - P(s|E \cup \{j\}) \geq P(s|E \cup \{i\}) - P(s|E) \geq 0,$$

(2)

for all $i, j \in N$, $i \neq j$, and $E \subseteq N \setminus \{i, j\}$. In our setting, “the function $P(s|E)$ having increasing differences” is equivalent to $P(s|E)$ being supermodular.\footnote{Notice that supermodularity is only imposed on $P(s|E)$ and not on $P(y|E)$ for $y \neq s$. Importantly, our specification of $P(s|E)$ allows for symmetric and independent tasks (as in Laux (2001), Bond and Gomes (2009), and Zhao (2008)), as well as asymmetric and interdependent tasks. Observe that supermodularity of $P(s|E)$ can be intuitively interpreted as that the agent’s effort levels in the tasks are complements for affecting our results.}

In Bond and Gomes (2009), the principal’s output function can be concave, linear or convex in the number of successful tasks. However, the tasks are symmetric and importantly independent, in the sense that an effort level in one task does not affect the probability of success of another task.
the principal — a high effort in one task, while inducing success in that task more likely, also increases the probability of success of another task, ceteris paribus.

In addition to the supermodularity condition in (2), we assume that signal $s$ is also the state with the highest likelihood ratio for the probability functions $P(y|N)$ and $P(y|K)$, where $K \subseteq N$ denotes the effort choice where the agent only works in all $k$ observable tasks:

$$(i) \quad \frac{P(s|N)}{P(s|\emptyset)} > \frac{P(y|N)}{P(y|\emptyset)}, \quad \text{and} \quad (ii) \quad \frac{P(s|N)}{P(s|K)} > \frac{P(y|N)}{P(y|K)} \quad \text{for all } y \in Y, y \neq s. \quad (3)$$

The maximum likelihood ratio assumption is common in the literature (e.g., Innes (1990), Laux (2001), Zhao (2008) and Bond and Gomes (2009)). It is worth noting that we typically do not require both $(i)$ and $(ii)$ in (3) to hold simultaneously:$^{15}$ When the principal only uses output signals in a contract, the relevant binding incentive constraint is shirking from $N$ to $\emptyset$ — in this case, $(i)$ should hold so that a positive wage is paid only after output signal $s$; On the other hand, when both output signals and effort observation in the $k$ observable tasks can be used to motivate effort $N$, the relevant binding constraint is shirking from $N$ to $K$ (this is true because of (1) and (2)), in which case $(ii)$ should hold so that output signal $s$ is again the most informative signal in detecting deviation. Our results (except Proposition 3 with wage comparisons) can thus be obtained with a weaker version of (3), namely, the maximum likelihood ratios are obtained at different output signals.

Finally, we describe the time and information structure in the principal’s contracting problem. The principal first proposes the wage schemes to the agents, who can then accept or reject the offer(s). If at least one agent rejects, the game ends. Otherwise, the agents simultaneously make effort choices. The resulting noisy output signals and the effort levels on the observable tasks are observed, along with the implied wage payoffs to the agents. We assume that the proposed wage schemes and the realized wage payments to the agents are verifiable.

3 All-or-Nothing Payments in Optimal Contracts

We now characterize the principal’s optimal contracts. Three natural scenarios are considered in the characterization: (1) unobservable effort observation (on the $k$ tasks), (2) observable and verifiable effort observation, and (3) observable but unverifiable effort observation. For the first two cases, it suffices to consider individual optimal contracts, and we consider an optimal contract that is based on the agents’ relative performances in the last case. Importantly, we show that a unifying feature that ties all the optimal contracts together is an all-or-nothing payment scheme.

---

$^{15}$We present a two-task example in Section 3.2 to further illustrate the two conditions (2) and (3). In particular, conditions $(i)$ and $(ii)$ in (3) can arise simultaneously.
3.1 Optimal Contract with Unobservable Inputs

Consider first the case where the input measures are not observable to the principal, hence only the noisy output signals are relevant in designing the optimal contract. Notice that this case is equivalent to the scenario where the observation on the $k$ effort choices is observable but unverifiable when the principal can only offer individual contracts.

The principal’s contracting problem is to minimize the expected wage payment, based only on the output signals, subject to the limited liability and the incentive constraints so that the agent chooses effort $N$, as described by $(P)$:

$$\max_{w(y)} - \sum_{y \in Y} P(y|N) w(y)$$  

$$(P) \quad w(y) \geq 0, \forall y \in Y; \quad (LL) \quad \sum_{y \in Y} P(y|N) w(y) - C(N) \geq \sum_{y \in Y} P(y|E) w(y) - C(E), \forall E \subseteq N. \quad (IC)$$

Here $w(y)$ is the wage payment given output signal $y$; $(LL)$ the limited liability constraint; and $(IC)$ the incentive constraint indicating that the agent prefers effort $N$ to any other effort. The main difficulty in solving $(P)$ lies in that the incentive constraint $(IC)$ contains a total of $(2^n - 1)$ individual constraints, representing various ways that the agent can deviate from the effort $N$. To circumvent the difficulty, we first consider a reduced program where $(LL)$ is replaced by an extremal deviation constraint where the agent prefers $N$ to $\emptyset$. We then show that the same optimal scheme solves both program $(P)$ and the reduced program. Proposition 1 presents the optimal contract.

**Proposition 1** Suppose that the effort observation on the $k$ tasks is not available. The principal’s optimal contract is to offer the agent wage scheme $\bar{w}(y)$, which solves program $(P)$:

$$\bar{w}(y) = \begin{cases} 
\frac{C(N) - C(\emptyset)}{P(y|N) - P(y|\emptyset)}, & \text{if } y = s; \\
0, & \text{otherwise.}
\end{cases}$$

The intuition of Proposition 1 is as follows: Given that the likelihood ratio $\frac{P(y|N)}{P(y|\emptyset)}$ is maximized when $y = s$, signal $s$ is hence the most informative state in detecting shirking from “N” to “∅” — here, as mentioned before, we only require condition (i) in (3). The principal should then pay the agent a positive amount only after seeing signal $s$. Given such a wage scheme, the agent’s payoff from an effort profile $E$ is:

$$u(\bar{w}, E) = \frac{C(N) - C(\emptyset)}{P(s|N) - P(s|\emptyset)} P(s|E) - C(E),$$

This is a standard approach in the literature (Laux (2001), Zhao (2008) and Bond and Gomes (2009)). We follow Bond and Gomes (2009) in calling this an extremal incentive constraint.
which is supermodular in $E$, or the agent views the effort choices in different tasks as complements under $\tilde{w}(y)$ in (4). As a result, the agent’s goal is perfectly aligned with that of the principal and exerting effort in all $n$ tasks is hence a best response.

As mentioned, the rent squeezing insight and the all-or-nothing payment scheme being optimal are first obtained in Laux (2001) in a similar multitask agency setting with symmetric independent tasks and constant linear effort costs. Proposition 1 generalizes these results by showing that the interesting features carry over to the optimal contract in a more general multitask contracting environment where tasks can be interdependent and asymmetric.

### 3.2 Optimal Contract with Observable and Verifiable Inputs

We now turn to the principal’s optimal contracting problem when both the output signals and effort observation on the $k$ tasks are observable and verifiable. Specifically, the principal’s least cost contract, which can now explicitly depend on both the output signals and the verifiable effort observation, solves program $(P')$:

$$\begin{align*}
\max_{w(y)} \quad & -\sum_{y \in Y} P(y|N) w(y) \\
\text{s.t.} \quad & w(y) \geq 0, \quad \forall \ y \in Y, \quad (LL) \\
& \sum_{y \in Y} P(y|N) w(y) - C(N) \geq \sum_{y \in Y} P(y|K) w(y) - C(K), \quad (IC_1) \\
& \sum_{y \in Y} P(y|N) w(y) - C(N) \geq 0. \quad (IC_2)
\end{align*}$$

Recall that $K$ is the effort choice where the agent works only in the $k$ observable tasks. The incentive system resembles a forcing contract where the agent is punished severely (a zero wage) if he shirks in any of the $k$ tasks, which is feasible as the effort observation in the $k$ tasks is now verifiable. Moreover, it suffices for the principal to deter the deviation of shirking in all $(n - k)$ unobservable tasks $(IC_1)$, for a similar reason as in Proposition 1. As $(IC_1)$ might be binding, we only require condition $(ii)$ in (3) to hold for this section. Condition $(IC_2)$ ensures a non-negative expected wage payment for the agent when he chooses effort $N$.

The principal’s optimal contract derived from the contracting problem $(P')$ is presented in Proposition 2.

**Proposition 2** Suppose that both the output signals and the effort observation on observable tasks are verifiable. The principal’s optimal contract with both performances is to offer the agent a positive wage $\tilde{w}(y)$ only if $y = s$ and the agent works in all $k$ observable tasks, where

$$\tilde{w}(s) = \begin{cases} 
\frac{C(N) - C(K)}{P(s|N) - P(s|K)}, & \text{if } \frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}, \\
\frac{C(N)}{P(s|N)}, & \text{if } \frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}. 
\end{cases}$$

\(^{17}\) Also see Proposition 1 in Zhao (2008) for a similar result, but with a different proof.
In the optimal contract with verifiable inputs, the principal uses the harshest punishment whenever she observes a shirk in an observable task: a zero wage for all y’s. When the agent indeed works in all k observable tasks, we distinguish the following two cases in obtaining the optimal wage scheme:

First, if it holds that $\frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}$, which happens when the output signals are noisy or not very correlated, or k is small, then only the incentive constraint ($IC_1$) is binding: Since now monitoring using both output signals and observation on the k tasks is not effective or precise enough, the agent has to receive some positive rent so as to be motivated to exert effort in all n tasks.

Second, if we have $\frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}$ (when the output signals are precise or highly correlated, or k is large), then incentive constraint ($IC_2$) is the only relevant incentive constraint. In this case, the output signals and the perfect observation in the k observable effort choices are jointly powerful enough to incentivize the agent to exert effort in all n tasks at a zero rent.

Observe that constraint ($IC_2$) in ($P'$) can also be regarded as the agent’s participation constraint. This constraint is not binding in Laux (2001) or Zhao (2008). In our general multitask setting, however, this condition may be binding as intuitively the rent left to the agent can be pushed very low (indeed, zero) with the additional verifiable effort observation and interdependent tasks, which generates informational and/or technological synergy among the tasks.

As described, multitask agency problems with partial effort observation have also been analyzed in Laux (2001), Zhao (2008) and Chen (2010), with the assumption of symmetric independent tasks with constant linear costs. Laux (2001) considers a scenario where the agent works on N type A projects (with verifiable noisy output signals only) and M type B projects (with verifiable perfect effort observation only) with independent tasks. Proposition 2 is more general in several aspects: First, our principal observes the agent effort choices in the k observable tasks, which also affect the overall output signals in our setting. In particular, Proposition 2 applies to scenarios where there is only an aggregate output signal for all (M + N) tasks. This is important for cases where the output signal (e.g., sales revenues) is generated by the agent’s efforts in an integrated way such that it is difficult to single out the specific “contribution” of each individual task. Second, our setting allows for asymmetric and interdependent tasks. As in Chen (2010), Proposition 2 can also be applied to show that if the principal is not restricted to using a forcing contract as in Zhao (2008), then the principal is better off under the optimal wage scheme $\tilde{w}(y)$ in (5) than the wage scheme $\tilde{w}(y)$ (in (4)), which only depends on the output signals. The following corollary illustrates this:

**Corollary 1** The principal’s expected wage payment in the optimal contract with mixed performances ($E\tilde{w}$) is lower than that in the optimal contract with output performance ($E\tilde{w}$): $E\tilde{w} \leq E\tilde{w}$. In particular, $E\tilde{w} < E\tilde{w}$ holds under one or both of the following conditions: (a) $\frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}$ and (b) strict supermodularity of $P(s|E)$.
or strict submodularity of $C(E)$.$^{18}$

Two brief remarks are in order for Corollary 1: First, conditions (a) and (b) can be intuitively interpreted as that there is indeed informational or technological synergy among the tasks or between the $k$ observable tasks and the $(n-k)$ unobservable tasks. Hence, the optimal contract with mixed performances is strictly better. Second, at an intuitive level, the simple wage comparison result is not surprising and comes from the fact that in a moral hazard setting, additional informative signals enable the principal to better address the agent’s incentive problem. This is also consistent with the standard results in the previous moral hazard literature.$^{19}$

We now present a two-task example to further illustrate our setting and our results obtained so far.

**An Example:** Consider a one-agent setting where the project has two identical but interdependent tasks. For each task, the agent chooses $C$ (work) or $D$ (shirk), and for expositional convenience, let effort costs be symmetric and linear: The cost for work on a single task is $c > 0$ and shirk is costless. Suppose that for each task $i \in \{1, 2\}$, the output signal $y_i$ takes two possible values, $y_i \in Y = \{s, f\}$, representing “success” and “failure” respectively, and for both tasks, we have $P(s|C) = P(f|D) = q \in \left(\frac{1}{2}, 1\right)$, indicating that for each task, output signal $s$ (resp., $f$) occurs more likely when the agent works (resp., shirks) on the task. We further assume that the (linear) correlation between the two output signals $y_1$ and $y_2$ is always $\rho$, which can be interpreted as the spillover effects of an effort in a task has on the outcome of the other task. Finally, we assume that in addition to $y_1$ and $y_2$, the principal can also observe the second effort choice but not that in the first task.

We first evaluate the supermodularity condition (2) and the maximum likelihood condition (3). Note that by the above specification, the probability distribution $P(y|E)$ can be expressed as ($E \in \{CC, CD, DC, DD\}$ and $y \in Y^2$):

\[
\begin{array}{cc}
\begin{array}{cc}
 s & f \\
 q^2 + \rho q (1-q) & (1-\rho) q (1-q) \\
 (1-\rho) q (1-q) & (1-q)^2 + \rho q (1-q) \\
 P(y|CC) & P(y|CD)
\end{array}
\end{array}
\]

Fig 1. Signal Distributions Conditional on $E = CC$ and $E = CD$.

The output signal distributions $P(y|DC)$ and $P(y|DD)$ can be defined analogously. Given $P(y|E)$, one can readily verify that if $q > \frac{1}{2}$ and $\rho \geq 0$, strict supermodularity

$^{18}$Strict supermodularity of $P(s|E)$ (resp., submodularity of $C(E)$) holds if inequality (2) (resp., inequality (1)) is strict for all pairs $i, j$ in $N$. Clearly, this is stronger than necessary for the purpose of Corollary 1, which remains valid if such inequality in (1) or (2) holds for at least one pair of tasks. We present condition (b) in its current form only for expositional convenience.

of $P\left(s|E\right)$ and the maximum likelihood ratio condition hold.\textsuperscript{20}

We next consider optimal individual contracts in this simple setting. The principal’s optimal expected wage payments in the optimal output-based contract and the optimal contract with mixed performances can be calculated as, respectively,

$$Ew(y) = \frac{2c \cdot P\left(s|CC\right)}{P\left(s|CC\right) - P\left(s|DD\right)}$$

and $E\tilde{w}(y) = \begin{cases} 
\frac{c \cdot P(s|CC)}{P(s|CC) - P(s|DC)}, & \text{if } \frac{P(s|CC)}{P(s|DC)} \leq 2 \\
2c, & \text{if } \frac{P(s|CC)}{P(s|DC)} > 2
\end{cases}$.

Notice that $E\tilde{w}(y) > E\tilde{w}(y)$ given strict supermodularity of $P\left(s|E\right)$. In addition, if $\frac{P(s|CC)}{P(s|DC)} > 2$, equivalently $(1 - \rho)(1 - q) < \frac{1}{3}$, the optimal mixed-performance contract specifies an expected wage $2c$. This implies that if the synergy between the output signals is large enough ($\rho$ close to 1) and/or if output signals are precise enough ($q$ close to 1), the optimal mixed-performance contract induces effort $E = CC$ at a zero rent for the agent.

### 3.3 Optimal Contract with Observable and Unverifiable Inputs

We now analyze optimal incentive schemes when the principal observes both performance measures, but only the noisy output signals are verifiable. In such a scenario, the observation on the $k$ effort choices cannot be used in an individual contract in a self-enforcing way: When incentive schemes depend on unverifiable performance measures, a moral hazard problem arises — The principal will always ex post misrepresent an agent’s performance to reduce final wage payments, as such misrepresentation cannot be verified and denied by a court. Such ex post opportunistic behavior from the principal would render the contract in Proposition 2 infeasible.

The literature has put forward two main approaches where such observable but unverifiable performance can still be exploited effectively. The first approach is to let the two parties arrange their transactions repeatedly so as to design a self-enforcing relational contract: If both parties engage in repeated transactions, myopic behavior such as the one mentioned above will be costly, as each party knows such behavior will be detected and punished in the future.\textsuperscript{21} This approach is clearly not feasible in our one-shot setting. The other approach is found in the literature of moral hazard with multiple agents, which suggests that when interacting with multiple agents, the principal can design a contract that is based on the agents’ relative performances (Bhattacharya (1983) and Malcomson (1984, 1986)). If the incentive scheme is based on the rankings of the agents’ performances, the potential adverse incentives of the

\textsuperscript{20}Specifically, we have $P\left(s|CC\right) - P\left(s|E\right) \geq P\left(s|E\right) - P\left(s|DD\right)$, where $E = DC$ or $CD$. Moreover, $\frac{P(s|CC)}{P(s|DD)} \geq \frac{P(s|CC)}{P(s|DD)}$ and $\frac{P(s|CC)}{P(s|DC)} \geq \frac{P(s|CC)}{P(s|DC)}$ hold for all $y \in Y^2$. In particular, condition “$q > \frac{1}{2}$ and $\rho \geq 0$” is imposed mainly due to the maximum likelihood condition. Supermodularity in this setting merely requires “$1 \geq 4q(1 - q)(1 - \rho)^2$”.

\textsuperscript{21}See Kvaløy and Olsen (2009) and references therein on this line of research.
principal disappears as the total compensation paid to the agents is independent of the final performance outcome, leaving no financial gain for the principal to falsify reporting the performance rankings. Following this insight, we design a relative performance contract. Notice that our relative performance contract importantly employs both the verifiable output signals and the unverifiable effort observation, which is different from usual rank order contracts in the literature.

We first describe the design of the relative performance contract, which specifies a game for the agents. The key issue of the design lies in carefully specifying the agents’ payoffs in the game so that the contract engenders a Nash equilibrium where the agents choose high effort in all n tasks. First, given our setting with risk-neutral agents, the principal can without loss of generality specify two rewards in the relative performance contract, w and l, where w is the reward for the winner(s) and l for the loser(s). With the limited liability constraint, we can hence set l = 0. The incentive scheme is then designed in two steps as follows:

1. Agents with output signal s are first singled out as potential recipients of the reward w. As long as there are at least two agents whose realized output signal profiles are s, the principal has to incur a wage payment w (the principal pays nothing if there is at most one agent with output signal s).

2. Among the agents with output signal s, prize w is awarded to the agent with the “best” effort choice in the k observable tasks — “best” in the sense of exerting effort on the largest number among the k tasks. In case of a tie, the winner of the prize w is chosen with equal probability.\(^{22}\)

The principal’s problem here is to find the minimal reward w so as to ensure the existence of a Nash equilibrium where all agents choose effort N. This is described by the following program (\(\mathcal{P}^R\)):

\[
\begin{align*}
\max_w & -w \\
\text{s.t. } U_i(N; N, \ldots, N; w) & \geq 0, \forall i \ (IC') \\
U_i(N; N, \ldots, N; w) & \geq U_i(E; N, \ldots, N; w) \ \forall i, \forall E \subseteq N, \ (IC^w)
\end{align*}
\]

where \(U_i(E; N, \ldots, N; w)\) is agent i’s expected utility given w and effort profile \((N, \ldots, N)\) of the other agents, when i chooses effort E,

\[
U_i(E; N, \ldots, N; w) = \left\{ \begin{array}{ll} 
\sum_{j=1}^{m-1} \frac{P(s|E)}{j+1} P(s|N)^j (1 - P(s|N))^{m-j-1} \cdot w - C(E), & \text{if } K \subseteq E, \\
0, & \text{otherwise,}
\end{array} \right.
\]

\(^{22}\)With risk-neutral agents, one can equivalently design the scheme by giving the winning agents equal shares of the prize w in case of a tie.
where $m$ is the number of agents ($m \geq 2$, $m \in \mathbb{N}$) and “$K \subseteq E$” implies that when choosing $E$, agent $i$ works at least in all the $k$ observable tasks. In deriving $U_i(E; N; \cdots; N; w)$, notice that agent $i$, given $(N; \cdots; N)$, receives positive payoffs if and only if (1) $i$ works at least in the $k$ observable tasks, and (2) $i$’s realized output signal is $s$, together with at least one other agent.

We now present the key result of this subsection: the optimal relative performance contract solving $(\mathcal{P}^R)$, again featuring all-or-nothing payments, is as good as the optimal individual contract identified in Proposition 2 for the principal:

**Proposition 3** Assume that only the output signals are verifiable. Then for the principal, the optimal relative performance contract solving $(\mathcal{P}^R)$ is ex ante equivalent (in expected wage payments) to the optimal individual contract $(\hat{w}_1(y), \hat{w}_2(y))$ in (5), the latter being only feasible when both the output signals and the effort observation on the $k$ tasks are verifiable.

We present several remarks for Proposition 3:

First, in our setting, when effort observation is unverifiable, an enforceable optimal individual contract is the one characterized in Proposition 1. Together with Corollary 1, Proposition 3 shows that a contract based on relative performance evaluations can indeed outperform optimal individual contracts when verifiability of direct effort observation is an issue. This is consistent with the previous literature on that rank-order contracts can correct the moral hazard issue of the principal. The novel aspect of Proposition 3 is to show that in a general multitask setting with multiple performance measures, some being verifiable, a relative performance scheme can be usefully adapted to employ the verifiable aspects of the performances, so as to decrease the principal’s expected incentive payments.

Following this point, Proposition 3 suggests one potential motivation for common practices such as outstanding employee awards frequently used in organizations: Some aspects of employees’ effort are observable, but may also be subjective and difficult to verify such as professionalism, dedication to the job, support for the organization, etc. In such environments, a simple and yet effective way to employ such information is augmenting the individual performance scheme with relative performance evaluations so that incentive payments depend on both the absolute individual performances and an ordinal ranking of the unverifiable and subjective evaluations. Such a scheme is self-enforcing and cost effective to the principal, especially when a strong synergy exists among the tasks and the observable effort is important for the principal.

Second, the crucial feature of our relative performance scheme is that the principal pays reward $w$ if and only if there are two or more agents having output signal $s$, and such a payment is made independently of the agents’ effort choices in the $k$ observable tasks. While identifying two or more output signal $s$ is verifiable and feasible in our setting, such a specification may seem awkward. The reason for such a specification is the following: The optimal contract in Proposition 2 is especially effective when $P(s; N) > C(N)$ and $C(K)$, as the agent is left with zero rents. If one were to specify “$w$ will
be awarded when there is at least one agent with output $s^i$, every agent would then choose no effort at all since effort $\emptyset$ always yields a small but positive payoff, give our full-support assumption on $P(s|E)$. Notice that the possibility of such a profitable deviation under zero rents also implies that our relative performance scheme dominates the scheme with paying a reward regardless of output signals, i.e., when output signals are unverifiable (also see our fourth remark below). One possible and natural interpretation of our specification, however, is that the principal can stipulate in the contract that whenever the prize $w$ is awarded, \textit{at least two agents will be chosen as the recipients}. This is indeed commonly seen in practice.

Third, evidently, our relative performance contract is not the unique contract to implement the result of Proposition 2: Since the agents choose effort simultaneously, the key incentive issue is that effort $N$ a best response for each agent, if all the other agents choose $N$. As our agents are risk-neutral, one can consequently design different relative performance schemes to achieve the same outcome (for example, one can design a scheme with multiple levels of non-zero prizes). Our relative performance contract is, in our opinion, attractive as it is simple and it achieves the best possible outcome for the principal in our setting.

Fourth, verifiability of the output signals is important for the equivalence result in Proposition 3. Indeed, if both output signals and perfect input observations are unverifiable, one can similarly construct a relative performance scheme that is based on ordinal rankings of the agents’ performances, but the principal now has to pay reward $w$ regardless of the final outputs of the agents. One can show that the corresponding rank order contract is dominated by the one characterized in Proposition 3 in that the principal pays a higher expected reward when the outputs are unverifiable. This stands in contrast to Lazear and Rosen (1981), but is consistent with the findings in Green and Stokey (1983) on that a relative performance incentive scheme may use the available information in a rather crude way. For detailed analysis on this, see the working paper Chen (2011).

Finally, it is important to note that one potential problem of our relative performance contract is that our contract design does not necessarily implement the effort profile $(N, \cdots, N)$ uniquely: there could be other Nash equilibria where the agents exert effort in some or none of the $n$ tasks. This issue is not new and has been documented in the literature. Our general framework, while elegantly characterizes a

\footnote{Observe that in this case, $U_i(\emptyset; N, \cdots, N; w) = \sum_{j=1}^{m-1} \frac{P(s|\emptyset)}{\binom{m-1}{j+1}} P(s|N)^j (1 - P(s|N))^{m-j-1}$, $w - C(\emptyset)$, which is always positive given the full-support assumption and $C(\emptyset) = 0$.}

\footnote{In Green and Stokey (1983), in the absence of a common shock, tournaments are dominated by individual contracts as they introduce extra randomness in the (risk-averse) agents’ payments which now depend on other agents’ idiosyncratic shocks. In our setting with risk-neutral agents and limited liability, rank-order contracts are dominated as they ignore some cardinal feature of output signals. While the two settings are not completely comparable, both results illustrate that rank-order contracts make inefficient use of information.}

\footnote{See Mookherjee (1984) and Demski and Sappington (1984) for various agency examples in correlated environments on this multiplicity problem. Ma, Moore and Turnbull (1988) and Ma}
general environment for all-or-nothing payment schemes, is mathematically inconvenient to fully describe the complete set of equilibria: The supermodular and submodular specifications are meant to capture a general setting where only the extremal incentive constraint is binding, so as to induce an all-or-nothing payment result. In proving Proposition 3, a similar phenomenon arises, but now the optimal reward \( w \) carries a non-trivial probability component (constant \( \Delta \) in the proof). To characterize the full set of Nash equilibria under this reward, one inevitably encounters the complex task of evaluating the change in probability (compared to \( \Delta \)) of obtaining \( w \) relative to the change in effort costs, where our non-separable specifications give us little hope in disentangling such technical complexity.

One possible argument for the multiplicity issue is that in our symmetric setting, it suffices to treat “everyone exerting full effort” as a focal point. However, such an argument may also rest on shaky ground without a corresponding equilibrium selection theory.\(^{26}\) This being said, a possible approach to mitigate the problem of agents coordinating on a wrong equilibrium is to introduce discrimination among the ex ante identical agents: The principal divide the reward evenly among the agents only if all the agents work in the observable tasks and obtain output \( s \). When agents coordinate on partial efforts in the \( k \) tasks (and obtain output \( s \)), the principal gives reward \( w \) to a single agent (and the specification of such agents can even depend on the specific mis-coordination so that no single agent regards (partial) shirking as a dominant strategy).\(^{27}\)

4 Conclusion and Discussion

Designing the right incentive plans for organizations is complicated by the existence of multiple performance measures, multiple activities involved in a specific job, and the possibility of using relative performance schemes when there are multiple employees. This paper provides a general multitask agency framework to analyze the principal’s optimal incentive schemes with the presence of multiple agents and multiple performance measures with different verifiability issues.

In our framework, the principal’s output is supermodular and an agent’s effort cost is submodular in the effort choices of the agent. Such a general specification applies to scenarios where an agent’s various activities contribute to a single output in an integrated and complementary way so that identifying the specific contribution

\(^{26}\)Indeed, as remarked by a referee, there is probably little hope to argue in favor of the equilibrium if agents obtain zero rents in the equilibrium (when \( P(s|N)/P(s|K) > C(N)/C(K) \)).

\(^{27}\)Such a discriminatory policy may seem arbitrary, but it could be a motivation for ranks and hierarchies among (identical) agents. See Winter (2004) for an interesting illustration of this idea.
of effort on an individual activity to the principal’s overall objective is difficult. More importantly, our framework enables us to systematically vary the characteristics of the effort observation in analyzing the principal’s optimal incentive schemes.

Under this framework, we characterize the principal’s optimal contracts in three natural scenarios: unobservable inputs, observable and verifiable inputs, and observable but unverifiable inputs. A common feature arising in the optimal contracts is an “all-or-nothing payment” incentive scheme. Our result contributes to the literature on multitasking agency problems with limited liability by embedding and generalizing several existing models (Laux (2001), Zhao (2008) and Chen (2010)).

Although our results are obtained in a fairly general framework, a remaining natural question is then “how far the proposed set-up resembles a ‘maximal domain’ for all-or-nothing payments?” It is without doubt that risk-neutrality (with limited liability) of the agents is crucial for our results so that the risk-sharing issue is irrelevant. What is less clear is the role of supermodularity of profits and submodularity of costs. We divide our discussion on this in two parts.

First, with respect to the advantage of combining tasks (though this is not our main focus and as mentioned, has already been pointed out in Laux (2001)), supermodularity and submodularity are not both necessary to realize “gains from multitasking”: Supermodularity of profits and submodularity of costs make the tasks natural complements for both the agent and the principal. It is hence intuitive to combine the tasks in an optimal individual contract. However, it is clear from the proof of Proposition 1 that if either supermodularity or submodularity (but not both) fails, as long as the synergy or complementarity of profits (resp., effort costs) is strong enough to dominate the countering effect of decreasing returns to scale in effort costs (resp., profits), then the same result arises. On this aspect, though it seems that we have merely provided a more general framework for achieving gains from multitasking, our result makes it clear that complementarities, informational or technological, of the tasks are indeed crucial for combining tasks to a single manager — the previous literature identifies the informational synergy for the principal, while this paper identifies a general form of complementarity for both parties.

Second, from a technical point of view, supermodularity of profits and submodularity of effort costs are again not necessary for an all-or-nothing payments result: When either condition (or both) fails, it may no longer be true that only the extremal incentive constraint is binding. Nevertheless, all-or-nothing payments may still arise when the principal pays the agent a positive wage only in the state of outcome that is the most informative in detecting shirking from $N$ to the preferred effort of the agent (i.e., the corresponding binding incentive constraint). A similar argument goes

---

28Our result is hence not a consequence of some knife-edge phenomenon. This is important for the submodularity assumption on effort costs: A common approach in the multitask literature is agents regarding efforts on various tasks as substitutes: a higher effort on one task crowds out that on other tasks. Our results are thus robust to small crowding-out effects of efforts, especially when the complementarities of the tasks are strong for the principal.
for the general case where the principal adopts a standard cost-and-benefit approach in determining the optimal effort choice to implement — though in this case, a new issue of why in our setting assigning these \( n \) tasks to the same agent in the first place would emerge.

As we have argued above, supermodularity of profits and submodularity of effort costs are not indispensable in driving our results. Nevertheless, we have chosen to work exclusively with these specifications in this paper, as in our opinion such a framework prescribes an intuitive economic environment, is mathematically elegant to work with, and is importantly consistent with the previous literature.

Appendix

Proof of Proposition 1. Consider first the following reduced program with \((IC)\) being replaced by the extremal incentive constraint where the agent prefers \( N \) to \( \emptyset \):

\[
\begin{align*}
\max_{w(y)} & - \sum_{y \in Y} P(y|N) w(y) \\
\text{s.t.} & \quad w(y) \geq 0, \quad \forall y \in Y \\
& \quad \sum_{y \in Y} P(y|N) w(y) - C(N) \geq \sum_{y \in Y} P(y|\emptyset) w(y) - C(\emptyset).
\end{align*}
\]

Construct the following Lagrangian, where \( \lambda \) (resp., \( \mu(y) \)) is the Lagrange multiplier of the reduced incentive (resp., limited liability) constraint in program \((P^f)\):

\[
L(w(y); \lambda, \mu(y)) = - \sum_{y \in Y} P(y|N) w(y) + \mu(y) w(y) + \lambda \left[ \sum_{y \in Y} (P(y|N) - P(y|\emptyset)) w(y) - C(N) + C(\emptyset) \right].
\]

The first-order necessary condition for \( w(y) \) can be derived as:

\[
-P(y|N) + \mu(y) + \lambda (P(y|N) - P(y|\emptyset)) = 0. \tag{A-1}
\]

First, it is easy to show that \( \lambda > 0 \): If not, then \((A-1)\) implies \( \mu(y) > 0 \forall y \). Complementary slackness \( \mu(y) w(y) = 0 \) and \( (C(N) - C(\emptyset)) > 0 \) further imply that the extremal incentive constraint is violated.

Next, suppose that \( \mu(y) = 0 \) (and hence \( w(y) > 0 \)) for some \( y \neq s \) and \( \mu(s) > 0 \). Applying \((A-1)\) for signal \( y \) and signal \( s \) respectively, we have:\(^{29}\)

\[
\frac{P(s|N)}{P(s|\emptyset)} = \frac{\lambda}{\lambda - 1} = \frac{P(y|N)}{P(y|\emptyset)},
\]

which is in contradiction to \((3)\). It follows that \( w(y) > 0 \) only if \( y = s \). Hence, the optimal wage scheme for program \((P^f)\) is:

\[
\bar{w}(y) = \begin{cases} 
\frac{C(N) - C(\emptyset)}{P(s|N) - P(s|\emptyset)}, & \text{if } y = s, \\
0, & \text{otherwise.}
\end{cases}
\]

\(^{29}\)Note that \( \lambda = 1 \) yields \( P(y|\emptyset) = 0 \), contradicting the full-support assumption on \( P \).
To show that the wage scheme $\bar{w}(y)$ also solves program $(P)$, it suffices to show that $\bar{w}(y)$ implies that

$$\sum_{y \in Y} P(y|N) \bar{w}(y) - C(N) \geq \sum_{y \in Y} P(y|E) \bar{w}(y) - C(E), \quad \forall \ E \subseteq N.$$ 

Or equivalently,

$$\frac{P(s|N)[C(N) - C(\emptyset)]}{P(s|N) - P(s|\emptyset)} - C(N) \geq \frac{P(s|E)[C(N) - C(\emptyset)]}{P(s|N) - P(s|\emptyset)} - C(E), \quad \forall \ E \subseteq N. \quad (A - 2)$$

Recursively applying the inequalities (1) and (2) (supermodularity of $P(s|E)$ and submodularity of $C(E)$), we have (denote $l$ as the cardinality of $E$, $|E| = l$):

$$\frac{P(s|N) - P(s|E)}{n - l} \geq \frac{P(s|E) - P(s|\emptyset)}{l}, \quad (A - 3)$$

$$\frac{C(N) - C(E)}{n - l} \leq \frac{C(E) - C(\emptyset)}{l}. \quad (A - 4)$$

Therefore,

$$\frac{[P(s|N) - P(s|E)]}{P(s|N) - P(s|\emptyset)} [C(N) - C(\emptyset)] \geq \frac{n - l}{n} [C(N) - C(\emptyset)] \geq C(N) - C(E),$$

which verifies $(A - 2)$ after rearranging the terms. ■

**Proof of Proposition 2.** We distinguish two cases:

**Case 1** $\frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}$.

Recall that $K$ is the effort choice where the agent *only* works in the $k$ observable tasks. Given $P(s|N) \leq \frac{C(N)}{C(K)} P(s|K)$, we now show that the optimal wage scheme is derived using *only* $(IC_1)$:

---

30For expositional convenience, let $E = \{1, 2, ..., l\}$ and $N\setminus E = \{l + 1, l + 2, ..., n\}$. Then

- $C(N) - C(E)$
- $= C(N) - C(N \setminus \{n\}) + C(N \setminus \{n\}) - C(N \setminus \{n, n - 1\}) + ... + C(N \setminus \{n, ..., l + 2\}) - C(E)$
- $\leq (n - l) [C(N \setminus \{n, n - 1, ..., l + 2\}) - C(E)]$
- $= (n - l) [C(E \cup \{l + 1\}) - C(E)]$,

while at the same time,

- $C(E) - C(\emptyset) = C(E) - C(E \setminus \{l\}) + C(E \setminus \{l\}) - C(E \setminus \{l, l - 1\}) + ... + C(\{1\}) - C(\emptyset)$
- $\geq l[C(E) - C(E \setminus \{l\})].$

Finally, $C(E \cup \{l + 1\}) - C(E) \geq C(E \setminus \{l\})$ (again, by submodularity) implies that $(A - 4)$ is true. Inequality $(A - 3)$ can be established in a similar fashion.
From \((IC_1)\), using a similar argument as in the proof of Proposition 1, the optimal wage scheme can be calculated as:

\[
\hat{w}'(y) = \begin{cases} 
\frac{C(N) - C(K)}{P(s|N) - P(s|K)}, & \text{if } y = s, \\
0, & \text{otherwise.}
\end{cases} \quad (A - 5)
\]

It is easy to show that \(\sum_{y \in Y} P(y|N)\hat{w}(y) - C(N) \geq 0\), or \((IC_2)\) is satisfied with such \(\hat{w}'(y)\) when \(\frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}\).

**Case 2** \(\frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}\).

Given \(P(s|N) > \frac{C(N)}{C(K)} P(s|K)\), the optimal wage scheme is determined only by \((IC_2)\), from which, the wage scheme can be calculated as:

\[
\hat{w}''(y) = \begin{cases} 
\frac{C(N) - C(\emptyset)}{P(y|N)}, & \text{if } y = s, \\
0, & \text{otherwise.}
\end{cases} \quad (A - 6)
\]

It is easily verified that \((IC_1)\) is satisfied under \(\hat{w}''(y)\).

Finally, the incentive constraints where the agent does not shirk in any of the \(k\) observable tasks can be dispensed with, by specifying a zero wage upon such an observation, as effort observation is verifiable. We conclude that the wage scheme \(\hat{w}(y)\), defined in (5) is the optimal wage scheme. ■

**Proof of Corollary 1.** The principal’s expected wage payments under \(\hat{w}(y)\) and \(\hat{w}(y)\) can be written as, respectively,

\[
E\hat{w} = \frac{P(s|N)[C(N) - C(\emptyset)]}{P(s|N) - P(s|\emptyset)},
\]

\[
E\hat{w} = \begin{cases} 
\frac{P(s|N)[C(N) - C(K)]}{P(s|N) - P(s|K)}, & \text{if } \frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}, \\
C(N), & \text{if } \frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}.
\end{cases}
\]

When \(\frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}\), then supermodularity of \(P(s|E)\) and submodularity of \(C(E)\) (or equivalently, \((A - 3)\) and \((A - 4)\)) directly imply that

\[
\frac{C(N) - C(\emptyset)}{C(N) - C(K)} \geq \frac{k}{n - k} \geq \frac{P(s|N) - P(s|\emptyset)}{P(s|N) - P(s|K)}, \text{ or } E\hat{w} \geq E\hat{w}.
\]

In addition, if the inequality (1) or (2) holds for at least one pair of tasks \(i, j\) in \(N\), then \(\frac{C(N) - C(\emptyset)}{C(N) - C(K)} > \frac{P(s|N) - P(s|\emptyset)}{P(s|N) - P(s|K)}\), or \(E\hat{w} > E\hat{w}\).

Next, when \(\frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}\), we have \(E\hat{w} > E\hat{w}\) as \(\frac{P(s|N)}{P(s|N) - P(s|\emptyset)} > 1\) by our full support assumption on \(P\). ■
Proof of Proposition 3. We first simplify the constraints \((IC')\) and \((IC'')\) in \((P^R)\), starting with the simpler \((IC')\), which is equivalent to

\[
(IC') : w \geq \frac{C(N)}{P(s|N)\Delta}, 
\]

where

\[
\Delta = \sum_{j=1}^{m-1} \frac{1}{j+1} \binom{m-1}{j} P(s|N)^j (1 - P(s|N))^{m-j-1}. 
\]

To interpret \(\Delta\), observe that \(\binom{m-1}{j} P(s|N)^j (1 - P(s|N))^{m-j-1}\) is the probability that there are \(j\) (among \((m-1)\)) agents having output signal \(s\) when all the other \((m-1)\) agents choose \(N\), while \(\frac{1}{j+1}\) indicates that agent \(i\) will share the reward \(w\) with such \(j\) agents when \(i\)'s output is also \(s\).

Constraint \((IC'')\), on the other hand, involves multiple incentive constraints:

First, given that all the other agents choose \(N\), if \(i\) shirks in some (or all) of the \(k\) observable tasks, \(i\)'s payoff is zero even with output \(s\): If \(i\) is the only agent with output \(s\), the principal pays 0 to the agents given our design; if another (or more) agent also obtains output \(s\), \(i\)'s payoff is again zero because of her incompetent effort on the \(k\) tasks. Hence, \((IC')\) is sufficient to encompass the cases where \(i\) (partially) shirks in the \(k\) tasks.

Consider now the set of constraints in \((IC'')\) where agent \(i\) at least works in all the \(k\) observable tasks: \(K \subseteq E \subseteq N\). In this case, agent \(i\)'s payoff is:

\[
U_i(E; N, \cdots, N; w) = \sum_{j=1}^{m-1} \frac{P(s|E)}{j+1} \binom{m-1}{j} P(s|N)^j (1 - P(s|N))^{m-j-1} w - C(E)
\]

\[
= \Delta P(s|E) w - C(E). 
\]

Since \(\Delta\) is a constant to \(i\), as in the proof of Proposition 1, it suffices to consider an extremal constraint \(U_i(N; N, \cdots, N; w) \geq U_i(K; N, \cdots, N; w)\)

which yields

\[
w \geq \frac{C(N) - C(K)}{[P(s|N) - P(s|K)]\Delta}. 
\]

Summarizing, the incentive constraints \((IC')\) and \((IC'')\) reduce to two simple inequalities \((A - 7)\) and \((A - 8)\).

As \(\Delta\) is a constant, we can then again discuss two familiar cases:

1. If \(\frac{P(s|N)}{P(s|K)} \leq \frac{C(N)}{C(K)}\), then only \((A - 8)\) is binding. Hence, the principal’s expected wage payment, denoted as \(E\tilde{w}_R\) is:

\[
E\tilde{w}_R = \frac{\tilde{P}[C(N) - C(K)]}{\Delta [P(s|N) - P(s|K)]}, 
\]

where
where $\tilde{P}$ is the probability that “two or more agents have realized output $s$”:

$$\tilde{P} = \sum_{j=2}^{m} \binom{m}{j} P(s|N)^j (1 - P(s|N))^{m-j}.$$  

With a simple change of the upper and lower bounds of summation, constant $\Delta$ can be equivalently written as

$$\Delta = \sum_{j=2}^{m} \frac{1}{j} \left( \binom{m-1}{j-1} P(s|N)^{j-1} (1 - P(s|N))^{m-j} \right)$$

$$= \frac{1}{m} \sum_{j=2}^{m} \binom{m}{j} P(s|N)^{j-1} (1 - P(s|N))^{m-j}. $$

Therefore,

$$E\tilde{w}_R = \frac{\tilde{P} [C(N) - C(K)]}{\Delta [P(s|N) - P(s|K)]} = mP(s|N) \frac{C(N) - C(K)}{P(s|N) - P(s|K)} = mE\tilde{w}. $$

2. If $\frac{P(s|N)}{P(s|K)} > \frac{C(N)}{C(K)}$, then (A-7) is the relevant constraint. The principal’s expected wage payment is:

$$E\tilde{w}_R = \frac{\tilde{P}C(N)}{\Delta P(s|N)} = mC(N) = mE\tilde{w}. $$

We conclude that the optimal relative performance contract is ex ante equivalent (in terms of expected wage payments) to the optimal individual contracts in Proposition 2 from the principal’s point of view. ■

References


